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Influence of Diffusion on Spin Waves in the Linear Model of Ferromagnetic Thin Films with Given ε, μ and σ

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At present a great attention is paid to investigations of thin ferromagnetic films. In this paper we also give a contribution to this problem. It summarises a part of the results achieved in the autor's thesis. This work starts from [1], where the influence of diffusion on spin waves in the linear model of a thin film consisting of two monoatomic ferromagnetic linear chains diffusing in each other has been examined.

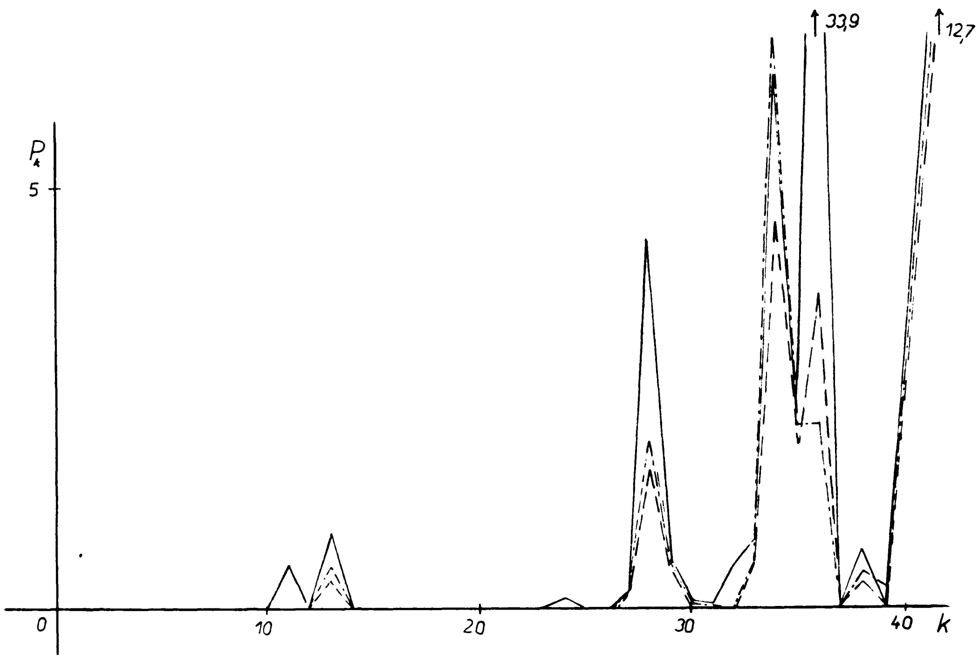

Fig. 1. The spin wave resonance peaks for the various conductivities (in arbitrary units). The solid curve corresponding to $\sigma = 0$ (after [1]), the dashed curve corresponding to $\sigma = 10^{16}\,s^{-1}$ and the dot-and-dashed curve corresponding to $\sigma = 3\,.\,10^{17}\,s^{-1}$ (after [4])

The system is no more periodic and, as a consequence, difficulties of mathematical character will appear. In determining the energy eigenvalues and eigenwave functions of spin waves the results of the work [1] based on the method of the second quantisation have been used here. The calculations in [1] and consequently in this paper are based on the Hamiltonian containing the Heisenberg exchange term, Zeeman term and the axial anisotropy term:

$$H = -\frac{1}{2}\sum_{j,j'=1}^{N} \mathcal{J}_{j,j'}(\vec{S}_j, \vec{S}_{j'}) - g\mu_0 H_e \sum_{j=1}^{N} S_j^z + \sum_{j=1}^{N} k_j (S_j^z)^2 ,$$

where $\mathcal{J}_{j,j'}$ is the exchange integral, $\vec{S}_j$ the spin operator, g the Lande factor, μ_0 the Bohr magneton, H_e z-component of the intensity of the magnetic field, k_j anisotropy constant. N is the number of the atoms in the linear chain.

