P. Žáček; Vladimír Vanýsek

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Motion of Cometary Particle With Variable Mass Under Central Fields of Forces

P. ŽÁČEK and V. VANÝSEK
Department of Astronomy and Astrophysics, Charles University, Prague*)

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The equations for trajectory, energy, radius and gas flow rate of a small cometary particle with variable mass are derived in this paper. These equations are valid for the central field of forces caused by the Sun and for zero initial particle velocity relative to the nucleus of comet. The influence of the nucleus on a particle is neglected.

Движение кометарной пилинки с переменной массой в центральном поле сил. В этой работе выведены уравнения для траектории, энергии и радиуса малой кометарной пилинки с переменной массой и для числа из ней испаряющихся молекул. Эти уравнения справедливы для центрального поля сил, вызываемого Солнцем и для нулевой первоначальной скорости пилинки относительно ядра кометы. Влиянием ядра на полинку пренебрегается.

Pohyb kometární částice s proměnnou hmotností v centrálním poli sil. V práci jsou odvozeny rovnice pro dráhu, energii, poloměr a výparnost malé kometární částice s proměnnou. hmotností. Tyto rovnice platí pro centrální pole sil, vyvolaných Sluncem a pro nulovou počáteční rychlost částice vzhledem k jádru komety. Vliv jádra na částici je zanedbán.

Introduction

The trajectories of cometary particles have been calculated by many authors under various assumptions. For example Bessel (1836), Finson and Probstein (1968) and Omarov (1973) have considered the motion of the particles with various sizes under the central field of forces. Omarov has studied the motion of a particle with variable mass. Delsemme and Miller (1971) have introduced the model of comet with the ice grains, which, consequently change its size and mass.

The equation of the trajectory of a particle with variable mass discussed in this paper is derived under these assumptions:

– the influence of cometary nucleus on a particle is neglected;
– the total force, which affects a particle is central relative to the Sun;

*) 150 00 Praha 5, Švédská 8, Czechoslovakia.

– the initial velocity of a particle relative to the nucleus is very low and can be supposed to be zero.

Those particles, which are emitted by nucleus under these conditions in different times, create in space a curve, which can be called "axial" curve of the cometary tail in respect to the other particles, which have non-zero initial velocities relative to the nucleus. For constant masses of particles the "axial" curve coincides with some "zero" syndyne.

1. Trajectory and energy of a particle with variable mass

Assuming central field of forces the equations of motion of a particle, which initial velocity relative to the nucleus is zero, is defined as

$$\ddot{X} = -\mu GM \cos \varphi / R^2 , \quad \ddot{Y} = -\mu GM \sin \varphi / R^2 , \quad \ddot{Z} = 0 \tag{1}$$

$$\mu = (F_g - F_r)/F_g , \quad F_g = mGM/R^2 . \tag{2}$$

The quantity F_g is the force of gravity, F_r is the sum of other central forces, which affect a particle, M is the mass of the Sun, G is the gravitational constant, m is the mass of a particle. The quantities X, Y, Z and R, φ, Z are Cartesian or cylindrical heliocentric coordinates of a particle, respectively. For above mentioned coordinates the equations (3) are valid:

$$X = R \cos \varphi , \quad Y = R \sin \varphi , \quad Z = 0 . \tag{3}$$

Since both the gravitational effect of the nucleus and the pressure of gas (which is evaporated by nucleus) on a particle is neglected, the coordinate Z is identically equal to zero. From the equations (1) and (3) it follows:

$$\ddot{R} - R\dot{\varphi}^2 = -\mu GM/R^2 \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} [(\dot{R}^2 + R^2\dot{\varphi}^2)/2 - h_1] = \mu GM\dot{R}/R^2 \tag{5}$$

$$R^2\dot{\varphi} = C \quad (\dot{\varphi} \neq 0) , \tag{6}$$

where C, h_1 are constants of integration (h_1 is the particle energy per unit mass in the moment of the separation of a particle from the nucleus).

