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ON HOLOMORPHICALLY PROJECTIVE MAPPINGS FROM MANIFOLDS WITH EQUIAFFINE CONNECTION ONTO KÄHLER MANIFOLDS

Irena Hinterleitner and Josef Mikeš

Abstract. In this paper we study fundamental equations of holomorphically projective mappings from manifolds with equiaffine connection onto (pseudo-) Kähler manifolds with respect to the smoothness class of connection and metrics. We show that holomorphically projective mappings preserve the smoothness class of connections and metrics.

1. Introduction

T. Otsuki and Y. Tashiro [23] introduced the concept of holomorphically projective mappings of Kähler manifolds which preserve the complex structure, and which are generalizations of geodesic mappings. These mappings are studied in many directions, see [2]–[29]. On the other hand, issues related to the mappings and almost complex structures are found in [3, 4, 6, 15, 22, 23, 24, 26, 29].

Fundamental equations for holomorphically projective mappings of (pseudo-) Kähler manifolds in a linear form were found by Domashev and Mikeš [5, 16, 18], see [26, pp. 210-220], [22, pp. 245-248]. In [7] I. Hinterleitner studied holomorphically projective mappings between e-Kähler manifolds in detail. It was shown that they preserve the smoothness class C^r $(r \geq 2)$ of the metric.

In the papers [1, 19] research on holomorphically projective mappings from manifolds with affine connections onto (pseudo-) Kähler manifolds was initiated.

In our paper, we present some new results obtained for holomorphically projective mappings from n-dimensional manifolds A_n with equiaffine connection ∇ and with covariantly almost constant structure F onto (pseudo-) Kähler manifolds $\bar{K}_n$ with metric $\bar{g}$ and with structure $\bar{F}$ from the point of view of differentiability of affine connections and metrics. Here we refine the results of [7, 1, 19]:

If $A_n \in C^{r-1}$ $(r \geq 2)$ admits holomorphically projective mappings onto $\bar{K}_n \in C^2$, then $\bar{K}_n \in C^r$.

Here $A_n \in C^{r-1}$ and $\bar{K}_n \in C^r$ denotes that $\nabla \in C^{r-1}$ and $\bar{g} \in C^r$, which means that in a common coordinate system $x = \{x^1, x^2, \dots, x^n\}$ their components

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$\Gamma^h_{ij}(x) \in C^{r-1}$ and $\bar{g}_{ij}(x) \in C^r$, respectivelly. We suppose that the differentiability degree r is equal to $0, 1, 2, \dots, \infty, \omega$, where $0, \infty$ and ω denotes continuous, infinitely differentiable, and real analytic functions, respectively.

The connection ∇ of A_n, as it is known, need not be the Levi-Civita conection of any metric and A_n need not be a (pseudo-) Riemannian manifold, i.e. there need not be a metric, see [6].

2. Definitions and basic results of F-planar mappings

In [1, 19] were studied holomorphically projective mappings from manifolds A_n with affine connection onto Kähler manifolds $\bar{K}_n$, which are special cases of F-planar mappings (Mikeš and Sinyukov [21], see [8, 17], [22, p. 213–238]).

We consider an n-dimensional manifold A_n with a torsion-free affine connection ∇, and an affinor structure F, i.e. a tensor field of type $(1,1)$.

Definition 1 (Mikeš, Sinyukov [21], see [22, p. 213])**.** A curve ℓ, which is given by the equations $\ell = \ell(t)$, $\lambda(t) = d\ell(t)/dt$ $(\not\equiv 0)$, $t \in I$, where t is a parameter, is called *F-planar*, if its tangent vector $\lambda(t_0)$, for any initial value t_0 of the parameter t, remains, under parallel translation along the curve ℓ, in the distribution generated by the vector functions λ and $F\lambda$ along ℓ.

In accordance with this definition, ℓ is F-planar, if and only if the following condition holds ([21], see [22, p. 213]): $\nabla_{\lambda(t)}\lambda(t) = \varrho_1(t)\lambda(t) + \varrho_2(t)F\lambda(t)$, where ϱ_1 and ϱ_2 are some functions of the parameter t.

