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ON THE QUEUE-SIZE DISTRIBUTION IN THE MULTI-SERVER SYSTEM WITH BOUNDED CAPACITY AND PACKET DROPPING

Oleg Tikhonenko and Wojciech M. Kempa

A multi-server $M/M/n$-type queueing system with a bounded total volume and finite queue size is considered. An AQM algorithm with the "accepting" function is being used to control the arrival process of incoming packets. The stationary queue-size distribution and the loss probability are derived. Numerical examples illustrating theoretical results are attached as well.

Keywords: AQM algorithms, loss probability, multi-server queueing system, queue-size distribution

Classification: 90B22, 60K25

1. PRELIMINARIES

Finite-buffer queueing models have wide applications in many real-life problems of technological or economical nature. Such systems can model the management process of accumulation and service of jobs, or be useful in logistics. As it seems, the main application concerns telecommunication networks where finite systems can describe the arrival/departure traffic of packets of data in the Internet routers (which have buffers of bounded capacity measured in bytes). In classical queueing systems the cases of single and batch (group) arrivals are being distinguished. In the latter situation the incoming packets (customers, calls etc.) occur in groups of random sizes. A generalization of a finite-buffer queueing system with group arrivals is a model in that the arriving packets occur individually but they have generally distributed volumes, and the total volume of the system (the sum of volumes of all present packets) is bounded (see e. g. [20, 21] and [22] for the theoretical results for such systems).

One of the most important problems occurring in the operation of the Internet router are losses of packets due to the buffer congestion. One of the possible solutions of this problem is the enlarging of the buffer volume. Of course, it can cause the prolongation of data transmission. Another approach is using one of the Active Queue Management (AQM) algorithms. The basic one was proposed in [10] and is called the Random Early Detection (RED). In the RED scheme the incoming packet can be blocked "by" a dropping function which deletes the packet with probability depending on the current queue size.

In fact, by using the AQM mechanism, the effect of reducing the queue size can be achieved in two ways:

- firstly, by the reduction of the buffer queue by using a dropping function;
- by the reduction of intensity of the input flow of packets as a reaction for dropping of the sent packet.

A linearly increasing dropping function was considered in [4]. In the literature other shapes of dropping functions can be found, e. g. an exponential (REM algorithm, see [1, 16]) and a quadratic one ([24]). A "gentle" version of a linear dropping function (GRED) was proposed in [11] and a dynamic RED algorithm (DRED) can be found in [2]. AQM algorithms in single-server models with finite buffer and single arrivals were studied in [5, 6, 7, 8] and [15]. In particular, in [7] the queue-size distribution was obtained for the $M/G/1/N$-type system with a dropping function. The problem of optimization of the shape of a dropping function to minimize the variance of the queue size was considered in [8]. Other variants and applications of AQM mechanisms can be found in [12, 13, 17, 18, 19] and [23].

In [14] the $M/M/1/N$ system with single and batch arrivals and a dropping function was investigated. The formulae for different steady-state characteristics were found: the queue-size distribution, the number of packets (group of packets) consecutively lost and the time between two successive losses. A generalization of the $M/M/1/N$ system with dropping was considered in [22], where the case of generally distributed packet volumes and the bounded total volume of the system was investigated. The representation for the stationary queue-size distribution was obtained there.

In the article we generalize results from [22] for the case of a multi-server system. Instead of the classical dropping function we introduce the "accepting" function (see [22]) that accepts the incoming packet with a probability depending on the occupied volume of the system at the pre-arrival instant, and (may be) on the volume of the arriving packet.

The paper is organized as follows. In the next Section 2 we give the mathematical description of the model, introduce some necessary notations and define the Markovian process describing the evolution of the original system. Section 3 contains main theoretical results. We write there down the system of Kolmogorov-type equations for the transient queue-size distribution and transform it to the form for the stationary state. Next, we find the explicit solution of the latter system. Moreover, as a corollary, we obtain there the formulae for the corresponding system without dropping. In the last Section 4 we present sample of illustrative numerical computations for different types of "accepting" functions and packet volume distributions.