If we denote

$$U = R^{-1} , \quad U' = \mathrm{d}U/\mathrm{d}\varphi , \quad U'' = \mathrm{d}^2U/\mathrm{d}\varphi^2 , \tag{7}$$

then we can write the Binet's formula in the form:

$$C^2U^2(U'' + U) = \mu GMU^2 \tag{8}$$

and for particle's energy per unit mass $h(\varphi)$ it holds:

$$h(\varphi) = [(CU')^2 + (CU)^2]/2 - \mu GMU = h_1 - GM \int_{\varphi_1}^{\varphi} \mu' U \, d\varphi \tag{9}$$

The latter term on the right side of the equation (9) is the change of energy due to evaporation of molecules from a particle; φ_1 is the true anomaly of the separation point of a particle from the nucleus.

The right side of the equation (5) has been integrated by parts:

$$\int \mu U' \, d\varphi = \mu U - \int \mu' U \, d\varphi \, .$$

The equations (8) and (9) are valid both for $\mu = \mu(\varphi)$ and for $\mu = \text{const} \leqq 1$. It is evident from the derivational scheme:

(1a) $\cos\varphi$ + (1b) $\sin\varphi \Rightarrow$ (4); (1a) $\dot{X}$ + (1b) $\dot{Y} \Rightarrow$ (5)
(4) $\dot{R}$ = (5) $\Rightarrow$ (6); $\dot{R} = -CU'$, $\ddot{R} = -C^2U^2U''$; $R\dot{\varphi}^2 = C^2U^3$.

The solution $U = U(\varphi)$ of the equation of motion (10) was looked for in the form (11) by variation of constants under the conditions (12) and (13):

$$U'' + U = \mu GM/C^2 \tag{10}$$

$$U = K_1 \cos\varphi + K_2 \sin\varphi \geqq 0 \tag{11}$$

$$K_1' \cos\varphi + K_2' \sin\varphi = 0 \tag{12}$$

$$U'' + U = -K_1' \sin\varphi + K_2' \cos\varphi = \mu GM/C^2 \tag{13}$$

From (12) and (13) it follows:

$$K_1' = -\mu GM \sin\varphi/C^2 \, , \quad K_2' = \mu GM \cos\varphi/C^2 \tag{14}$$

$$K_1 = -GM \int_{\varphi_1}^{\varphi} \mu \sin\varphi \, d\varphi/C^2 + K_4 \tag{15}$$

$$K_2 = \quad GM \int_{\varphi_1}^{\varphi} \mu \cos\varphi \, d\varphi/C^2 + K_3 \, . \tag{16}$$

The constants K_3, K_4 are determined by the initial conditions (17), (18) (for the moment t_1 of the particle's separation from the nucleus):

$$R_c(t_1) = R_p(t_1) = R_1 \, , \quad \varphi_c(t_1) = \varphi_p(t_1) = \varphi_1 \tag{17}$$

$$\dot{R}_c(t_1) = \dot{R}_p(t_1) = \dot{R}_1 \, , \quad \dot{\varphi}_c(t_1) = \dot{\varphi}_p(t_1) = \dot{\varphi}_1 \tag{18}$$

The $R_c(t)$, $\varphi_c(t)$, and $R_p(t)$, $\varphi_p(t)$ are the coordinates of the nucleus or the particle (in the time t), respectively. The constant C must be the same one both for the nucleus and for the particle; see (6), (17), (18) and the conditions in the introduction.

If we suppose that

$$U_c = GM/C^2 + K\cos\varphi_c = P^{-1}(1 + e\cos\varphi_c) = R_c^{-1}\,, \tag{19}$$

where $P = C^2/GM$ (polar radius for $\varphi_c = \pi/2$) and

$e = KP$ is the numerical eccentricity of the cometary trajectory, then we can write (see (6), (12), (17) to (19)):

$$PU_p(t_1) = PR_1^{-1} = K_4P\cos\varphi_1 + K_3P\sin\varphi_1 = 1 + e\cos\varphi_1 \tag{20}$$

$$C^{-1}P\dot{R}_1 = K_4P\sin\varphi_1 - K_3P\cos\varphi_1 = e\sin\varphi_1\,, \tag{21}$$

from (20) and (21) $K_3P = \sin\varphi_1$, $K_4P = e + \cos\varphi_1$, because $\dot{R}_c = CP^{-1}e\sin\varphi_c$, $\dot{R}_p = -CU_p' = C(K_1\sin\varphi_p - K_2\cos\varphi_p)$.