We shall find the energy spectrum in the spin-wave approximation limiting ourselves to interaction between the nearest neighbours. The diagonalisation leads to the search of the eigenvalues of the matrix (1), corresponding to the energy eigenvalues of the spin waves, which have been computed numerically.

$$\begin{bmatrix} \mathcal{J}_{1,2}S_2 - 2k_1S_1, & -\mathcal{J}_{1,2}\sqrt{(S_1S_2)}, & 0, \ldots\ldots, 0 \\ -\mathcal{J}_{2,1}\sqrt{(S_1S_2)}, & \mathcal{J}_{2,1}S_1 + \mathcal{J}_{2,3}S_3 - 2k_2S_2, & -\mathcal{J}_{2,3}\sqrt{(S_2S_3)}, \; 0, \ldots, 0 \\ & \vdots & \\ 0, \ldots, 0, -\mathcal{J}_{i,i-1}\sqrt{(S_iS_{i-1})}, & \mathcal{J}_{i,i-1}S_{i-1} + \mathcal{J}_{i,i+1}S_{i+1} - 2k_iS_i, & - \mathcal{J}_{i,i+1}\sqrt{(S_iS_{i+1})}, \ldots, 0 \\ & \vdots & \\ 0, \ldots\ldots, 0, & -\mathcal{J}_{N,N-1}\sqrt{(S_NS_{N-1})}, & \mathcal{J}_{N,N-1}S_{N-1} - 2k_NS_N \end{bmatrix} \quad (1)$$

These results as well as the programs for the numerical calculation are taken from [1].

One of the problems of the work [1] was to find the resonance peaks of the spin resonance curves. For the calculation of the probability of creation of a spin waves per unit time interval the time dependent perturbation theory in the first approximation has been used following [1], where the perturbation has the form of the Zeeman term. The form of the resonance curves for our model is shown in Fig. 1.

These influence of the boundary conditions, of the conductivity σ, the dielectrical constant ε and of the magnetic permeability μ on the time dependent external magnetic field is not considered in this work. In order to state how much the spinwave resonance is sensitive to these parameters, we have calculated the magnetic field within the linear chain from the Maxwell's equations respecting the boundary conditions.

For the probability P_k of the creation of a spinwave corresponding to the wave vector k per unit of time we obtained relation

$$P_k = (B_1^k)^2 + (B_2^k)^2 , \tag{2}$$

where

$$B_1^k = \sum_{j=1}^{N} P_1(R_{jz}) \sqrt{S_j}\, u_j^k , \qquad B_2^k = \sum_{j=1}^{N} P_2(R_{jz}) \sqrt{S_j}\, u_j^k ,$$

$$P_1(r_z) = \frac{A_1(B_1(r_z) + C_1(r_z)) + A_2(B_2(r_z) + C_2(r_z))}{A_1^2 + A_2^2} ,$$

$$P_2(r_z) = \frac{A_2(B_1(r_z) + C_1(r_z)) - A_1(B_2(r_z) + C_2(r_z))}{A_1^2 + A_2^2} ,$$

$$A_1 = \cos(aR)\,[(4c\hbar/|E_k|) \,.\, a \cosh(\beta R) - 2(\varepsilon + \mu) \sinh(\beta R)] + \sin(aR) \times \times [(4c\hbar/|E_k|) \,.\, \beta \sinh(\beta R) + (8\pi\hbar/|E_k|) \,.\, \sigma \cosh(\beta R)] ,$$

$$A_2 = \cos(aR)\,[(8\pi\hbar/|E_k|)\, \sigma \sinh(\beta R) + (4c\hbar/|E_k|) \,.\, \beta \cosh(\beta R)] + \sin(aR) \times \times [2(\varepsilon + \mu) \cosh(\beta R) - (4c\hbar/|E_k|) \,.\, a \sinh(\beta R)] ,$$