2. QUEUEING MODEL AND AUXILIARY RESULTS

Let us consider a multi-server queueing system, denoted by $M/M/n/(m,V)$, in which successive packets arrive according to a Poisson process with intensity a and are served individually with the exponential distribution of service time, with mean μ^{-1}. As it is usually being done, we assume that sequences of successive interarrival and service times are totally independent random variables. We assume that the ith arriving packet has a volume ζ_i, being a random variable with a general-type distribution function $L(\cdot)$, and the total volume of the system, i.e. the sum of volumes of all packets present in the system, is bounded by a non-random value V.

Moreover, we assume that the system contains n identical independent servers and one finite buffer with m places. Thus, the total number of packets present in the system is bounded by $m+n$ (we can also analyze the case of $m=\infty$, if $V<\infty$).

Let $r(\cdot)$ be a right-hand continuous non-increasing function defined on the interval $[0,V]$, and such that $r(0)\le 1$ and $r(V)\ge 0$. We call $r(\cdot)$ the "accepting" function. Assume that the arriving packet is characterized by a volume x, while the total volume of the system at the pre-arrival epoch equals y.

Later on, we shall analyze two following types of packet dropping system $M/M/n/(m,V)$ "behavior": 1) the incoming packet is "qualified" for service with probability $r(x+y)$ and deleted with probability $1-r(x+y)$; 2) the incoming packet is "qualified" for service with probability $r(y)$ and deleted with probability $1-r(y)$. Moreover, each packet will be lost if $x+y>V$, or if the number of packets present in the system at the pre-arrival epoch equals $m+n$.

Let us denote by $\eta(t)$ the number of packets present in the system at time t and by $\sigma(t)$ – the total volume of the system at time t. Then, if the packet arriving at time t is dropped, we have $\eta(t)=\eta(t^-)$ and $\sigma(t)=\sigma(t^-)$. In the case of acceptance of the arriving packet we have $\eta(t)=\eta(t^-)+1$ and $\sigma(t)=\sigma(t^-)+x$.

Define

$$P_k(t)=\mathbf{P}\{\eta(t)=k\},\quad p_k=\mathbf{P}\{\eta=k\}, \tag{1}$$

where $k=0,\dots,m+n$, and η stands for the number of packets present in the system in the stationary state.

It is easy to note that the evolution of the original queueing system can be described by using the following Markovian process (see [22]):

$$\big(\eta(t),\,\zeta_i(t),\,i=1,\dots,\eta(t)\big), \tag{2}$$

where $\zeta_i(t)$ denotes the volume of the ith packet present in the system at time t. Note that $\sigma(t)=\sum_{i=1}^{\eta(t)}\zeta_i(t)$. Here we take the assumption that the arriving packets are numbered successively as they occur. Of course, if $\eta(t)=0$ then also $\sigma(t)=0$.

If we define additionally

$$G_k(y,t)dy=\mathbf{P}\{\eta(t)=k,\,\sigma(t)\in[y,y+dy)\},\quad k=1,\dots,n+m, \tag{3}$$

then we have

$$P_k(t)=\mathbf{P}\{\eta(t)=k\}=\int_0^V G_k(y,t)\,\mathrm{d}y,\quad k=1,\dots,n+m. \tag{4}$$

To supplement necessary notations let us denote by p_{loss} the stationary loss probability i.e. the probability that the incoming packet is lost (due to the buffer overflow or dropping "via" the function $r(\cdot)$). Let $\rho=\frac{a}{n\mu}$ be the traffic load of the system. Lastly, by $A^{k*}(\cdot)$ we denote the k-fold Stieltjes convolution of a function $A(\cdot)$, which is defined on the interval $[0,\infty)$, with itself i. e.