The equation of particle's trajectory has the forms (22) and (23):

$$U_p = R_p^{-1} = P^{-1}\left[1 + ec_\varphi + c_\varphi\int_{\varphi_1}^{\varphi}(1-\mu)\,s_\varphi\,\mathrm{d}\varphi - s_\varphi\int_{\varphi_1}^{\varphi}(1-\mu)\,c_\varphi\,\mathrm{d}\varphi\right] \geqq 0 \tag{22}$$

$$U_p = R_p^{-1} = P^{-1}\left[ec_\varphi + c_{\varphi-\varphi_1} - c_\varphi\int_{\varphi_1}^{\varphi}\mu s_\varphi\,\mathrm{d}\varphi + s_\varphi\int_{\varphi_1}^{\varphi}\mu c_\varphi\,\mathrm{d}\varphi\right] \geqq 0\,, \tag{23}$$

where the following symbols were used: $c_\varphi = \cos\varphi$, $s_\varphi = \sin\varphi$, $c_{\varphi-\varphi_1} = \cos(\varphi - \varphi_1)$, $\varphi = \varphi_p$; P, e have the same significance as in the equation (19). For $\mu = \mu_1 = \text{const} \leqq 1$ it follows from (22) or (23):

$$R_p = P[\mu_1 + (1-\mu_1)\cos(\varphi - \varphi_1) + e\cos\varphi]^{-1}\,,\quad \varphi = \varphi_p\,. \tag{24}$$

If we assume the equation (24) in the form (25)

$$R_p(t) = P_p[1 + e_p\cos(\varphi_p(t) - \varphi_0)]^{-1}\,, \tag{25}$$

then from the conditions (17), (18) we have:

$$P_p\mu_1 = P \tag{26}$$

$$\mu_1 e_p = [\mu_1^2 + e^2 - 1 + 2(1-\mu_1)(1 + e\cos\varphi_1)]^{1/2} \tag{27}$$

$$\varphi_0 = \varphi_1 - \arcsin(e\sin\varphi_1/\mu_1 e_p) \tag{28}$$

The right side of equations (26), (27) have the non-zero values also for $\mu_1 = 0$.

For $\mu = \mu(\varphi)$ the P_p, e_p are variables. We can evaluate the energy constant h_1 from the equation (9):

$$h_1 = [(1 + e^2)/2 - \mu_1 + (1-\mu_1)e\cos\varphi_1]\,G^2M^2/C^2\,,\quad \mu_1 = \mu(\varphi_1)\,. \tag{29}$$

The equations of the "axial" curve have the form (Finson and Probstein, 1968):

$$\xi = R_{p2}\cos(\varphi_{c2} - \varphi_{pc}) - R_{c2}\,,\quad \eta = R_{p2}\sin(\varphi_{c2} - \varphi_{p2})\,, \tag{30}$$

where ξ, η are Cartesian cometocentric coordinates of the points on this curve at the moment of observation t_2, R_{c2}, φ_{c2} are the known heliocentric coordinates of the nucleus at t_2, R_{p2}, φ_{p2} are the unknown helicoentric coordinates of points on the "axial" curve at t_2. The R_{p2}, φ_{p2} can be calculated from conditions (31), (32) and (33) for various values φ_1, t_1; see (6), (17), (18).

The quantity τ is the time of particle's flight from nucleus.

$$t_2 = t_1 + \tau \tag{31}$$

$$R_p^{-1}(\varphi) > 0 \quad \text{for} \quad \pi > \varphi_{c2} \geqq \varphi_{p2} \geqq \varphi \geqq \varphi_1 > -\pi$$

$$R_c^{-1}(\varphi) > 0 \quad \text{for} \quad \pi > \varphi_{c2} \geqq \varphi \geqq \varphi_1 > -\pi \tag{32}$$

$$\int_{\varphi_1}^{\varphi_{c2}} R_c^2(\varphi)\, \mathrm{d}\varphi = \int_{\varphi_1}^{\varphi_{p2}} R_p^2(\varphi)\, \mathrm{d}\varphi = \tau(GMP)^{1/2} = \tau C \tag{33}$$