We suppose two spaces A_n and $\bar{A}_n$ with torsion-free affine connection ∇ and $\bar{\nabla}$, respectively. Affine structures F and $\bar{F}$ are defined on A_n, resp. $\bar{A}_n$.

Definition 2 (Mikeš, Sinyukov [21], see [22, p. 213])**.** A diffeomorphism f between manifolds with affine connection A_n and $\bar{A}_n$ is called an *F-planar mapping* if any F-planar curve in A_n is mapped onto an $\bar{F}$-planar curve in $\bar{A}_n$.

Due to the diffeomorphism f we always suppose that ∇, $\bar{\nabla}$, and the affinors F, $\bar{F}$ are defined on M where $A_n = (M, \nabla, F)$ and $\bar{A}_n = (M, \bar{\nabla}, \bar{F})$. The following holds.

Theorem 1. *An F-planar mapping f from A_n onto $\bar{A}_n$ preserves F-structures (i.e. $\bar{F} = a\,F + b\,\mathrm{Id}$, a,b are some functions), and is characterized by the following condition*

$$P(X,Y) = \psi(X)\cdot Y + \psi(Y)\cdot X + \varphi(X)\cdot FY + \varphi(Y)\cdot FX \tag{1}$$

for any vector fields X, Y, where $P = \bar{\nabla} - \nabla$ is the deformation tensor field of f, ψ and φ are some linear forms.

This theorem was proved by Mikeš and Sinyukov [21] for finite dimension $n > 3$, a more concise proof of this theorem for $n > 3$ and also a proof for $n = 3$ was given by I. Hinterleitner and Mikeš [8], see [22, p. 214].

We introduce the following classes of F-planar mappings from manifolds A_n with affine connection ∇ onto (pseudo-) Riemannian manifolds $\bar{V}_n$ with metric $\bar{g}$:

Definition 3 (Mikeš [17], see [22, p. 225])**.**

1. An F-planar mapping of a manifold A_n with affine connection onto a (pseudo-) Riemannian manifold $\bar{V}_n$ is called an F_1*-planar mapping* if the metric tensor satisfies the condition

$$\bar{g}(X, FX) = 0\,, \quad \text{for all} \quad X\,. \tag{2}$$

2. An F_1-planar mapping $A_n \to \bar{V}_n$ is called an F_2*-planar mapping* if the one-form ψ is gradient-like, i.e.

$$\psi(X) = \nabla_X \Psi\,, \tag{3}$$

where Ψ is a function on A_n.

3. An F_1-planar mapping $A_n \to \bar{V}_n$ is called an F_3*-planar mapping* if the one-forms ψ and φ are related by

$$\psi(X) = \varphi(FX)\,. \tag{4}$$

Remark. F-planar curves and F_1-planar mappings are a generalization of quasi-geodesic curves, resp. mappings by A. Z. Petrov [24], which he used for space-times.

3. Definitions and basic results of holomorphically projective mappings onto Kähler manifolds

(Pseudo-) Kähler manifolds were first considered by P.A. Shirokov and independently these manifolds were studied by E. Kähler, see [22, p. 68].

Definition 4. A (pseudo-) Riemannian manifold $\bar{K}_n = (M, \bar{g}, \bar{F})$ is called a (*pseudo-*) *Kähler manifold* if beside the tensor $\bar{g}$, a tensor field $\bar{F}$ of type (1,1) is given on M, such that the following conditions hold:

$$\text{(a)}\ \ \bar{F}^2 = -\,\mathrm{Id}, \qquad \text{(b)}\ \ \bar{g}(X, \bar{F}X) = 0 \text{ for all } X\,, \qquad \text{(c)}\ \ \bar{\nabla}\bar{F} = 0\,. \tag{5}$$

We remark that the formulas (1) – (4) hold for holomorphically projective mappings between (pseudo-) Kähler manifolds, see [4, 5, 16, 18, 22, 26]. For this reason we give the following definition

Definition 5. An F-planar mapping A_n onto a Kähler manifold $\bar{K}_n$ is called a *holomorphically projective mapping*, if it is F_3-planar.