$$A^{0*}(y) \equiv 1, \qquad A^{k*}(y) = \int_0^y A^{(k-1)*}(y-x)\,\mathrm{d}A(x), \quad k = 1,2,\dots.$$

3. STATIONARY DISTRIBUTION IN THE SYSTEM WITH PACKET DROPPING

Under the notations introduced in Section 2 for the first type of system "behavior" we can build the following system of Kolmogorov-type equations for the transient queue-size distribution $P_k(t)$, $k = 0,\dots,n+m$, in the $M/M/n/(m,V)$ system with "accepting" function $r(\cdot)$:

$$P_0'(t) = -aP_0(t)\int_0^V r(x)\,\mathrm{d}L(x) + \mu P_1(t); \tag{5}$$

$$P_1'(t) = aP_0(t)\int_0^V r(x)\,\mathrm{d}L(x) - a\int_0^V G_1(y,t)\int_0^{V-y} r(x+y)\,\mathrm{d}L(x)\,\mathrm{d}y - \mu P_1(t) + 2\mu P_2(t); \tag{6}$$

$$P_k'(t) = a\int_0^V G_{k-1}(y,t)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y - a\int_0^V G_k(y,t)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y - k\mu P_k(t) + (k+1)\mu P_{k+1}(t),$$
$$k = 2,\dots,n-1; \tag{7}$$

$$P_k'(t) = a\int_0^V G_{k-1}(y,t)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y - a\int_0^V G_k(y,t)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y - n\mu P_k(t) + n\mu P_{k+1}(t),$$
$$k = n,\dots,n+m-1; \tag{8}$$

$$P_{n+m}'(t) = a\int_0^V G_{n+m-1}(y,t)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y - n\mu P_{n+m}(t). \tag{9}$$

Using the same notations, taking in (5) – (9) the limit $t \to \infty$, we obtain the following corresponding system for the stationary probabilities p_k, $k = 0,\dots,n+m$:

$$0 = -ap_0\int_0^V r(x)\,\mathrm{d}L(x) + \mu p_1; \tag{10}$$

$$0 = ap_0\int_0^V r(x)\,\mathrm{d}L(x) - a\int_0^V g_1(y)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y - \mu p_1 + 2\mu p_2; \tag{11}$$

$$0 = a\int_0^V g_{k-1}(y)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y$$
$$-\,a\int_0^V g_k(y)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y - k\mu p_k + (k+1)\mu p_{k+1},$$
$$k = 2,\dots,n-1; \tag{12}$$
$$0 = a\int_0^V g_{k-1}(y)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y$$
$$-\,a\int_0^V g_k(y)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y - n\mu p_k + n\mu p_{k+1},$$
$$k = n,\dots,n+m-1; \tag{13}$$
$$0 = a\int_0^V g_{n+m-1}(y)\int_0^{V-y} r(y+x)\,\mathrm{d}L(x)\,\mathrm{d}y - n\mu p_{n+m}, \tag{14}$$

where

$$g_k(y) = \lim_{t\to\infty} G_k(y,t). \tag{15}$$

For brevity in writing, let us denote

$$R(z) = \int_0^z r(V-z+x)\,\mathrm{d}L(x). \tag{16}$$

Now, the system (10) – (14) can be rewritten in the form

$$0 = -ap_0R(V) + \mu p_1; \tag{17}$$
$$0 = ap_0R(V) - a\int_0^V g_1(y)R(V-y)\,\mathrm{d}y - \mu p_1 + 2\mu p_2; \tag{18}$$
$$0 = a\int_0^V g_{k-1}(y)R(V-y)\,\mathrm{d}y - a\int_0^V g_k(y)R(V-y)\,\mathrm{d}y$$
$$-\,k\mu p_k + (k+1)\mu p_{k+1}, \qquad k = 2,\dots,n-1; \tag{19}$$
$$0 = a\int_0^V g_{k-1}(y)R(V-y)\,\mathrm{d}y - a\int_0^V g_k(y)R(V-y)\,\mathrm{d}y$$
$$-\,n\mu p_k + n\mu p_{k+1}, \qquad k = n,\dots,n+m-1; \tag{20}$$
$$0 = a\int_0^V g_{n+m-1}(y)R(V-y)\,\mathrm{d}y - n\mu p_{n+m}. \tag{21}$$

