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STABILITY ANALYSIS FOR NEUTRAL STOCHASTIC SYSTEMS WITH MIXED DELAYS

Huabin Chen and Peng Hu

This paper is concerned with the problem of the exponential stability in mean square moment for neutral stochastic systems with mixed delays, which are composed of the retarded one and the neutral one, respectively. Based on an integral inequality, a delay-dependent stability criterion for such systems is obtained in terms of linear matrix inequality (LMI) to ensure a large upper bounds of the neutral delay and the retarded delay by dividing the neutral delay interval into multiple segments. A new Lyapunov–Krasovskii functional is constructed with different weighting matrices corresponding to different segments. And the developed method can well reduce the conservatism compared with the existing results. Finally, an illustrative numerical example is given to show the effectiveness of our proposed method.

Keywords: neutral stochastic time-delay systems, delay decomposition approach, exponential stability, linear matrix inequality (LMI)

Classification: 93D09, 93E03

1. INTRODUCTION

Time-delay systems have been widely investigated since time-delay is frequently encountered in many dynamical systems, including chemical or process control systems and networked control systems [9, 10, 14–15, 22] and the reference therein. Besides, due to the much more conservatism of the delay-independent conditions compared with delay-dependent ones when the time-delay is small, pursuing the delay-dependent stability criteria is much theoretical and practical value. Recently, the stability issue for neutral time-delay systems has also been attracted much attention and some valuable results about the delay-dependent stability criteria of such systems have been reported in [6, 12–13, 16–17, 23] and the references therein.

Recently, the problems on the stability analysis, the H_∞-control design and the H_∞ filter design of neutral stochastic time-delay systems have been considered in [2–4, 11, 18, 21] and the references therein. By utilizing the free-weighting matrix technique and the bounding technique, some LMI-based sufficient conditions ensuring the exponential stability in mean square moment and H_∞ control for neutral stochastic time-delay systems were given [3]; Resorting to the mode transformation method coupled with the bounding technique, the exponential stability in mean square moment for such systems was analyzed in [11]. But, the results [3, 11] are much more conservative since the bounding technique on cross term can bring much conservatism while using the free-weighting matrix technique or the mode transformation technique. By only establishing the generalized Finsler's Lemma, Chen, et al. in [4] have discussed the exponential stability in mean square moment of such systems and the obtained stability criteria are much less conservative than ones in [3, 11].

As pointed out in [10], the maximum allowable bound of the time delay is larger, the obtained conditions are less conservative. Over the past decades, many valuable results were mainly involved into the less conservative stability conditions for time-delay systems by using the delay decomposition approach proposed in [7, 10], for example, time-delay systems [5, 17], the Markov systems with interval time-varying delay [8], discrete time-delay systems [9], neutral time-delay systems [12, 16], switched time-delay systems [19], stochastic time-delay systems [20], fuzzy time-delay systems [22], and singular time-delay systems [23], etc. However, the obtained results in [5, 7–9, 12, 16–17, 19–20, 22] can not be applied into considering the stability analysis for neutral stochastic systems with mixed delays since the existence of stochastic perturbation and the presence of the neutral item can make the problem be complicated. Thus, how to obtain the delay-dependent stability criteria for uncertain neutral stochastic linear systems with mixed delays by using this useful approach still remains an interesting and challenging problem.

In this paper, an integral inequality is firstly established for neutral differential equation, and then a new Lyapunov–Krasovskii functional is constructed with different weighting matrices corresponding to different segments, the delay-dependent exponential stability criterion for neutral stochastic systems with mixed delays is derived in terms of linear matrix inequality (LMI). Finally, one numerical example is provided to illustrate the effectiveness of the obtained result.