$$B_1(r_z) + C_1(r_z) = \cos[a(r_z - R)] \,.\, [(2c\hbar/|E_k|) \,.\, a \cosh[\beta(r_z - R)] + + 2\varepsilon \sinh[\beta(r_z - R)]] + \sin[a(r_z - R)] \,.\, [(2c\hbar/|E_k|)\, \beta \sinh[\beta(r_z - R)] - - (8\pi\hbar/|E_k|)\, \sigma \cosh[\beta(r_z - R)]] ,$$

$$B_2(r_z) + C_2(r_z) = \cos[a(r_z - R)] \,.\, [(2c\hbar/|E_k|)\, \beta \cosh[\beta(r_z - R)] - - (8\pi\hbar/|E_k|)\, \sigma \sinh[\beta(r_z - R)]] - \sin[a(r_z - R)]\, [2\varepsilon \cosh[\beta(r_z - R)] + + (2c\hbar/|E_k|)\, a \sinh[\beta(r_z - R)]] ,$$

$$a = \frac{|E_k| \sqrt{(\varepsilon\mu)}}{c\hbar} \sqrt{\left[\frac{1}{2} + \sqrt{\left(\frac{1}{4} + \frac{4\pi^2\hbar^2\sigma^2}{E_k^2 \varepsilon^2}\right)}\right]} ,$$

and

$$\beta = - \frac{|E_k| \sqrt{(\varepsilon\mu)}}{c\hbar} \sqrt{\left[-\frac{1}{2} + \sqrt{\left(\frac{1}{4} + \frac{4\pi^2\hbar^2\sigma^2}{E_k^2 \varepsilon^2}\right)}\right]}$$

where R is the length of the linear chain, E_k and u_j^k are the eigenvalues and the eigenfunctions of the matrix (1). Assuming $\sigma = 0$, $\varepsilon = \mu = 1$ we obtain from (2)

$$P_k = \left| \sum_{j=1}^{N} \sqrt{S_j}\, u_j^k \right|^2$$

is accordance with [1].

In order to be able to perform the numerical calculations we need to know the permitivity ε, the magnetic permeability μ and the conductivity σ for the expected frequences of the magnetic field ($10^{12} - 10^{14}$ s^{-1}). Numerical data for the conductivity have been taken from [2], [3].

With regard to the lack of information on the permittivity we have limited ourselves to $\varepsilon = 1$ and investigated only the influence of the conductivity on the resonance curves. We also take $\mu = 1$.

It is evident, that the energy lost will take place in the thin ferromagnetic film due to the eddy currents. In the more detailed analysis we have stated that it is possible to neglect these losses for the conductivities (10^{12} — 10^{17} s^{-1}) and the thickness of the films as considered in our paper.

For the numerical calculations (and the comparison of results of both papers) following values have been chosen:
$\alpha = 3 \,.\, 10^{-8}$ cm for the lattice constant
and for the electric conductivity the values
1. $\sigma = 3 \,.\, 10^{17}$ s^{-1} (which is the value of the conductivity of Fe for the temperature —195 °C, approximately equal also to 70Ni30Fe, 85Ni15Fe a Ni),
2. $\sigma = 10^{16}$ s^{-1} (corresponding to the conductivity of the permalloy γ-phase 30Ni70Fe for the temperature 0 °C, by 100 °C of the γ-phase 30Ni70Fe and 35Ni65Fe, by 400 °C of the γ-phase 15Ni85Fe and 20Ni80Fe, by 850 °C of the γ-phase 5Ni95Fe). The theory has been applied to the linear chain (taken from [1]) in the form

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O – corresponding to the spin $S = 2$, X to the spin $S = 1$. The results are demonstrated in the Fig. 1.

Per calculations proved that the spinwave resonance depends on the conductivity of the films. Some peaks may be suppressed and some other ones may appear. However, in general as found in [1] character of the spinwave resonance curves has been preserved. Substantial changes may be expected in thicker films. The detailed calculations, tables and graphs may be found in the thesis [4].

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