The equations of the system (17) – (21) lead to the following theorem:

Theorem 3.1. The stationary probabilities p_k, $k = 0,\dots,m+n$, in the $M/M/n/(m,V)$-type queueing system with packet dropping are given by the following formulae:

$$p_k = \begin{cases} \frac{(n\rho)^k}{k!}R^{k*}(V)p_0, & k = 1,\dots,n, \\ \frac{n^n\rho^k}{n!}R^{k*}(V)p_0, & k = n+1,\dots,n+m, \end{cases} \tag{22}$$

where ρ is the traffic load, and

$$p_0 = \Big[\sum_{k=0}^{n} \frac{(n\rho)^k}{k!} R^{k*}(V) + \frac{n^n}{n!} \sum_{k=n+1}^{n+m} \rho^k R^{k*}(V)\Big]^{-1}. \tag{23}$$

P r o o f. Let N_k be defined by the relation:

$$N_k = \begin{cases} \frac{(n\rho)^k}{k!}, & k = 1, \dots, n, \\ \frac{n^n \rho^k}{n!}, & k = n+1, \dots, n+m. \end{cases}$$

Let C be a certain constant. It is easy to verify, by directly substitution, that the functions $g_k(y)$ defined by the identity

$$g_k(y)\,\mathrm{d}y = CN_k\,\mathrm{d}R^{k*}(y),$$

whence we obviously have

$$p_k = \int_0^V g_k(y)\,\mathrm{d}y = CN_k R^{k*}(V),$$

where $R(\cdot)$ was defined in (16), satisfy the system of equations (17) – (21).

The value of constant $C = p_0$ can be easily found using the normalization condition $\sum_{k=0}^{n+m} p_k = 1$. □

For the second type of system "behavior" (see Section 2) we have to replace the system (10) – (14) by the following equations:

$$0 = -ap_0 r(0) L(V) + \mu p_1; \tag{24}$$

$$0 = ap_0 r(0) L(V) - a\int_0^V g_1(x) r(x) L(V-x)\,\mathrm{d}x - \mu p_1 + 2\mu p_2; \tag{25}$$

$$0 = a\int_0^V g_{k-1}(x) r(x) L(V-x)\,\mathrm{d}x - a\int_0^V g_k(x) r(x) L(V-x)\,\mathrm{d}x \\ - k\mu p_k + (k+1)\mu p_{k+1}, \qquad k = 2, \dots, n-1; \tag{26}$$

$$0 = a\int_0^V g_{k-1}(x) r(x) L(V-x)\,\mathrm{d}x - a\int_0^V g_k(x) r(x) L(V-x)\,\mathrm{d}x \\ - n\mu p_k + n\mu p_{k+1}, \qquad k = n, \dots, n+m-1; \tag{27}$$

$$0 = a\int_0^V g_{n+m-1}(x) r(x) L(V-x)\,\mathrm{d}x - n\mu p_{n+m}. \tag{28}$$

After introducing the relation

$$R(z) = r(V-z) L(z), \tag{29}$$

we can rewrite the equations (24) – (28) in the form that coincides with the system (17) – (21). Therefore, in this case we obtain the solution in the form (22) – (23), where $R(z)$ is determined by the relation (29).