Notations: In this paper, R^n and $R^{m\times n}$ are the n-dimensional Euclidean space and the set of real $m\times n$ matrix, respectively. $|\cdot|$ denotes the Euclidean norm. $(\Omega,\ \Im,\ P)$ is a completed probability space, where Ω is the sample space, $\Im$ is a σ-algebra of subsets of Ω, and P is the probability measure. For a real symmetric matrix X, $X>0$ $(X\geq 0)$ means that X is positive definite (positive semi-definite). The superscript $''T''$ denotes the transpose of a matrix or a vector. Denote by $L^2_{\Im_0}([-r,0];R^n)$ $(r>0)$ is the family of all $\Im_0$-measurable, $C([-r,0];R^n)$-valued random variables $\xi=\{\xi(\theta):\ -r\leq\theta\leq 0\}$ such that $\sup_{\theta\in[-r,0]}E|\xi(\theta)|^2<+\infty$, where $E(\cdot)$ stands for the mathematical expectation operator. Matrices, it not explicitly stated, are assumed to have compatible dimensions. $LV(\cdot)$ presents the It$\widehat{o}'$s differential operator.

2. MAIN RESULTS

2.1. Problem formulation and some preliminaries

In this paper, we consider the neutral stochastic systems with mixed delays [4]:

$$d[x(t)-Cx(t-\tau)]=[Ax(t)+Bx(t-h)]\,\mathrm{d}t+[Fx(t)+Gx(t-h)]\,\mathrm{d}B(t),\quad t\geq 0,\quad (1)$$

and the initial value $x_0(\theta)=\varphi\in L^2_{\Im_0}([-2\sigma,0];R^n),\ \theta\in[-2\sigma,0]$, $\sigma=\max\{\tau,h\}$. And $x(t)\in R^n$ is the state vector with the Euclidean norm $|\cdot|$ in R^n, and $B(t)$ is one-dimensional Brownian motion defined on the complete probability space $(\Omega,\Im,P)$. The delay τ and h are two positive scalars which represent the neutral delay and the retarded delay, respectively. The matrices: $A,\ B,\ C,\ F,\ G$ are five known constant ones.

Lemma 1. (Gu, et al. [10]) For any constant matrix $Z > 0,\ Z \in R^{n\times n}$ and a scalar $\gamma > 0$, if there exists a vector function $\omega :\ [0,\gamma] \to R^n$ such that the following integral are well defined, then the following inequality holds:

$$\gamma \int_0^\gamma \omega^T(s) Z \omega(s)\,\mathrm{d}s \geq \Big(\int_0^\gamma \omega^T(s)\,\mathrm{d}s\Big) Z \Big(\int_0^\gamma \omega(s)\,\mathrm{d}s\Big).$$

Lemma 2. For $r > 0$ and for two scalars: $a,\ b$ with $0 \leq a < b \leq r$, let n-dimensional vector functions $x(t),\ \widetilde{f}(t)$ and a matrix $D \in R^{n\times n}$ satisfy the neutral differential equation:

$$\frac{\mathrm{d}[x(t) - Dx(t-r)]}{\mathrm{d}t} = \widetilde{f}(t), \quad t \geq 0, \tag{2}$$

where the initial condition $x(\theta) = \psi(\theta) \quad (\theta \in [-r, 0])$. For any constant matrix $W = W^T > 0,\ W \in R^{n\times n}$, if the following integral is well defined, then

$$-(b-a)\int_{t-b}^{t-a} \widetilde{f}^T(s) W \widetilde{f}(s)\,\mathrm{d}s \leq \eta(t)\Omega\eta^T(t), \tag{3}$$

where $\eta(t) = [x^T(t-a)\ \ x^T(t-b)\ \ x^T(t-a-r)\ x^T(t-b-r)]$ and

$$\Omega = \begin{bmatrix} -W & W & WD & -WD \\ W^T & -W & -WD & WD \\ D^TW & -D^TW & -D^TWD & D^TWD \\ -D^TW & D^TW & D^TWD & -D^TWD \end{bmatrix}.$$

Proof. From (2), we have

$$\int_{t-b}^{t-a} \widetilde{f}(s)\,\mathrm{d}s = x(t-a) - Dx(t-a-r) - x(t-b) + Dx(t-b-r). \tag{4}$$

