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Description of Stress-Strain Curves on High Temperature Deformed Steels

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Introduction

In response to the growing interest on the work hardening and softening processes of the metallic material under the heat loading in the zone of plastic deformation, a theoretical and laboratory study has been conducted to examine and to define in mathematical form the shape of stress-strain curve.

The stress-strain curve of a polycrystalline metal [1, 2] under any simple loading has the shape shown diagrammatically in current pictures. The three most usual types of test are the tension of a rod, the compression of a short cylindrical block, and the twisting of a thin-walled tube or a prismatic bar. The result of such a test is represented by plotting the mean stress σ (tensile, compressive or shear) acting over the current (initial) cross-sectional area, against some measure of the total strain. The amount of deformation is usually measured as equivalent (logarithmic = true) strain.

Over the years, a number of investigations defined the overall response of a material in terms of some generalized equation of state

$$\sigma = f(T, \varepsilon, \dot{\varepsilon}) \quad (1)$$

e.g. on the basis of relaxation's study proposed [3]

$$\sigma = \sigma_0 \cdot B_1 \exp(n_1 T) \cdot B_2 \varepsilon^{n_2} \cdot \exp(n_3 \varepsilon) \cdot B_3 \dot{\varepsilon}^{n_4} \cdot B_4 \tau^{n_5} \cdot \exp(n_6 \tau) \quad (2)$$

where B , to B_4 and n_1 to n_6 are constants, T , ε , $\dot{\varepsilon}$, τ are temperature ($^{\circ}\text{C}$ or K), strain, strain rate (s^{-1}) and dwell period between deformations in (s), respectively. Other possibility is [4]

$$\sigma = \sigma_0 \cdot \varepsilon^n \cdot \dot{\varepsilon}^m \cdot \exp(C + A/T) \quad (3)$$

At present the description resulted in [5]

$$\sigma = A \cdot \varepsilon^b \cdot \exp(-c\varepsilon^d) \cdot \dot{\varepsilon}^{fT} \cdot \exp(g/T) \quad (4)$$

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Stress-strain curves

PEAK STRESS

Characteristic stresses (original σ_{ss} – stress at steady-state condition) may be correlated with temperature and strain rate. At low stresses ($\alpha\sigma_{ss} < 0.8$) and high stresses ($\alpha\sigma_{ss} > 1.2$) lead these equations in common form [6]

$$\dot{\varepsilon}_{ss} = A \exp(-Q/(RT)) [\sinh(\alpha\sigma_{ss})]^n \quad (5)$$

where A , α , n are constants and Q is activation energy in (J/mol), and all data may be correlated using the Zener-Hollomon parameter $Z = \dot{\varepsilon} \exp(Q/(RT))$. Relationship (5) has been applied successfully with σ_p – stress at peak (STRESSP) and transformed into [7, 8]

$$\sinh(\alpha\sigma_p) = (Z/A)^{1/n} \quad (6)$$

$$\sigma_p = \frac{1}{\alpha} \arg \sinh \left(\frac{\dot{\varepsilon} \cdot \exp(Q/(RT))}{A} \right)^{1/n} \quad (7)$$

PEAK STRAIN

Temperature compensated strain rate Z is related to time compensated strain rate W as [9]

$$W = A_p \cdot Z^{-a_p} \quad (8)$$

Hence, using $\varepsilon_p = W_p \cdot Z$ [10] and eq. (8)

$$\ln \varepsilon_p = \ln A_3 + (1 - a_3) \ln \dot{\varepsilon} + (1 - a_3) \frac{Q}{RT} \quad (9)$$

is obtained where A_p , A_3 , a_3 are constants, $W_p = t_p \exp(-Q/(RT))$ and t_p is time to peak in (s). By gradual linear regression analysis we become the slope $(1 - a_3)$. For expressing the thermally and strain rate activated process on peak strain ε_p a newly suggested and verified form have been obtained as follows [10, 11]

$$\varepsilon_p = \dot{\varepsilon}^a (P + \sqrt{P^2 + 1}) \quad (10)$$

where P is temperature dependent, $P = Y + X/T$, Y , X are constants, a has the same meaning as $(1 - a_3)$ eq. (9) and with rearrangement is obtained

$$\varepsilon_p = \dot{\varepsilon}^a \cdot \exp \arg \sinh(Y + X/T) \quad (11)$$