Taking $L(y) = 0$ for $y < 1$, and $L(y) = 1$ for $y \geq 1$ (all the arriving packets have constant volumes equal 1) and assuming that $V = m+n$ and $r(0), \dots, r(m+n-1)$ are numbers which are equal to probabilities to be accepted for service for an arriving packet that meets at the arriving epoch $0, \dots, m+n-1$ other packets in the system respectively, we obtain the "classical" system of the $M/M/n/m$-type with AQM as a special case of the second type of system "behavior". For this case we obtain the following corollary:

Corollary 3.2. For the stationary probabilities p_k in the $M/M/n/m$-type queueing system with packet dropping the following representations are true:

$$p_k = \begin{cases} \frac{(n\rho)^k}{k!} p_0 \prod_{i=0}^{k-1}(1-d_i), & 0 < k \leq n, \\ \frac{n^n \rho^k}{n!} p_0 \prod_{i=0}^{k-1}(1-d_i), & n < k \leq n+m, \end{cases} \tag{30}$$

where

$$p_0 = \left[\sum_{k=0}^{n} \frac{(n\rho)^k}{k!} \prod_{i=0}^{k-1}(1-d_i) + \frac{n^n}{n!} \sum_{k=n+1}^{n+m} \rho^k \prod_{i=0}^{k-1}(1-d_i) \right]^{-1} \tag{31}$$

and $d_i = 1 - r(i)$, $i = 0, 1, \dots, m+n-1$, denotes here the "typical" dropping function (see e. g. [4]).

P r o o f. In this case the equations (24) – (28) take the following form:

$$0 = -ap_0 r(0) + \mu p_1; \tag{32}$$
$$0 = ap_{k-1} r(k-1) - ap_k r(k) - k\mu p_k + (k+1)\mu p_{k+1}, \ k = 1, \dots, n-1; \tag{33}$$
$$0 = ap_{k-1} r(k-1) - ap_k r(k) - n\mu p_k + n\mu p_{k+1}, \ k = n, \dots, n+m-1; \tag{34}$$
$$0 = ap_{n+m-1} r(n+m-1) - n\mu p_{n+m}. \tag{35}$$

It can be proved by direct substitution that the solution of the system (32) – (35) has the form

$$p_k = \begin{cases} C \frac{(n\rho)^k}{k!} \prod_{i=0}^{k-1} r(i), & 0 < k \leq n, \\ C \frac{n^n \rho^k}{n!} \prod_{i=0}^{k-1} r(i), & n < k \leq n+m, \end{cases}$$

where $C = p_0$ is some constant value.

It is clear that the formulae (30) – (31) follow from the last relation and the normalization condition. □

Let us note that the formulae (30) – (31) significantly generalize the result obtained in [14] for the $M/M/1/m$-type queueing system with packet dropping.

As a corollary from Theorem 3.1 or from the relations (22) – (23) we also derive the following result for both types of system behavior:

Corollary 3.3. For the stationary queue-size distribution $\widehat{p}_k$, $k = 0, \dots, n+m$, in the $M/M/n/(m, V)$-type system without dropping of packets ($r(x) \equiv 1$, $x \in [0, V]$) the following representations hold true:

$$\widehat{p}_k = \begin{cases} \frac{(n\rho)^k}{k!}\widehat{p}_0 L^{k*}(V), & k = 1, \ldots, n, \\ \frac{n^n \rho^k}{n!}\widehat{p}_0 L^{k*}(V), & k = n+1, \ldots, n+m, \end{cases} \tag{36}$$

where $L(\cdot)$ stands for the distribution function of the packet volume.

Moreover

$$\widehat{p}_0 = \left[\sum_{k=0}^{n} \frac{(n\rho)^k}{k!} L^{k*}(V) + \frac{n^n}{n!} \sum_{k=n+1}^{n+m} \rho^k L^{k*}(V)\right]^{-1}. \tag{37}$$

Remark 3.4. Let us note that for some special forms of the distribution function $L(\cdot)$ of volumes of the arriving packets, the Stieltjes convolutions $L^{k*}(V)$, occurring in (36) and, in consequence, probabilities $\widehat{p}_k$, can be found analytically.