2. Repulsive forces and size changes of particles

We take into consideration the effect of two repulsive forces on the particle with variable mass, moving in the tail of comet: the radiation pressure and reactive force, induced by non-uniform heating of the particle surface by the Sun and because $\dot{v} = F/m - v_0\dot{m}/m$, then

$$1 - \mu = \beta_1 a_1/a - R_p^2 b v_0 \dot{m}/mGM\,, \tag{34}$$

$$\beta_1 = 3Q_{pr}E_s/16\pi cGM\varrho a_1\,; \tag{35}$$

see (2), Finson and Probstein (1968), Dohanyi (1978). $\beta_1 a_1/a$ is the effect of radiation pressure, $a = a(t)$ is the radius of a particle, $a_1 = a(t_1)$, ϱ is the density of a particle, Q_{pr} is the efficiency for radiation pressure, $E_s = 3.82 \,.\, 10^{26}$ watts is the power radiated by the Sun, c is the speed of light. The second term on the right side of equation (34) is the effect of reactive force, induced by the more intensive evaporation of molecules on the sun-ward side of a particle. R_p is the distance of a particle from the Sun, v_0 is the mean thermal velocity of molecules, escaping from the particle, b is the factor of asymetry; we can say, it holds approximately that $b = (N_+ - N_-)/(N_+ + N_-)$, where N_+ is the number of molecules, which are escaping towards the Sun. N_- is the number of molecules, which are escaping in the opposite direction. m, $\dot{m}$ are the mass of the particle and its time derivative, G, M see equations (1), (2).

For $b = 0$ (symmetrical case of evaporation of molecules from a particle or for constant particle radius) the equation (34) has the form (36):

$$1 - \mu = \beta_1 a_1/a = 3Q_{pr}E_s/16\pi cGM\varrho a\,, \quad b = 0\,. \tag{36}$$

For $b \neq 0$ we assume, that $\varrho = \text{const}$, from which it follows $\dot{m}/m = 3\dot{a}/a$,

$$\dot{a} + a(1 - \mu)\, GM/3R_p^2 bv_0 = \beta_1 a_1 GM/3R_p^2 bv_0 \,. \tag{37}$$

If $Q_{pr} \doteq \text{const}$ ($a > \lambda$; λ is the effective wavelength of solar radiation), then

$$a = a_1 \exp\left(-\int_{t_1}^{t} \beta_2 \,\mathrm{d}t\right)\left[\int_{t_1}^{t} \beta_3 \exp\left(\int_{t_1}^{t} \beta_2 \,\mathrm{d}t\right)\mathrm{d}t + 1\right], \tag{38}$$

where

$$\beta_2 = (1 - \mu)\, GM/3R_p^2 bv_0 \,, \quad \beta_3 = \beta_1 GM/3R_p^2 bv_0 \,. \tag{39}$$

If $Q_{pr} = Q_{pr}(a)$ is the known function, then we can write:

$$\dot{a} = [\beta_1 a_1 - a(1 - \mu)]\, GM/3R_p^2 bv_0 \,, \quad \dot{a} = a' C U_p^2 \tag{40}$$

$$\int_{a_1}^{a} [\beta_1 a_1 - a(1 - \mu)]^{-1} \,\mathrm{d}a = (GM/3C) \int_{\varphi_1}^{\varphi} (bv_0)^{-1} \,\mathrm{d}\varphi \,. \tag{41}$$

The ratio of the force of solar radiation pressure to the force of gravity on the particles is given approximately $\beta_1 = 5.73 \,.\, 10^{-4}\, Q_{pr}(\varrho a_1)^{-1}$ (if ϱ and a_1 are in SI units). For the perfectly absorbing particles the β_1 is increasing function with decreasing radius and if the diffractions effects are neglected then $\beta \gg 1$ for all absorbing spheres with radii $a < 10^{-7}$ m. However for natural dielectrics (ice, silicates), the β_1 is somewhat more complicated function of the particle radius (Schwehm 1976). β_1 reaches a maximum of about 0.5 for silicates as andesite or obsidian around $a \sim 5 \,.\, 10^{-7}$ m and 0.8 for ice. For small particles the interaction with the radiation is dominated by diffraction effects and complicated internal refraction and interference effects must be taken in consideration and only Mie theory must be applied.

Since Q_{pr} is not trivial function of a in the following discussion is used only illustrative approximation. It is assumed that only small fraction of the solar radiation extincted on the particles is consumed for the evaporation and excess of the kinetic energy of the molecules. It implies, that substance of the particle must have the small one, but non-zero imaginary part of refractive index, then the material is some kind of natural dielectrics. If we know the particle radius as a function of time, $a = a(t)$, then we can calculate the gas flow rate of particle Z_p from the equation

$$S Z_p m_M = -\dot{m} \,, \tag{42}$$

where S is the superficial area of particle, m_M is the mass of molecule, $\dot{m}$ is the time derivative of particle mass. Esspecially we have

$$Z_p = -\dot{a}\varrho / m_M \,. \tag{43}$$

Energy for evaporation of molecules is determined by the absorption of solar radiation.