By analysis of formulas (4) and (5c) we find that $\nabla\bar{F} = 0$. After differentiation of (5a), using (5c), and $\bar{F} = a\,F + b\,\mathrm{Id}$ (see Theorem 1), we find that $\bar{F} = \pm F$. Thus the following theorem holds.

Theorem 2. *If $A_n = (M, \nabla, F)$ admits holomorphically projective mappings onto a (pseudo-) Kähler manifold $\bar{K}_n = (M, \bar{g}, \bar{F})$, then $\bar{F} = \pm F$ and the structure F is a covariantly constant almost complex structure, i.e. $F^2 = -\,\mathrm{Id}$ and $\nabla F = 0$.*

From formulas (4) and (5a) follows that for holomorphically projective mappings $f\colon A_n \to \bar{K}_n$:

$$\varphi(X) = -\psi(FX) \quad \text{for all} \quad X\,.$$

From Theorem 2 and formulas (1), (5b) follows the following theorem.

Theorem 3. *Let $A_n = (M, \nabla, F)$ be a manifold M with affine connection ∇ and with a covariantly constant complex structure F. A diffeomorphism f from A_n onto a Kähler manifold $\bar{K}_n = (M, \bar{g}, F)$ is a holomorphically projective mapping if and only if*

$$(6)\qquad \begin{aligned} &\text{a)}\ \ P(X,Y) = \psi(X)\cdot Y + \psi(Y)\cdot X - \psi(FX)\cdot FY - \psi(FY)\cdot FX\,;\\ &\text{b)}\ \ \bar{g}(FX,FX) = g(X,X)\,, \end{aligned}$$

holds for any X, Y, where $P = \bar{\nabla} - \nabla$ is the deformation tensor field of f, ψ is a linear form.

In local notation formulas (6) have the following form:

$$(7)\qquad \text{a)}\quad \bar{\Gamma}^h_{ij}(x) = \Gamma^h_{ij}(x) + \delta^h_i\psi_j + \delta^h_j\psi_i - \delta^h_{\bar{i}}\psi_{\bar{j}} - \delta^h_{\bar{j}}\psi_{\bar{i}}\,, \qquad \text{b)}\quad \bar{g}_{\bar{i}\bar{j}} = g_{ij}\,,$$

where Γ^h_{ij}, $\bar{\Gamma}^h_{ij}$, $\bar{g}_{ij}$, ψ_i and F^h_i are the components of ∇, $\bar{\nabla}$, $\bar{g}$, ψ and F, respectively. Here and in the following we will use the conjugation operation of indices in the way

$$A^{\cdots}_{\cdots\ \bar{i}\ \cdots} = A^{\cdots}_{\cdots\ \alpha\ \cdots}\ F^{\alpha}_{i} \quad\text{and}\quad A^{\cdots\ \bar{i}\ \cdots}_{\cdots} = A^{\cdots\ \alpha\ \cdots}_{\cdots}\ F^{i}_{\alpha}\,.$$

Equations (7) are equivalent to the equations

$$(8)\quad \text{a)}\quad \nabla_k\bar{g}_{ij} = 2\psi_k\bar{g}_{ij} + \psi_i\bar{g}_{jk} + \psi_j\bar{g}_{ik} + \psi_{\bar{i}}\bar{g}_{\bar{j}\,k} + \psi_{\bar{j}}\bar{g}_{\bar{i}\,k}\,, \qquad \text{b)}\quad \bar{g}_{\bar{i}\bar{j}} = g_{ij}\,.$$

After contraction of (7) we obtain $\psi_i = \dfrac{1}{n+2}\,(\partial_i\sqrt{|\det\bar{g}|} - \Gamma^{\alpha}_{\alpha i})$, where $\partial_i = \dfrac{\partial}{\partial x^i}$.