It is possible e. g. for uniformly distributed volumes of packets (see e. g. [9]). Besides, obviously, in the case of exponential distributions $L(\cdot)$ of packet volumes, $L^{k*}(\cdot)$ is a k-Erlang distribution function.

Note that for $V = \infty$ and $m < \infty$ we obtain the well-known formulae for stationary probabilities $p_k = \mathbf{P}\{\eta = k\}$, $k = 0, \ldots, n+m$, in the classical system $M/M/n/m$ ([3]), and for all systems under consideration (with and without AQM) we have the following relationship for the loss probability:

$$p_{\text{loss}} = 1 - (n\rho)^{-1} \sum_{k=1}^{n-1} k p_k - \rho^{-1}\Big(1 - \sum_{k=0}^{n-1} p_k\Big). \tag{38}$$

4. NUMERICAL RESULTS

In this section we present sample of numerical results illustrating the formulae (22), (23) and (36), (37). In computations we use the three following "accepting" functions:

- $r(v) = \frac{V^2 - v^2}{V^2}, \quad 0 \le v \le V;$
- $r(v) = \frac{V-v}{V}, \quad 0 \le v \le V;$
- $r(v) = \left(\frac{V-v}{V}\right)^2, \quad 0 \le v \le V.$

Besides, we consider three different types of distribution functions describing the volumes of the arriving packets: deterministic (in which the volume of each arriving packet is constant and equal one), exponential and the Erlang one.

All numerical results are obtained using the *Mathematica* environment.

4.1. Deterministic distribution of the packet volume

Let us take into consideration the system in which all the arriving packets have constant volumes equal one i. e. we have

$$L(x) = \begin{cases} 0, & x < 1, \\ 1, & x \ge 1. \end{cases}$$

k	$\widehat{p}_k$	p_k
0	1.589460×10^{-7}	4.428151×10^{-6}
1	1.907352×10^{-6}	5.313781×10^{-5}
2	1.144411×10^{-5}	3.156386×10^{-4}
3	4.577645×10^{-5}	1.212052×10^{-3}
4	1.831058×10^{-4}	4.411870×10^{-3}
5	7.324232×10^{-4}	0.0148239
6	2.929693×10^{-3}	0.0444717
7	0.0117188	0.113847
8	0.0468751	0.232249
9	0.187500	0.334438
10	0.750001	0.254173

Tab. 1 Queue-size distributions for the system without and with dropping for deterministic packet volume distribution and $\rho = 4$.

k	$\widehat{p}_k$	p_k
0	6.743111×10^{-10}	2.998568×10^{-8}
1	1.416053×10^{-8}	6.296993×10^{-7}
2	1.486856×10^{-7}	6.545725×10^{-6}
3	1.040799×10^{-6}	4.398727×10^{-5}
4	7.285594×10^{-6}	2.801989×10^{-4}
5	5.099916×10^{-5}	1.647570×10^{-3}
6	3.569941×10^{-4}	8.649740×10^{-3}
7	2.498959×10^{-3}	0.0387508
8	0.0174927	0.138340
9	0.122449	0.348618
10	0.857143	0.463662

Tab. 2. Queue-size distributions for the system without and with dropping for deterministic packet volume distribution and $\rho = 7$.

In fact, such a model for $n = 3$, $m = 7$ and $V = 10$ corresponds to the usual $M/M/3/7$-type system.

Besides, let us take $n = 3$, $m = 7$ and $V = m + n = 10$. Moreover, choose

$$r(v) = \frac{V^2 - v^2}{V^2}, \quad 0 \le v \le V$$

as the "accepting" function. In Table 1 we present stationary queue-size distributions for the system without and with packet dropping for $\rho = 4$. These two distributions are compared graphically in Figure 1 (in brighter colour the case of the system without packet dropping). Similarly, the case of $\rho = 7$ is presented in Table 2 and Figure 2.

