

# Applications of Mathematics

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## Book Reviews

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## BOOK REVIEWS

*I. Nowak*: RELAXATION AND DECOMPOSITION METHODS FOR MIXED INTEGER NONLINEAR PROGRAMMING. Birkhäuser Verlag, Basel, 2005. ISBN 3-7643-7238-9 (ISBN-13: 978-3-7643-7238-5, e-ISBN: 3-7643-7374-1), hardcover, xvi+213 pages, price EUR 92,00.

In his monograph, the author treats mixed integer nonlinear programs (MINLPs), that is, nonlinear optimization problems containing both continuous and discrete variables. The problems become even more challenging if they are nonconvex.

The book comprises 14 chapters organized into two parts, namely *Basic Concepts* and *Algorithms*. In a short introductory chapter, a general nonconvex MINLP is defined (Problem  $\mathfrak{P}$ ). The goal is to minimize a nonlinear function  $f(x, y)$  where  $x$  is a vector of continuous variables belonging to a bounded  $m$ -dimensional interval and  $y$  is a vector of integer variables belonging to a bounded  $n$ -dimensional interval. The variables are further constrained by  $g(x, y) \leq 0$  and  $h(x, y) = 0$  where  $g$  and  $h$  are nonlinear vector functions. As shown in the next chapter, it is important to properly reformulate Problem  $\mathfrak{P}$  to facilitate its solution. Also, the notion of the block-separable optimization problem (BSOP) is defined.

Relaxations of Problem  $\mathfrak{P}$  are introduced in Chapter 3. These are based either on the convexification of sets and functions appearing in Problem  $\mathfrak{P}$  or on Lagrangian-based dual problems. Decompositions are the subject of Chapter 4. Decomposition methods solve a BSOP by splitting it into several smaller subproblems that are coupled through a master problem. Various methods are described in the framework of four decomposition principles: dual methods, primal cutting-plane methods, column generation, and Benders decomposition. In a separate chapter, special attention is paid to relaxations of all-quadratic programs where  $f$  as well as the constraints are quadratic functions.

Chapter 6 can be viewed as a continuation of Chapter 3. Indeed, it presents several methods for generating convex underestimating relaxations. The next chapter focuses on cut, lower bounds, and box reduction methods that are designed to improve polyhedral and nonlinear relaxations.

Chapter 8 begins with an overview of local and (for quadratic programs) global optimality criteria known in smooth optimization. Using these results, the author infers a new global optimality criterion that can be used to split off a given local minimizer from the feasible set. For quadratic programs with particular constraints, special global optimality criteria are given.

The theory-oriented part of the monograph ends with a short Chapter 9 that presents concepts of using MINLPs in optimal control and stochastic programming.

The next chapter is also short; it gives a concise overview of global optimization methods.

Particular methods classified as deformation heuristics form Chapter 11. The idea behind them is to design a parameter-driven transformation that changes a difficult optimization problem into a sequence of relaxed problems that are easier to solve and that converge to the original problem.

Two other heuristics (i.e., methods not verifying global optimality) for solving MINLPs are presented in Chapter 12 whereas the next chapter introduces branch-and-bound algorithms that are able to systematically search for a global solution and to prove global optimality.

After the last chapter devoted to LAGO (Lagrangian Global Optimization, an object-oriented library developed by the author and his co-workers), two appendices follow. In the first, the author presents his views on the future perspectives of MINLP solvers development. In the second, details on MINLP problems used in numerical experiments are listed.

Chapters 11–13 contain tables and graphs illustrating various performance features of the methods described. The reader, however, can find algorithms and performance tables in the theory-oriented part, too (in Chapters 4, 5, and 7).

The bibliography section contains more than 200 items. The book ends with an index section.

This self-contained monograph is rich in content, provides the reader with a wealth of information, and motivates his or her further interest in the subject. The book offers fairly comprehensive description of the MINLP theory and algorithms. Its somewhat condensed style will probably please readers from the field but interested readers not familiar with the subject might find reading some parts of the book demanding.

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