Moreover, if ∇ is an equiaffine connection ([6], [22, p. 35]) then a function G exists for which $\Gamma^{\alpha}_{\alpha i} = \partial_i G$. In this case

$$(9)\qquad \psi_i = \partial_i\Psi, \quad \Psi = \frac{1}{n+2}\,(\sqrt{|\det\bar{g}|} - G).$$

Because the holomorphically projective mapping is F_3-planar, after elementary modifications we have the following theorem ([17], [22]):

Theorem 4. *Let A_n be a manifold with an equiaffine connection which satisfies the assumption of Theorem 3. A manifold A_n admits holomorphically projective mappings onto $\bar{K}_n$ if and only if a regular symmetric tensor a^{ij} and a vector λ^i satisfy the following equations:*

$$(10)\qquad \text{a)}\quad \nabla_k a^{ij} = \lambda^i\delta^j_k + \lambda^j\delta^i_k + \lambda^{\bar{i}}\delta^{\bar{j}}_k + \lambda^{\bar{j}}\delta^{\bar{i}}_k\,, \qquad \text{b)}\quad a^{\bar{i}\bar{j}} = a^{ij}\,.$$

From equations (8) we obtain (10) by the relations

$$a^{ij} = \mathrm{e}^{-2\Psi}\bar{g}^{ij}\,, \qquad \lambda^i = -\mathrm{e}^{-2\Psi}\bar{g}^{i\alpha}\psi_\alpha\,,$$

where $\|\bar{g}^{ij}\| = \|\bar{g}_{ij}\|^{-1}$. On the other hand from equations (10) we obtain (8) by the relations

$$\bar{g}^{ij} = \mathrm{e}^{2\Psi}a^{ij}\,, \qquad \Psi = \ln\sqrt{|\det\tilde{g}|} - G\,, \qquad \|\tilde{g}_{ij}\| = \|a^{ij}\|^{-1}\,.$$

Evidently, the results of Section 3 hold if

$$A_n = (M,\nabla,F) \in C^0 \qquad \big(\Gamma^h_{ij}(x) \in C^0,\ F^h_i(x) \in C^1\big)$$

and

$$\bar{K}_n = (M,\bar{g},F) \in C^1 \qquad \big(\bar{g}_{ij}(x) \in C^1\big)\,.$$

4. Holomorphically projective mappings from $A_n \in C^1$ onto $\bar{K}_n \in C^2$

Let $A_n = (M, \nabla, F)$ be a manifold M with an equiaffine connection ∇ and with a covariantly constant complex structure F and let A_n admit a holomorphically projective mapping onto the Kähler manifold $\bar{K}_n = (M, \bar{g}, F)$. We suppose that

$$A_n \in C^1 \left(\Gamma^h_{ij}(x) \in C^1,\ F^h_i(x) \in C^1\right) \text{ and } \bar{K}_n \in C^2 \left(\bar{g}_{ij}(x) \in C^2\right).$$

From $\nabla_j F^h_i = 0$ it follows that $F^h_i \in C^2$ and its integrability condition has the form $R^{\bar{h}}_{ijk} = R^h_{\bar{i}jk}$, where R^h_{ijk} is the cuvature tensor on A_n.

We shall investigate the integrability condition of equation (10). Let us differentiate it covariantly by x^l and then alternate it w.r. to the indices k and l. From the Ricci identity we find the following:

$$\begin{aligned}(11)\quad &\nabla_l \lambda^i \delta^j_k + \nabla_l \lambda^j \delta^i_k - \nabla_k \lambda^i \delta^j_l - \nabla_k \lambda^j \delta^i_l + \\ &\nabla_l \lambda^{\bar{i}} \delta^{\bar{j}}_k + \nabla_l \lambda^{\bar{j}} \delta^{\bar{i}}_k - \nabla_k \lambda^{\bar{i}} \delta^{\bar{j}}_l - \nabla_k \lambda^{\bar{j}} \delta^{\bar{i}}_l = -a^{\alpha i} R^j_{\alpha kl} - a^{\alpha j} R^i_{\alpha kl}\,.\end{aligned}$$