Efficiency factor for the absorption Q_{abs} of a larger spherical particle is given by the relation

$$Q_{abs} = 1 + 2[(\varkappa + 1)\exp(-\varkappa) - 1]\,\varkappa^{-2} \doteq 1 - \exp(-\varkappa_0 a)\,, \tag{44}$$

where $\varkappa/2a$ is the linear absorption coefficient which is, of course, function of the refractive index $m_c = n_{re} - in_i$, where n_i is in this case very small ($\leqslant 0.1$). The factor of the absorption coefficient $\varkappa$ is given by relation

$$\varkappa = 8\pi n_i a/\bar{\lambda}\,, \tag{45}$$

where $\bar{\lambda}$ is the mean wavelength of the maximum of the solar spectrum. The absorption coefficient $\varkappa_0$ is defined with a sufficient approximation

$$\varkappa_0 = 0.55\varkappa/a\,. \tag{46}$$

The numerical factor in equation (46) varies in limits 2/3 for $a \to \infty$ and 1/2 for $a \to 0$.

We can estimate the gas flow rate and radius of a particle as a functions of time, (if we substitute (47) into (43)):

$$Z_p \doteq Z_N[1 - \exp(-\varkappa_0 a)]\,, \tag{47}$$

$$a \doteq \varkappa_0^{-1} \ln\left[(\exp(\varkappa_0 a_1) - 1)\exp\left(-\varkappa_0 m_M \varrho^{-1} \int_{t_1}^{t} Z_N\,\mathrm{d}t + 1\right)\right]. \tag{48}$$

where a is the radius of a particle, Z_N is the gas flow rate of nucleus (with the same chemical composition and structure of material as well as particle and in the same distance from the Sun as a particle).

The equation (47) was chosen from this reason: for very large particles the Z_p is practically independent on a radius of a particle, which for very small particles is proportional to particle radius (Kaplan and Pikelner, 1963). On the other hand, for the function $Z_p(a)$ in equation (47) the radius can be explicitly calculated (see equation (48)).

Law of conservation of thermal energy can be written in the form (Delsemme and Miller, 1971):

$$Z_N = [E_s(1 - A_0)\,U_p^2/16\pi - \sigma(1 - A_1)\,T^4]/L_M \tag{49}$$

where E_s is the total luminosity of the Sun, A_0, A_1 are the values of albedo in the visible or infrared spectral regions, respectively, σ is the Stefan-Boltzmann constant, L_M is the latent heat (of evaporation or sublimation) per one molecule, T is the mean temperature of particle, $U_p = R_p^{-1}$. For Z_N also it holds (Kaplan and Pikelner, 1963):

$$Z_N = C_Z \exp(-A_Z/T)\,T^{B_Z}\,, \tag{50}$$

where constants A_Z, B_Z, C_Z can be calculated or measured for given physical properties of the particle.

If we compare both terms for Z_N from equations (49) and (50), we shall obtain the function $U = U(T)$; see (7). Under assumption that $\mu = \mu(\varphi)$ i.e. also $U = U(\varphi)$, it is possible numerically evaluate $Z_N(\varphi)$; then

$$\int_{t_1}^{t} Z_N \, \mathrm{d}t = \int_{\varphi_1}^{\varphi} Z_N(\varphi) \, C^{-1} U_p^{-2} \, \mathrm{d}\varphi$$

(see (6)) and from equation (48) it follows $a \doteq a(\varphi)$.

Equations (37) and (6) with a given function of $\mu = \mu(\varphi)$, may lead to the estimation of b as function of $(\varphi - \varphi_1)$ or $(t - t_1)$.

Relative error of Z_p is less than 10 percent in equation (47); (see (45)). More exact results may be obtained by the equation (47) $Z_p = Z_N Q_a$, however the evaluation of particle radius will be more complicated.