STRESS

For a fixed T , the stress-strain curves are progressively higher at medium $\varepsilon/\varepsilon_p$ for a higher strain rate. Although these curves are lower for decreasing Z value. The function which describes best the reduced flow stress is [12]

$$\ln \left(\frac{\sigma}{\sigma_p} \right) = c \left[\ln \left(\frac{\varepsilon}{\varepsilon_p} \right) + \left(1 - \frac{\varepsilon}{\varepsilon_p} \right) \right] \quad (12)$$

where c is a positive constant between zero and unity, which decreases as the curves are higher. The value of c is determined through plots of $[-\ln(\sigma/\sigma_p)]$ vs. $[-\ln(\varepsilon/\varepsilon_p) - (1 - \varepsilon/\varepsilon_p)]$. These should be linear but are never perfectly so.

The evaluation procedure

The result of continuous torsion tests are taken into the calculation – as torque moment and strain at various temperatures ($T \in (800; 1000)^\circ\text{C}$ and strain rates $\dot{\epsilon} \in (6; 1000) \text{ min}^{-1}$) from hot deformed HSLA steel. The specimens were 50 mm long having a diameter of 6 mm.

1. STRESSPC (STRESS at Peak Calculated)

The data can be fitted by

$$(13) \quad \sigma = 90 \arg \sin h \left(\frac{\dot{\epsilon} \exp \frac{365\,400}{RT}}{8 \cdot 10^{13}} \right)^{0,2}$$

using personal computer AT.

2. STRAINPC (STRAIN at Peak Calculated)

$$(14) \quad \epsilon_{PC} = \dot{\epsilon}^{0,142} \cdot \exp \arg \sin h \left(-6,9 + \frac{7\,180}{T} \right)$$

3. STRESS (STRESS Experimental compared with STRESS Calculated)

$$(15) \quad \sigma_c = \sigma_{PC} \left[\frac{\epsilon}{\epsilon_{PC}} \exp \left(1 - \frac{\epsilon}{\epsilon_{PC}} \right) \right]^c$$

Discussion

The value of exponent c , wich is variable, slightly complicate common use of equation (15). Taking into account that the resulting c is adequate for temperature,

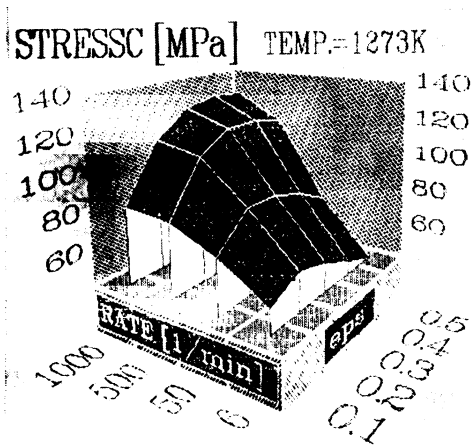


Fig. 1. Effect of strain rate from 6–1000 min^{-1} and true strain from 0.1 to 0.5 on STRESS Calculated (eq. 17).