Contracting (11) by the indices j and k, we obtain

$$(12)\qquad (n-1)\ \nabla_l \lambda^i - \nabla_{\bar{l}} \lambda^{\bar{i}} = \mu \cdot \delta^i_l + \nabla_\alpha \lambda^{\bar{\alpha}} \cdot \delta^{\bar{i}}_l - a^{\alpha i} R_{\alpha l} - a^{\alpha\beta} R^i_{\alpha\beta l}\,,$$

where $\mu \overset{\text{def}}{=} \nabla_\alpha \lambda^\alpha$, $R_{ij} \overset{\text{def}}{=} R^\alpha_{i\alpha j}$ is the Ricci tensor, which is symmetric for the equiaffine connection ∇.

When we contract (12) with F^l_i and then use properties of the Riemann and the Ricci tensors, we can see $\nabla_\alpha \lambda^{\bar{\alpha}} = 0$. We apply the conjugation operation bar on the indices i and l, and subtract (12) from the result. After some calculations we have

$$n \cdot (\nabla_{\bar{l}} \lambda^{\bar{i}} - \nabla_l \lambda^i) = (a^{\alpha i} R_{\alpha l} + a^{\alpha\beta} R^i_{\alpha\beta l}) - (a^{\alpha \bar{i}} R_{\alpha \bar{l}} + a^{\alpha\beta} R^{\bar{i}}_{\alpha\beta\bar{l}})\,,$$

and from (12) we find

$$(13)\qquad n\, \nabla_l \lambda^i = \mu\, \delta^i_l - a^{\alpha\beta} T^i_{l\alpha\beta}\,,$$

where

$$T^i_{l\alpha\beta} \overset{\text{def}}{=} \frac{n-1}{n} (\delta^i_\beta R_{\alpha l} + R^i_{\alpha\beta l}) + \frac{1}{n} (\delta^{\bar{i}}_\beta R_{\alpha\bar{l}} + R^{\bar{i}}_{\alpha\beta\bar{l}})\,.$$

5. Holomorphically projective mappings from $A_n \in C^r$ $(r \geq 2)$ onto $\bar{K}_n \in C^2$

Let $A_n = (M, \nabla, F)$ be a manifold M with an equiaffine connection ∇ and with a covariantly constant complex structure F (i.e. $F^2 = -\operatorname{Id}$ and $\nabla F = 0$), which admits holomorphically projective mappings onto the Kähler manifold $\bar{K}_n = (M, \bar{g}, F)$. We suppose that

$$A_n \in C^{r-1}\ (\Gamma^h_{ij}(x) \in C^{r-1},\ r \geq 2,\ F^h_i(x) \in C^1) \text{ and } \bar{K}_n \in C^2\ (\bar{g}_{ij}(x) \in C^2)\,.$$

From $\nabla_j F^h_i = 0$ it follows that $F^h_i \in C^r$. We proof the following theorem

Theorem 5. *If $A_n \in C^{r-1}$ $(r \geq 2)$ admits holomorphically projective mappings onto $\bar{K}_n \in C^2$, then $\bar{K}_n \in C^r$.*

The proof of this theorem follows from the following lemmas.

Lemma 1 (see [11])**.** *Let $\lambda^h \in C^1$ be a vector field and ϱ a function. If $\partial_i \lambda^h - \varrho\, \delta_i^h \in C^1$, then $\lambda^h \in C^2$ and $\varrho \in C^1$.*

Lemma 2. *If $A_n \in C^2$ admits a holomorphically projective mapping onto $\bar{K}_n \in C^2$, then $\bar{K}_n \in C^3$.*

Proof. In this case equations (10) and (13) hold. According to our assumptions, $\Gamma_{ij}^h \in C^2$ and $\bar{g}_{ij} \in C^2$. By a simple check-up we find $\Psi \in C^2$, $\psi_i \in C^1$, $a^{ij} \in C^2$, $\lambda^i \in C^1$ and $R^h_{ijk}, R_{ij} \in C^1$.