By using Lemma 1, it implies

$$-(b-a)\int_{t-b}^{t-a} \widetilde{f}^T(s) W \widetilde{f}(s)\,\mathrm{d}s \leq -\Big[\int_{t-b}^{t-a} \widetilde{f}^T(s)\,\mathrm{d}s\Big] W \Big[\int_{t-b}^{t-a} \widetilde{f}(s)\,\mathrm{d}s\Big]. \tag{5}$$

Thus, substituting (4) into the right side of (5), the desired result can be obtained. □

2.2. Stability analysis

In this subsection, we obtain the result on the exponential stability in mean square moment for systems (1) in terms of LMI in [1]. For the sake of simplicity, the following notations are adopted:

$$f(t) = Ax(t) + Bx(t-h) \quad \text{and} \quad g(t) = Fx(t) + Gx(t-h). \tag{6}$$

And in order to obtain the desired result, we need to construct the following new LKF:

$$\begin{aligned} V(t,x) &= \zeta^T(t)P\zeta(t) + \int_{t-\tau_m}^{t-\tau_0} \chi^T(s)H\chi(s)\,\mathrm{d}s + \int_{t-h}^{t-\tau_0} \eta^T(s)Q\eta(s)\,\mathrm{d}s \\ &\quad + \sum_{i=1}^{2m-1} \int_{t-\tau_i}^{t-\tau_{i-1}} \varphi^T(s)T_i\varphi(s)\,\mathrm{d}s \\ &\quad + h\int_{-h}^{0}\int_{t+\theta}^{t} y^T(s)Ry(s)\,\mathrm{d}s\,\mathrm{d}\theta + \frac{\tau}{m}\sum_{j=1}^{m}\int_{-\tau_j}^{-\tau_{j-1}}\int_{t+\theta}^{t} y^T(s)S_jy(s)\,\mathrm{d}s\,\mathrm{d}\theta, \end{aligned}$$

for any $t \geq 2\sigma$, where $\zeta(t) = x(t-\tau_0) - Cx(t-\tau_m)$, $y(t)\,\mathrm{d}t = f(t)dt + g(t)\,\mathrm{d}B(t)$, $m > 0$ denotes the number of divisions of the interval $[-\tau, 0]$, $\tau_i = \frac{i\tau}{m}$, $(i = 0, 1, 2, \dots, m)$, $\chi(t) = [x^T(t-\tau_0)\ x^T(t-h)\ x^T(t-\tau_m)]^T$, $\eta(t) = [x^T(t-\tau_0)\ \ x^T(t-\tau_m)]^T$, $\varphi(t) = [x^T(t-\tau_0)\ \ x^T(t-\tau_1)]^T$ and the matrices:

$$\begin{aligned} P &> 0,\ H = \begin{bmatrix} H_1 & H_2 & H_4 \\ H_2^T & H_3 & H_5 \\ H_4^T & H_5^T & H_6 \end{bmatrix} > 0,\ Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} > 0, \\ T_i &= \begin{bmatrix} T_i^{11} & T_i^{12} \\ (T_i^{12})^T & T_i^{22} \end{bmatrix} > 0,\ i = 1, 2, \dots, 2m-1,\ R > 0 \text{ and } S_k > 0,\ k = 1, 2, \dots, m, \end{aligned}$$

are of appropriate dimensions.

Remark 1. If $m = 1$ and $T_i = 0$ $(i = 1, 2, \dots, 2m-1)$ in (7), the LKF (7) is degrated into (25) defined in [4]. However, it should be pointed out that the proposed method in [4] is extremely difficult to consider the stability analysis for neutral stochastic time-delay systems (1) by constructing LKF (7).

Remark 2. In [12, 16–17], the authors have considered the stability analysis for neutral time-delay system by using the delay-decomposition technique and some less conservative delay-dependent stability criteria were obtained, but the results in [12, 16–17] can not be used to ensure the stability for neutral stochastic time-delay systems since the existence of stochastic perturbation and the presence of the neutral item can make the problem be complicated. In this paper, after establishing Lemma 2, the delay-decomposition technique can well study our concerned problem.