By means of the equations (38) or (48) we can estimate the lifetime of particle, which is approximately equal to the difference of integrational limits $\tau_p = t_3 - t_1$ (see (33)), where the final radius $a_f(t_3)$ is equal to a molecular radius a_M. This must be regarded as formal results for the upper limit of life-times only, because for very small radii the equations (38), (45) and (48) are invalid. On the other hand, if Z_p decreases very fast, the radius of a particle will be always larger than a_M. The estimation of particle lifetime $\tau_p = a_1 \varrho / Z_p m_M$ (Delsemme and Miller, 1971) is exact only, if Z_p is entirely independent on radius of a particle. It is not true for small particles (see (45), (47)) and it holds $\tau_p = (a_1 \varrho / Z_1 m_M) \ln (a_1 / a_M)$, when $Z_p / a = Z_1 / a_1 = \text{const.}$

Special assumptions:

If we use as approximate model with following assumptions:

$$1 - \mu = 1 - \mu_1 = \text{const}\,, \quad Q_{pr} = \text{const}\,, \quad v_0 b R_p^2 = v_1 b_1 R_1^2 = \text{const}\,, \tag{51}$$

where $R_1 = P(1 + e \cos \varphi_1)^{-1}$, $v_1 = v_0(R_1)$, $b_1 = b(R_1)$, then the equation (38) has the form

$$a = a_1 \left\{ \beta_1/(1 - \mu_1) + [1 - \beta_1/(1 - \mu_1)] \exp\left[- \frac{(1 - \beta_1)\, GM}{3R_1^2 b_1 v_1} (t - t_1) \right] \right\}, \tag{52}$$

and for Z_p it holds (after (43)):

$$Z_p = - \frac{\dot{a}\varrho}{m_M} = \frac{\varrho G M a_1 [1 - \mu_1 - \beta_1]}{3R_1^2 b_1 v_1 m_M} \exp\left[- \frac{(1 - \mu_1)\, GM}{3R_1^2 b_1 v_1} (t - t_1) \right] \tag{53}$$

If we substitute the following numerical values (for $t = t_1$) into (53), $b_1 = 1/3 = (N_+ - N_-)/(N_+ + N_-)$ i.e. $N_-/N_+ = 1/2$. ($\varrho = 10^3\ \text{kg m}^{-3}$; $G =$

$= 6 \,.\, 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$; $M = 2 \,.\, 10^{30}\,\mathrm{kg}$; $R_1 = 10^{11}\,\mathrm{m}$; $v_1 = 10^3\,\mathrm{m\,s}^{-1}$; $m_M =$ $= 3 \,.\, 10^{-26}\,\mathrm{kg}$; $a_1 = 10^{-2}\,\mathrm{m}$); we obtain $Z_p(t_1) = 10^{21}$ molecules $\mathrm{m}^{-2}\,\mathrm{s}^{-1}$; if $1 - \mu_1 = 2{,}5 \,.\, 10^{-1}$).

Discussion of the equation (52):

Since $a(t)$ and $Z_p(t)$ are exponential function the equation (52) is valid for those small particles, which are larger than the effective wavelength of solar radiation.

1) $0 \neq \beta_1 < 1 - \mu_1$: the radiation pressure at t_1 is not zero, but it is less than the total repulsive force. The particle will only diminish its radius (by evaporation) on a final value a_f:

$$a_f = \beta_1 a_1/(1 - \mu_1) = 3Q_{pr}E_s/16\pi c G M \varrho(1 - \mu_1)\,. \tag{54}$$

2) $\beta_1 = 1 - \mu_1 \neq 0$, the particle radius will be not changed. Particle is accelerated from the nucleus by radiation pressure only.

3) The case $\beta_1 > 1 - \mu_1$ is not real (evaporated molecules should accelerated the particle towards the Sun).

4) $0 \doteq \beta_1 < 1 - \mu_1$: Radiation pressure is at t_1 negligible. The particle will be fully evaporated.

5) $\beta_1 = 0 = 1 - \mu_1$: Very large particles. Due to the indeterminable form of $\beta_1/(1 - \mu_1)$ the equation (52) cannot be used.