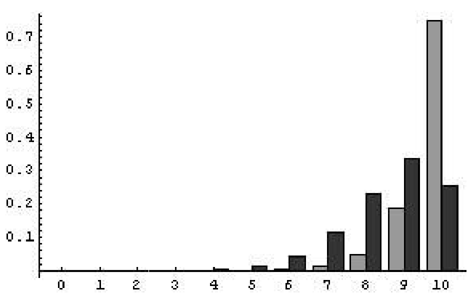

Fig. 1. Queue-size distributions for the system without and with dropping for deterministic packet volume distribution and $\rho = 4$.

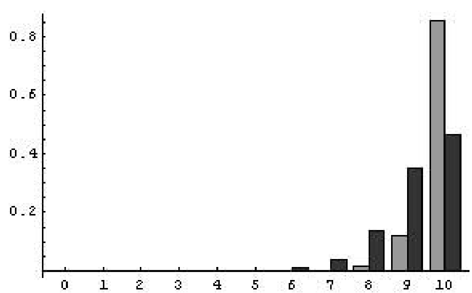

Fig. 2. Queue-size distributions for the system without and with dropping for deterministic packet volume distribution and $\rho = 7$.

k	$\widehat{p}_k$	p_k
0	1.118552×10^{-3}	0.0913135
1	6.666093×10^{-3}	0.439043
2	0.0193200	0.358058
3	0.0352484	0.0951918
4	0.0591917	0.0147866
5	0.0901206	1.495264×10^{-3}
6	0.123716	1.058562×10^{-4}
7	0.153222	5.527017×10^{-6}
8	0.171859	2.213164×10^{-7}
9	0.175487	7.005938×10^{-9}
10	0.164051	1.796249×10^{-10}

Tab. 3. Queue-size distributions for the system without and with dropping for exponential packet volume distribution and $\rho = 2$.

k	$\widehat{p}_k$	p_k
0	4.250296×10^{-7}	0.0161122
1	6.332486×10^{-6}	0.193672
2	4.588275×10^{-5}	0.394868
3	2.092774×10^{-4}	0.262445
4	8.785849×10^{-4}	0.101917
5	3.344160×10^{-3}	0.0257654
6	0.0114770	4.560112×10^{-3}
7	0.0355356	5.952369×10^{-4}
8	0.0996450	5.958716×10^{-5}
9	0.254371	4.715691×10^{-6}
10	0.594486	3.022633×10^{-7}

Tab. 4. Queue-size distributions for the system without and with dropping for exponential packet volume distribution and $\rho = 5$.

4.2. Exponential distribution of the packet volume

Now for the first type of system "behavior" let us consider exponentially distributed volumes of the incoming packets i. e.

$$L(x) = 1 - e^{-0.5x}, \quad x > 0.$$

Moreover, let us take, as previously, $n = 3$, $m = 7$ and $V = 10$, and define the "accepting" function as follows:

$$r(v) = \frac{V - v}{V}, \quad 0 \le v \le V.$$

In Table 3 we present stationary queue-size distributions for the system with and without packet dropping for $\rho = 2$. These two distributions are compared graphically in Figure 3 (as previously, in brighter colour the case of the system without packet dropping). The case of $\rho = 5$ is presented in Table 4 and Figure 4.

4.3. Erlang distribution of the packet volume

Lastly (also for the first type of system "behavior"), let us assume that the volume of the arriving packet has 2-Erlang distribution with parameter $\lambda = 1$. So, we have

$$L(x) = 1 - e^{-x}(1 + x), \quad x > 0.$$

Let us take, as in the previous two examples, $n = 3$, $m = 7$ and $V = 10$, and define the

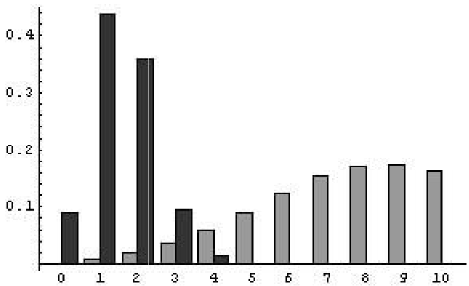

Fig. 3. Queue-size distributions for the system without and with dropping for exponential packet volume distribution and $\rho = 2$.