Remark 3. From (6) and $y(t)\,\mathrm{d}t = f(t)\,\mathrm{d}t + g(t)\,\mathrm{d}B(t)$, systems (1) can be written as the following form:

$$\frac{\mathrm{d}[x(t) - Cx(t-\tau)]}{\mathrm{d}t} = y(t).$$

Consequently, based on the LKF (7) and Lemma 2, we can derive the following result:

Theorem 3. For a positive integer number m and two given scalars $\tau > 0$, $h > 0$, neutral stochastic systems with mixed delays (1) is exponentially stable in mean square moment, if there exist matrices $P > 0$, $H > 0$, $Q > 0$, $T_i > 0$ $(i = 1, 2, \dots, 2m-1)$,

$R > 0$, $S_k > 0$ $(k = 1, 2, \ldots, m)$ and an appropriate dimensional matrix N, such that the linear matrix inequality (LMI) holds

$$\Upsilon = \begin{bmatrix} \Upsilon^1 & \Upsilon^2 \\ (\Upsilon^2)^T & \Upsilon^3 \end{bmatrix} < 0, \tag{8}$$

where

$$\begin{aligned}
\Upsilon^1 &= (\Upsilon^1_{ij})^{(2m+3)\times(2m+3)},\\
\Upsilon^1_{1,1} &= P^TA + A^TP + F^TPF + H_1 + Q_1 + T_1^{11} - R - S_1,\\
\Upsilon^1_{1,2} &= PB + F^TPG + H_2 + R,\\
\Upsilon^1_{1,3} &= T_1^{12} + S_1,\\
\Upsilon^1_{1,m+2} &= A^TPC + H_4 + Q_2 + RC + S_1C,\\
\Upsilon^1_{1,m+3} &= -S_1C,\\
\Upsilon^1_{1,2m+3} &= -RC,\\
\Upsilon^1_{2,2} &= H_3 - Q_1 - R + G^TPG,\\
\Upsilon^1_{2,m+2} &= H_5 - B^TPC - RC,\\
\Upsilon^1_{2,2m+3} &= RC - Q_2,\\
\Upsilon^1_{j,j} &= T^{22}_{j-2} - T^{11}_{j-2} + T^{11}_{j-1} - T^{22}_{j-3} - S_{j-2} - S_{j-1},\\
\Upsilon^1_{j,j+1} &= -T^{12}_{j-2} + T^{12}_{j-1} + S_{j-1},\\
\Upsilon^1_{j,j+3} &= -S_{j-2}C,\\
\Upsilon^1_{j,m+j} &= S_{j-2}C + S_{j-1}C,\\
\Upsilon^1_{j,j+m+1} &= -S_{j-1}C,\\
\Upsilon^1_{m+j,m+j} &= T^{22}_{m+j-2} - T^{22}_{m+j-3} - T^{11}_{m+j-2} + T^{11}_{m+j-1} - C^TS_{j-1}C - C^TS_{j-2}C,\\
\Upsilon^1_{m+j,m+j+1} &= -T^{12}_{m+j-2} + T^{12}_{m+j-1} + C^TS_{j-1}C,\ j = 3, 4, \ldots, m+1,\\
\Upsilon^1_{m+2,m+2} &= H_6 - H_1 + Q_3 + T^{22}_m - T^{11}_m + T^{11}_{m+1} - T^{22}_{m-1} - C^TS_1C - C^TRC - S_m,\\
\Upsilon^1_{m+2,m+3} &= -T^{12}_m + T^{12}_{m+1} + C^TS_1C,\\
\Upsilon^1_{m+2,2m+1} &= -S_mC,\\
\Upsilon^1_{m+2,2m+2} &= S_mC - H_4,\\
\Upsilon^1_{m+2,2m+3} &= -H_2 + C^TRC,\\
\Upsilon^1_{2m+2,2m+2} &= -H_6 - T^{12}_{2m-1} - C^TS_mC,\\
\Upsilon^1_{2m+2,2m+3} &= -H_5^T,\\
\Upsilon^1_{2m+3,2m+3} &= -C^TRC - Q_3 - H_3,
\end{aligned}$$

$T_0^{22} \equiv 0,\ T_{2m}^{11} \equiv 0,$

$$\begin{aligned} \Upsilon^2 &= (\Upsilon^2_{ij})^{(2m+3)\times(m+1)} = \begin{bmatrix} N^TA & N^TB & 0 & \cdots & 0 \end{bmatrix}^T, \\ \Upsilon^3 &= h^2R + \frac{\tau^2}{m^2}\sum_{j=1}^{m} S_j - N^T - N, \end{aligned}$$

and other items in the matrix Υ^1 are zeros.