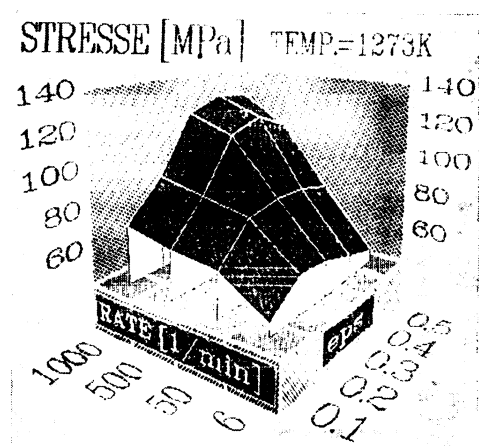
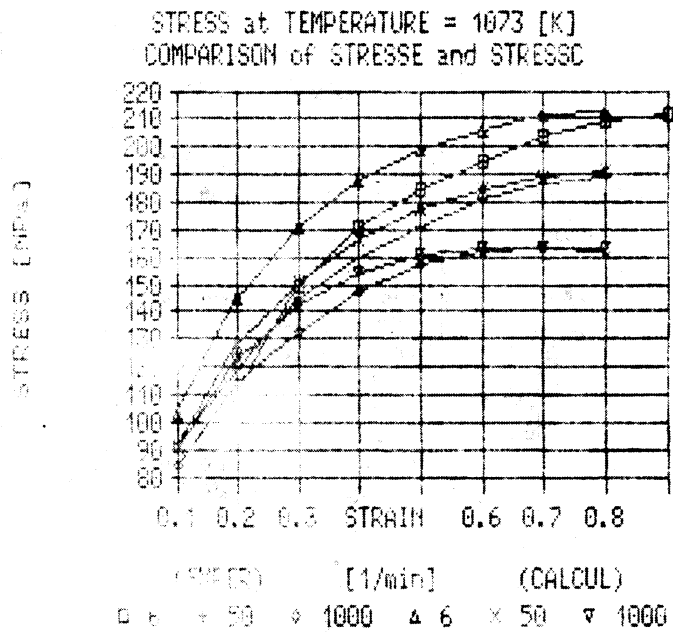
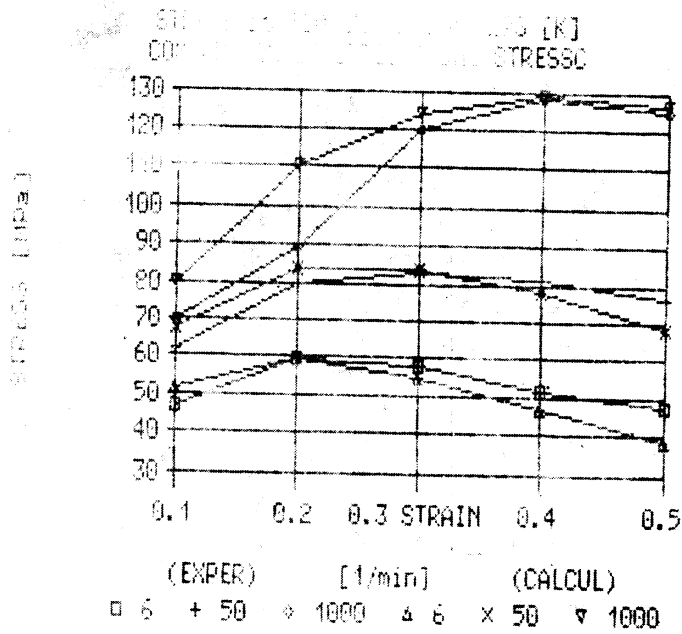


Fig. 2. Effect of strain rate from 6–1000 min^{-1} and true strain from 0.1 to 0.5 on STRESS Experimental.



a)



b)

Fig. 3. Comparison of STRESSE and STRESSC for a ... $T = 1073$ K, various strain and strain rates b ... $T = 1273$ K.

TEMPERATURE T = 1123 /K/ RATE $\dot{\epsilon}$ = 6 /min-1/
=====

eps= .1	STRESS (EXPER.) = 68 /MPa/	(CALCUL.) = 78 /MPa/
eps= .2	STRESS (EXPER.) = 96 /MPa/	(CALCUL.) = 100 /MPa/
eps= .3	STRESS (EXPER.) = 105 /MPa/	(CALCUL.) = 107 /MPa/
eps= .4	STRESS (EXPER.) = 104 /MPa/	(CALCUL.) = 106 /MPa/
eps= .5	STRESS (EXPER.) = 98 /MPa/	(CALCUL.) = 100 /MPa/
eps= .6	STRESS (EXPER.) = 94 /MPa/	(CALCUL.) = 93 /MPa/
eps= .7	STRESS (EXPER.) = 89 /MPa/	(CALCUL.) = 84 /MPa/
eps= .8	STRESS (EXPER.) = 88 /MPa/	(CALCUL.) = 76 /MPa/

TEMPERATURE T = 1123 /K/ RATE $\dot{\epsilon}$ = 50 /min-1/
=====

eps= .1	STRESS (EXPER.) = 78 /MPa/	(CALCUL.) = 86 /MPa/
eps= .2	STRESS (EXPER.) = 105 /MPa/	(CALCUL.) = 117 /MPa/
eps= .3	STRESS (EXPER.) = 128 /MPa/	(CALCUL.) = 133 /MPa/
eps= .4	STRESS (EXPER.) = 137 /MPa/	(CALCUL.) = 139 /MPa/
eps= .5	STRESS (EXPER.) = 139 /MPa/	(CALCUL.) = 140 /MPa/
eps= .6	STRESS (EXPER.) = 135 /MPa/	(CALCUL.) = 137 /MPa/
eps= .7	STRESS (EXPER.) = 130 /MPa/	(CALCUL.) = 132 /MPa/
eps= .8	STRESS (EXPER.) = 124 /MPa/	(CALCUL.) = 125 /MPa/