3. Behaviour of $1 - \mu = (1 - \mu_1)\,[1 + H(\varphi - \varphi_1)]$

The equation of trajectory a particle (22) or (23) will be expressed by elementary functions of φ, if $1 - \mu$ is polynomial in $(\varphi - \varphi_1)$. For illustration we introduce only the linear function, i.e. the case, when

$$1 - \mu = (1 - \mu_1)\,[1 + H(\varphi - \varphi_1)]\,, \tag{55}$$

where the change

$$\frac{\mathrm{d}}{\mathrm{d}\varphi}(1 - \mu) = (1 - \mu_1)\,H = C U_p^2 \frac{\mathrm{d}}{\mathrm{d}t}(1 - \mu) \gtrless 0\,, \tag{56}$$

is constant one. Then the equation for trajectory is written in the form:

$$U_p = R_p^{-1} = (1 - \mu_1)\,(c_{\varphi-\varphi_1} - H(\varphi - \varphi_1) + H s_{\varphi-\varphi_1}) + \mu_1 + e c_\varphi]\,P^{-1}\,, \tag{57}$$

as we can find out by substitution of the expression (55) for $1 - \mu$ into (22) and by integration. In the equation (57) the following symbols were used:

$$c_{\varphi-\varphi_1} = \cos(\varphi - \varphi_1)\,, \quad s_{\varphi-\varphi_1} = \sin(\varphi - \varphi_1)\,, \quad c_\varphi = \cos\varphi\,;$$

P, e are the parameter and eccentricity of the trajectory of cometary nucleus, $\mu_1 = \mu(\varphi_1)$, φ_1 is the true anomaly of the particle's separation point from the nucleus. The equation (57) is in consistence with the equation of energy (9), in which the value of h_1 from the formula (29) was used.

By this manner we can calculate the trajectory for the repulsive forces with higher powers of dependence on $\varphi - \varphi_1$, or for some expressions from Taylor's expansion of another function.

If $H < 0$ in the equation (55) the particle trajectory has an asymptote (58):

$$R = P_A/\cos(\varphi - \varphi_A), \quad P_A = R(\varphi = \varphi_A), \quad \varphi_3 = \varphi_A + \pi/2, \tag{58}$$

where φ_3 corresponds its direction. The constant φ_3 is determined by the condition

$$1 - \mu(\varphi_3) = 0, \quad \text{i.e.} \quad H = -1/(\varphi_3 - \varphi_1); \tag{59}$$

From the equation (58) it follows $U(\varphi_3) = 0$ i.e. (see (57)):

$$\begin{aligned} \mu_1 + (1 - \mu_1)\cos(\varphi_3 - \varphi_1) + e\cos\varphi_3 &= \\ = H(1 - \mu_1)[\varphi_3 - \varphi_1 - \sin(\varphi_3 - \varphi_1)]. \end{aligned} \tag{60}$$

From the conditions (59) and (60) it follows the equation (61):

$$1 - \mu_1 = -(1 + e\cos\varphi_3)/[\cos(\varphi_3 - \varphi_1) - \sin(\varphi_3 - \varphi_1)/(\varphi_3 - \varphi_1)], \tag{61}$$

where it must be $0° < \varphi_3 - \varphi_1 < 257°$, if π corresponds to 180°. If the values of φ_3, φ_1 are know then the evaluation of initial value of $1 - \mu_1$ is almost trivial. For the unknown value of φ_3 the equation (61) is transcendental. $1 - \mu$, for the case $H < 0$, decreases more quickly with the distance from the Sun, than the force of gravity. It implies from the condition (59). The reactive force, defined by the second term on the right side of the equation (34) decreases more quickly with the distance from the Sun, than the force of gravity, too. It is evident from the comparison of the equations (34), (49), (50), (42) and (47).

4. Conclusion

The equations for trajectory and energy of particle with variable mass, derived in this paper, hold, under introduced conditions almost generally and they have in some extend the advantage, that they are expressed by means of elements of cometary nucleus trajectory. For the equation of trajectory mentioned above it is not necessary calculate the osculation elements, which are variables.

The equations for radius of particle and its gas flow rate give a real possibility to estimate the behaviour of these quantities with respect to the course of the whole central repulsive force and on the trajectory of particle and how many molecules have been evaporated by the particle during its flight.

For number of molecules, evaporated by particle during the time $t - t_1$, it holds in general (see (42)):

$$\int_{t_1}^{t} Z_p S \, \mathrm{d}t = [m(t_1) - m(t)]/m_M \,, \tag{62}$$

where $m(t_1)$, $m(t)$ are values of mass of a particle – at times t_1 and t. The relation (62) can be used for determination of the density of molecules in the tail of a comet.

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