Proof. From LKF (7), by using the Itô's formula, we have

$$\mathrm{d}V(t,x) = LV(t,x)\,\mathrm{d}t + 2[x(t) - Cx(t-\tau)]^T Pg(t)\,\mathrm{d}B(t), \tag{9}$$

where

$$\begin{aligned} LV(t,x) &= 2\zeta^T(t)Pf(t) + g^T(t)Pg(t) + \chi^T(t-\tau_0)T\chi(t-\tau_0) \\ &\quad -\chi^T(t-\tau_m)T\chi(t-\tau_m) + \eta^T(t-\tau_0)Q\eta(t-\tau_0) - \eta^T(t-h)Q\eta(t-h) \\ &\quad + \sum_{i=1}^{2m-1}[\varphi^T(t-\tau_{i-1})T_i\varphi(t-\tau_{i-1}) - \varphi^T(t-\tau_i)T_i\varphi(t-\tau_i)] \\ &\quad + h^2y^T(t)Ry(t) - h\int_{t-h}^{t} y^T(s)Ry(s)\,\mathrm{d}s + \left(\frac{\tau}{m}\right)^2\sum_{j=1}^{m} y^T(t)S_jy(t) \\ &\quad -\frac{\tau}{m}\sum_{j=1}^{m}\int_{t-\tau_j}^{t-\tau_{j-1}} y^T(s)S_jy(s)\,\mathrm{d}s. \end{aligned} \tag{10}$$

Applying Lemma 2, we can obtain

$$-h\int_{t-h}^{t} y^T(s)Ry(s)\,\mathrm{d}s \le \eta_0^T(t)\Omega_0\eta_0(t), \tag{11}$$

and

$$-\frac{\tau}{m}\sum_{j=1}^{m}\int_{t-\tau_j}^{t-\tau_{j-1}} y^T(s)S_jy(s)\,\mathrm{d}s \le \sum_{j=1}^{m}\eta_j^T(t)\Omega_j\eta_j(t), \tag{12}$$

where $\eta_0(t) = [x^T(t-\tau_0)\ \ x^T(t-h)\ \ x^T(t-\tau_m)\ \ x^T(t-\tau_m-h)]^T$,
$\eta_j(t) = [x^T(t-\tau_{j-1})\ \ x^T(t-\tau_j)\ \ x^T(t-\tau_{j+m-1})\ \ x^T(t-\tau_{j+m})]^T$ $(j = 1,2,\ldots,m)$,
the matrices

$$\Omega_0 = \begin{bmatrix} -R & R & RC & -RC \\ R^T & -R & -RC & RC \\ C^TR & -C^TR & -C^TRC & C^TRC \\ -C^TR & C^TR & C^TRC & -C^TRC \end{bmatrix},$$

and

$$\Omega_j = \begin{bmatrix} -S_j & S_j & S_jC & -S_jC \\ S_j^T & -S_j & -S_jC & S_jC \\ C^TS_j & -C^TS_j & -C^TS_jC & C^TS_jC \\ -C^TS_j & C^TS_j & C^TS_jC & -C^TS_jC \end{bmatrix}, \quad j = 1,2,\ldots,m.$$