TEMPERATURE T = 1123 /K/ RATE $\dot{\epsilon}$ = 500 /min-1/
=====

eps= .1	STRESS (EXPER.) = 80 /MPa/	(CALCUL.) = 90 /MPa/
eps= .2	STRESS (EXPER.) = 120 /MPa/	(CALCUL.) = 127 /MPa/
eps= .3	STRESS (EXPER.) = 138 /MPa/	(CALCUL.) = 148 /MPa/
eps= .4	STRESS (EXPER.) = 155 /MPa/	(CALCUL.) = 161 /MPa/
eps= .5	STRESS (EXPER.) = 165 /MPa/	(CALCUL.) = 168 /MPa/
eps= .6	STRESS (EXPER.) = 168 /MPa/	(CALCUL.) = 170 /MPa/
eps= .7	STRESS (EXPER.) = 168 /MPa/	(CALCUL.) = 169 /MPa/
eps= .8	STRESS (EXPER.) = 160 /MPa/	(CALCUL.) = 166 /MPa/

TEMPERATURE T = 1123 /K/ RATE $\dot{\epsilon}$ = 1000 /min-1/
=====

eps= .1	STRESS (EXPER.) = 100 /MPa/	(CALCUL.) = 93 /MPa/
eps= .2	STRESS (EXPER.) = 142 /MPa/	(CALCUL.) = 133 /MPa/
eps= .3	STRESS (EXPER.) = 162 /MPa/	(CALCUL.) = 158 /MPa/
eps= .4	STRESS (EXPER.) = 175 /MPa/	(CALCUL.) = 173 /MPa/
eps= .5	STRESS (EXPER.) = 182 /MPa/	(CALCUL.) = 183 /MPa/
eps= .6	STRESS (EXPER.) = 186 /MPa/	(CALCUL.) = 187 /MPa/
eps= .7	STRESS (EXPER.) = 189 /MPa/	(CALCUL.) = 189 /MPa/
eps= .8	STRESS (EXPER.) = 186 /MPa/	(CALCUL.) = 188 /MPa/

TABLE
COMPARISON OF EXPERIMENTAL
AND CALCULATED RESULTS

t [C]	ω [min^{-1}]	STRESSPE	STRESSPC
1000	6	59.7273	59.8895
1000	50	85.456	84.985
1000	500	117.617	118.151
1000	1000	129.256	129.024
900	6	92.507	96.2548
900	50	120.986	128.329
900	500	147.327	166.37
900	1000	164.48	178.114
850	6	107.203	121.149
850	50	139.976	155.578
850	500	169.993	194.926
850	1000	188.983	206.872
800	6	142.12	150.634
800	50	164.48	186.434
800	500	192.046	226.309
800	1000	213.487	238.773

PEAK STRAINS

temperature [K]	rate of twisting [min^{-1}]	strainp	
		STRAINPE	STRAINPC
1273	6	.197537	.1903
	50	.251786	.257165
	500	.366307	.35636
	1000	.413775	.393024
1173	6	.261415	.266365
	50	.315296	.359915
	500	.461657	.498369
	1000	.549565	.549144
1123	6	.329538	.334584
	50	.466217	.451474
	500	.621253	.624558
	1000	.70535	.687928
1073	6	.443752	.441804
	50	.666054	.594724
	500	.875773	.820567
	1000	.840429	.906523

thus a mathematically correct relationship should be of the form

$$(16) \quad c = A - B/T \quad c = 1,5157 - 998/T$$

and eq. (15) transformed to

$$(17) \quad \sigma = \sigma_p \left[\frac{\varepsilon}{\varepsilon_p} \exp \left(1 - \frac{\varepsilon}{\varepsilon_p} \right) \right]^{A-B/T}.$$

Conclusion

Comparison of experimental – STRESSPE – and calculated – STRESSPC – values are given in tables together with STRAINPE and STRAINPC results. The delivered figures 1 to 3 illustrate differences between STRESSPE and STRESSPC according equations (15), (17), respectively.

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