From the above-mentioned conditions we easily convince ourselves that from equation (13) follows $\partial_l \lambda^i - \mu/n \in C^1$. From Lemma 1 follows that $\lambda^i \in C^2$, $\mu \in C^1$. Differentiating (10) twice we convince ourselves that $a_{ij} \in C^3$, and, evidently, also $\Psi \in C^3$ and $\bar{g}_{ij} \in C^3$. □

Further we covariantly differentiate (13) by x^m, and after alternation of the indices l and m and application of the Ricci identities and (10) we obtain:

$$(14) \qquad -n\lambda^\alpha R^i_{\alpha lm} = \delta^i_l \nabla_m \mu - \delta^i_m \nabla_l \mu - a^{\alpha\beta}(\nabla_m T^i_{l\alpha\beta} - \nabla_l T^i_{m\alpha\beta}) - \lambda^\alpha \Theta^i_{\alpha lm}\,,$$

where

$$\Theta^i_{\alpha lm} \stackrel{\text{def}}{=} T^i_{l\alpha m} + T^i_{lm\alpha} + T^i_{l\bar{\alpha}m} + T^i_{lm\bar{\alpha}} - T^i_{m\alpha l} - T^i_{ml\alpha} - T^i_{m\bar{\alpha}l} - T^i_{ml\bar{\alpha}}\,.$$

We contract formula (14) w.r. to the indices i and m, and we get

$$(15) \qquad (n-1)\,\nabla_l \mu = n\,\lambda^\alpha R_{\alpha l} - a^{\alpha\beta}(\nabla_\gamma T^\gamma_{l\alpha\beta} - \nabla_l T^\gamma_{\gamma\alpha\beta}) - \lambda^\alpha \Theta^\gamma_{\alpha l\gamma}\,.$$

The following theorem is the result of previous computations and Theorem 1.

Theorem 6. *Let A_n $(\in C^r, r \geq 2)$ be an equiaffine space with affine connection and let be defined a covariantly constant affinor F^h_i such that $F^h_\alpha F^\alpha_i = -\delta^h_i$. Then A_n admits a holomorphically projective mapping onto a Kählerian space $\bar{K}_n$ $(\in C^2)$ if and only if the following system of linear differential equations of Cauchy type is solvable with respect to the unknown functions a^{ij}, λ^i and μ:*

$$\nabla_k a^{ij} = \lambda^i \delta^j_k + \lambda^j \delta^i_k + \lambda^{\bar{i}} \delta^{\bar{j}}_k + \lambda^{\bar{j}} \delta^{\bar{i}}_k\,;$$

$$(16) \qquad n\,\nabla_l \lambda^i = \mu\,\delta^i_l - a^{\alpha\beta} T^i_{l\alpha\beta}\,;$$

$$(n-1)\,\nabla_l \mu = n\,\lambda^\alpha R_{\alpha l} - a^{\alpha\beta}(\nabla_\gamma T^\gamma_{l\alpha\beta} - \nabla_l T^\gamma_{\gamma\alpha\beta}) - \lambda^\alpha \Theta^\gamma_{\alpha l\gamma}\,,$$

where the matrix (a^{ij}) should further satisfy $\det\|a^{ij}\| \neq 0$ *and the algebraic conditions*

$$(17) \qquad a^{ij} = a^{ji}\,; \quad a^{\bar{i}\bar{j}} = a^{ij}\,.$$

Here T and Θ are tensors which are explicitly expressed in terms of objects defined on A_n, i.e. the affine connection A_n and the affinor F^h_i.

This theorem is a generalization of results in [5, 7, 1, 16, 19], see [18, 22, 26].

The system (16) does not have more than one solution for the initial Cauchy conditions $a^{ij}(x_o) = a_o^{ij}$, $\lambda^i(x_o) = \lambda_o^i$, $\mu(x_o) = \mu_o$ under the conditions (17). Therefore the general solution of (14) does not depend on more than $N_o = 1/4\,(n+1)^2$ parameters. The question of existence of a solution of (14) leads to the consideration of integrability conditions, which are linear equations w.r.t. to the unknowns a^{ij}, λ^i and μ with coefficient functions defined on the manifold A_n.

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