And from the Newton–Leibnitz formula [3, 13], it derives

$$2y^T(t)N^T\{[f(t)-y(t)]\,\mathrm{d}t+g(t)\,\mathrm{d}B(t)\}=0. \tag{13}$$

Substituting (11) – (13) into (10), it follows

$$LV(t,x)\le \xi^T(t)\Upsilon\xi(t)+2[x(t)-Cx(t-\tau)]^TPg(t)\,\mathrm{d}B(t)+2y^T(t)N^Tg(t)\,\mathrm{d}B(t), \tag{14}$$

where

$$\begin{aligned}\xi(t) \;=\; & [x^T(t-\tau_0)\; x^T(t-h)\; x^T(t-\tau_1)\; x^T(t-\tau_2)\; \ldots \\ & \ldots\; x^T(t-\tau_{2m-1})\; x^T(t-\tau_{2m})\; x^T(t-\tau_m-h)\; y^T(t)]^T.\end{aligned}$$

Consequently, it gives from (8) and (14) that

$$LV(t,x)\le$$

$$-\lambda(|x(t)|^2+|x(t-\tau)|^2)+2[x(t)-Cx(t-\tau)]^TPg(t)\,\mathrm{d}B(t)+2y^T(t)N^Tg(t)\,\mathrm{d}B(t), \tag{15}$$

where $\lambda=\lambda_{\min}(-\Upsilon)>0$. And from the definition of LKF (7), there exist positive scalars δ_1, δ_2 and δ_3 such that

$$V(t,x)\le \delta_1(|x(t)|^2+|x(t-\tau)|^2)+\delta_2\int_{t-\sigma}^{t}|x(s)|^2\,\mathrm{d}s+\delta_3\int_{t-\sigma}^{t}|x(s-\tau)|^2\,\mathrm{d}s.$$

Thus, for any $\alpha>0$, we obtain

$$\begin{aligned} d[e^{\alpha t}V(t,x)] \;\le\; & e^{\alpha t}\Big[(\alpha\delta_1-\lambda)|x(t)|^2+(\alpha\delta_1-\lambda)|x(t-\tau)|^2+\alpha\delta_2\int_{t-\sigma}^{t}|x(s)|^2\,\mathrm{d}s \\ & +\alpha\delta_3\int_{t-\sigma}^{t}|x(t-\tau)|^2\,\mathrm{d}s\Big]\,\mathrm{d}t+e^{\alpha t}\Big[2[x(t)-Cx(t-\tau)]^TPg(t)\;\mathrm{d}B(t) \\ & +2y^T(t)N^Tg(t)\,\mathrm{d}B(t)\Big]. \end{aligned} \tag{16}$$

And then, integrating both sides of (16) from 0 to t and two items: $\int_0^t e^{\alpha s}[x(s)-Cx(s-\tau)]^TPg(s)\,\mathrm{d}B(s)$, $\int_0^t e^{\alpha s}y^T(s)N^Tg(s)\,\mathrm{d}B(s)$ are martingale, we have

$$\begin{aligned} & E\{e^{\alpha t}V(t,x_t)\}-E\{V(0,x_0)\}\le \\ & \quad (\alpha\delta_1-\lambda)E\Big\{\int_0^t e^{\alpha s}|x(s)|^2\,\mathrm{d}s\Big\}+(\alpha\delta_1-\lambda)E\Big\{\int_0^t e^{\alpha s}|x(s-\tau))|^2\,\mathrm{d}s\Big\} \\ & \quad +\alpha\delta_2E\Big\{\int_0^t\int_{s-\sigma}^{s}e^{\alpha u}|x(u)|^2\,\mathrm{d}u\,\mathrm{d}s\Big\}+\alpha\delta_3E\Big\{\int_0^t\int_{s-\sigma}^{s}e^{\alpha u}|x(u-\tau)|^2\,\mathrm{d}u\,\mathrm{d}s\Big\}. \end{aligned} \tag{17}$$