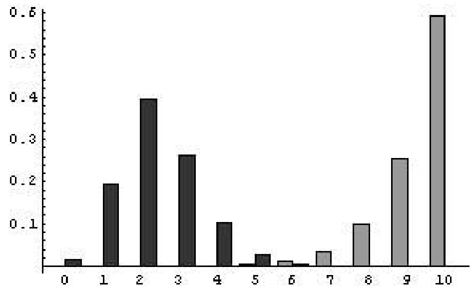

Fig. 4. Queue-size distributions for the system without and with dropping for exponential packet volume distribution and $\rho = 5$.

"accepting" function as

$$r(v) = \left(\frac{V - v}{V} \right)^2, \quad 0 \leq v \leq V.$$

Stationary queue-size distributions for the "pure" system and in the case of packet dropping are given in Tables 5 and 6 for $\rho = 3$ and $\rho = 6$, respectively. The distributions are visualized in Figures 5 and 6.

k	$\widehat{p}_k$	p_k
0	1.430501×10^{-4}	0.119582
1	1.286809×10^{-3}	0.710302
2	5.733648×10^{-3}	0.164611
3	0.0162146	5.447440×10^{-3}
4	0.0406591	5.815649×10^{-5}
5	0.0847935	2.541149×10^{-7}
6	0.142296	5.315823×10^{-10}
7	0.190811	5.948273×10^{-13}
8	0.205854	3.866425×10^{-16}
9	0.180904	1.479858×10^{-19}
10	0.131304	2.029519×10^{-20}

Tab. 5. Queue-size distributions for the system without and with dropping for 2-Erlang packet volume distribution and $\rho = 3$.

k	$\widehat{p}_k$	p_k
0	4.515084×10^{-7}	0.0533097
1	8.123097×10^{-6}	0.633308
2	7.238835×10^{-5}	0.293536
3	4.094244×10^{-4}	0.0194279
4	2.053312×10^{-3}	4.148208×10^{-4}
5	8.564263×10^{-3}	3.625120×10^{-6}
6	0.0287441	1.516676×10^{-8}
7	0.0770889	3.394245×10^{-11}
8	0.166333	4.412573×10^{-14}
9	0.292344	3.377787×10^{-17}
10	0.424382	9.264787×10^{-18}

Tab. 6. Queue-size distributions for the system without and with dropping for 2-Erlang packet volume distribution and $\rho = 6$.

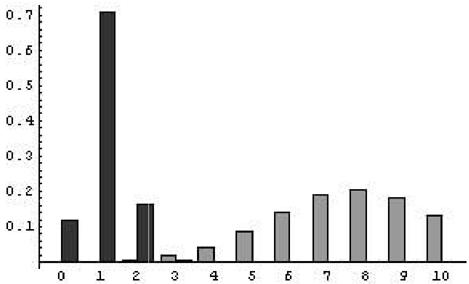

Fig. 5. Queue-size distributions for the system without and with dropping for 2-Erlang packet volume distribution and $\rho = 3$.

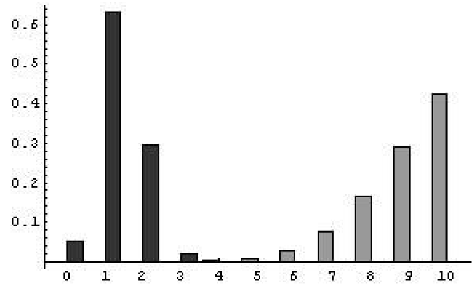

Fig. 6. Queue-size distributions for the system without and with dropping for 2-Erlang packet volume distribution and $\rho = 6$.

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