Consequently, by changing the integration sequence, the following inequalities hold:

$$\int_0^t\int_{s-\sigma}^{s}e^{\alpha u}|x(u)|^2\,\mathrm{d}u\,\mathrm{d}s\le \sigma e^{\alpha\sigma}\int_0^t e^{\alpha s}|x(s)|^2\,\mathrm{d}s+\sigma^2e^{\alpha\sigma}\sup_{\theta\in[-2\sigma,0]}|\psi(\theta)|^2, \tag{18}$$

and

$$\int_0^t \int_{s-\sigma}^s e^{\alpha u}|x(u-\tau)|^2 \, \mathrm{d}u \, \mathrm{d}s \le \sigma e^{\alpha\sigma} \int_0^t e^{\alpha s}|x(s-\tau)|^2 \, \mathrm{d}s + \sigma^2 e^{\alpha\sigma} \sup_{\theta\in[-2\sigma,0]} |\psi(\theta)|^2. \quad (19)$$

Substituting (18) – (19) into (17), it obtains

$$E\{e^{\alpha t}V(t,x_t)\} - E\{V(0,x_0)\} \le$$

$$2\sigma^2 e^{\alpha\sigma} E\{\sup_{\theta\in[-2\sigma,0]} |\psi(\theta)|^2\} + (\alpha\delta_1 - \lambda + \alpha\delta_2\sigma e^{\alpha\sigma})E\Big\{\int_0^t e^{\alpha s}|x(s)|^2 \, \mathrm{d}s\Big\}$$
$$+(\alpha\delta_1 - \lambda + \alpha\delta_3\sigma e^{\alpha\sigma})E\Big\{\int_0^t e^{\alpha s}|x(s-\tau))|^2 \, \mathrm{d}s\Big\}. \quad (20)$$

Choose $\alpha > 0$ such that

$$\alpha\delta_1 - \lambda + \alpha\delta_2\sigma e^{\alpha\sigma} < 0, \;\; \text{and} \;\; \alpha\delta_1 - \lambda + \alpha\delta_3\sigma e^{\alpha\sigma} < 0. \quad (21)$$

Therefore, from (20) – (21), it follows

$$e^{\alpha t} EV(t,x) \le M, \quad (22)$$

where $M = E\{V(0,x_0)\} + 2\sigma^2 e^{\alpha\sigma} \; E\{\sup_{\theta\in[-2\sigma,0]} |\psi(\theta)|^2\} > 0$.

Due to the fact that $\lambda_{\min}(P)|x(t)|^2 \le V(t,x(t))$, it deduces from (22) that

$$E|x(t)|^2 \le \frac{M}{\lambda_{\min}(P)} e^{-\alpha t}, \quad \alpha > 0, \quad t \ge 0,$$

which implies that systems (1) is exponential stability in mean square moment.

□

Remark 3. The exponential stability in mean square moment of systems (1) has been discussed with the aids of the generalized Finsler Lemma [4]. In contrast to the result given in [4], Theorem 3 gives some better ones since the simulation results given in section 3 can show that the conservatism of Theorem 3 will be reduced as the m increases.

Remark 4. It is well known that the input control can usually stabilize the unstable systems [10]. Recently, in [14–15], Jerzy has discussed the controllability for stochastic time-delay systems. Thus, inspired by ideas proposed in [14–15], we will make an attempt to consider the controllability for neutral stochastic time-delay systems in our later works.

3. AN ILLUSTRATIVE NUMERICAL EXAMPLE

Consider the following neutral stochastic systems [4]:

$$d[x(t) - Cx(t-\tau)] = [Ax(t) + Bx(t-h)]\,\mathrm{d}t + [Fx(t) + Gx(t-h)]\,\mathrm{d}B(t), \quad t \geq 0, \quad (23)$$

with the following parameters

$$C = \begin{bmatrix} -0.6 & 0.0 \\ 0.0 & 0.5 \end{bmatrix}, \ A = \begin{bmatrix} -0.4 & 0.2 \\ 0.0 & -0.5 \end{bmatrix}, \ B = \begin{bmatrix} -0.5 & 0.0 \\ 0.0 & 0.3 \end{bmatrix}, \ F = G = \begin{bmatrix} 0.2 & 0.0 \\ 0.0 & 0.2 \end{bmatrix}.$$

The comparison about the maximal allowable upper bounds of the neutral delay τ for given the various values on retarded delay h, and the comparison about the maximal allowable upper bounds of the retarded delay h for different values on the neutral delay τ for systems (23) can be derived based on Theorem 3 in this paper and Theorem 2 in [4] are listed in Table 1 and Table 2, respectively. It is easily seen from Table 1 and Table 2 that the conservatism of Theorem 3 is reduced as the m increases and Theorem 3 is obviously better than Theorem 2 in [4]. When $\tau = 1.5713$ and $h = 3.5$, Figure 1 displays the behavior of the solution $x(t) = [x_1^T(t) \;\; x_2^T(t)]^T$ to systems (23).

h	1.5	2.0	2.5	3.0	3.5
[4]	1.6009	1.4937	1.3731	1.2455	1.1173
$m = 1$	1.6009	1.4937	1.3731	1.2455	1.1173
$m = 2$	2.0118	1.8883	1.7545	1.6163	1.4767
$m = 4$	2.1206	1.9934	1.8571	1.7158	1.5713

Tab. 1. The maximal allowable upper bounds on the neutral delay τ for different values on the retarded delay h for systems (23).

τ	1.5	2.0	2.5	3.0	3.5
[4]	1.9729	1.2541	1.2541	1.2541	1.2541
$m = 1$	1.9729	1.2541	1.2541	1.2541	1.2541
$m = 2$	3.4161	1.5507	1.2541	1.2541	1.2541
$m = 4$	3.7455	1.9751	1.2541	1.2541	1.2541

Tab. 2. The maximal allowable upper bounds on the retarded delay h for different value on the neutral delay τ for systems (23).

4. CONCLUSION

In this paper, with a new augmented Lyapunov–Krasovskii functional (LKF) defined and the delay-decomposition method utilized, the delay-dependent exponential stability criterion for neutral stochastic linear systems with delays is derived in forms of linear matrix inequalities (LMIs), which involves fewer matrix variables and has less conservatism. Finally, an illustrative numerical example is provided to show the effectiveness of the obtained result.

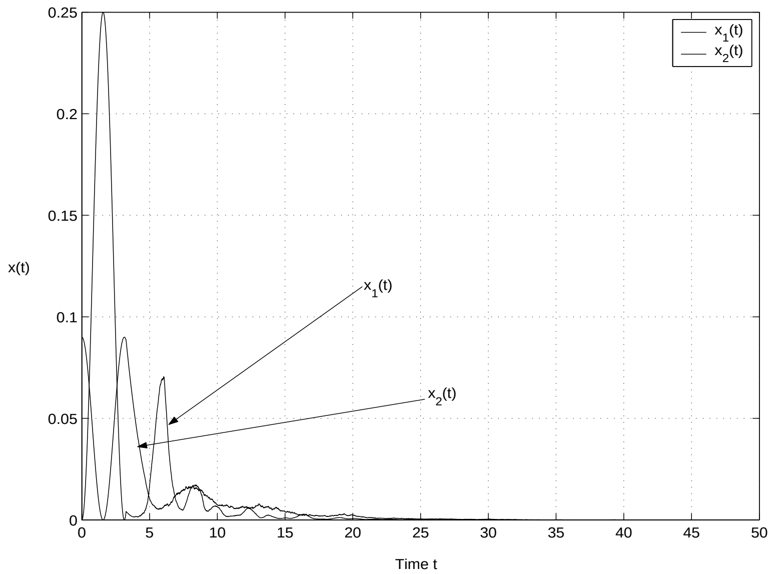

Fig. 1. The behavior of the solution $x(t)$ to systems (23).

Here, we mainly concentrate on the stability analysis for neutral stochastic systems with mixed constant delays. Recently, in [8, 16], the stability of linear systems with time-varying delay and the stability of neutral linear systems with distributed delay have been considered by using the delay decomposition method, respectively. Since the existence of the stochastic perturbation and the presence of the neutral item can make the problem be complicated, the problems on the stability analysis for neutral stochastic systems with time-varying delay and the stability analysis for neutral stochastic systems with distributed delay will be very interesting, which will be considered in our later works. Besides, as pointed out in Remark 4, the controllability for neutral stochastic linear systems with delay will be also worthy of being investigated.

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