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ALMOST PERIODIC SOLUTIONS FOR A CLASS OF DISCRETE SYSTEMS WITH ALLEE-EFFECT

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Abstract. In this paper, using Mawhin's continuation theorem of the coincidence degree theory, we obtain some sufficient conditions for the existence of positive almost periodic solutions for a class of delay discrete models with Allee-effect.

Keywords: discrete system; coincidence degree; almost periodic solution; Allee-effect

MSC 2010: 34K14, 92D25

1. Introduction

In the past few years, there has been increasing interest in studying dynamical characteristics such as stability, persistence and periodicity of ecological systems. In paper [4], the author proposed the single species model with Allee-effect

$$\dot{N}(t) = N(t)[a(t) - b(t)N^p(t - \sigma(t)) - c(t)N^q(t - \tau(t))], \tag{1.1}$$

where $a \in C(\mathbb{R}, \mathbb{R})$ and $b, c, \sigma, \tau \in C(\mathbb{R}, [0, +\infty))$ are ω-periodic functions with $\int_0^\omega a(t)\,\mathrm{d}t > 0$ and $\int_0^\omega (b(t) + c(t))\,\mathrm{d}t > 0$, $p \leqslant q$ are positive constants. Using the method of coincidence degree, the author obtained some conditions ensuring the existence of at least one positive periodic solution for (1.1). His results show that delays have no influence on the existence of a positive periodic solution of (1.1). Later on, (1.1) has been extensively studied. For instance, in paper [6], the authors investigated the permanence and attractivity of (1.1) by some analytic technique

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using a suitable Lyapunov functional. For more results related to (1.1), one can refer to [24], [21], [25] and the references cited therein.

Naturally, upon considering long-term dynamical behaviors, it is possible for the various components of biological and physical environment (reproduction rates, resource regeneration, etc.) of a population model to be periodic with rationally independent periods. Therefore, the study of almost periodic behavior is considered to be more accordant with reality. Recently, there are two main approaches to investigating the existence and stability of the almost periodic solutions of differential systems: one is using the fixed point theorem, Lyapunov functional method and differential inequality techniques (see [5], [14], [7]); the other is using functional hull theory and Lyapunov functional method (see [18], [16], [17]). We always apply the latter way to studying the almost periodic solutions for ecological systems, especially for discrete systems, in which we need first to study the persistence of the systems considered. In [23], [2], [22], applying the method of coincidence degree theory which is different from the previous results, the authors studied the almost periodic solutions for some classes of Lotka-Volterra systems. However, all of them only considered the continuous models.

In reality, the discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have non-overlapping generations. Also, since discrete time models can also provide efficient computational models of continuous models for numerical simulations, it is reasonable to study discrete time population models governed by difference equations (see [1], [11], [19], [20], [15], [13], [9], [27], [10]). Moreover, many authors have used Mawhin's continuation theorem to study the existence of periodic solutions to population models ([20], [15], [13], [9], [27], [10]). For example, in [20], the authors considered the discrete nonlinear delay population model with Allee effect

$$x(n+1) = x(n)\exp\{a(n) + b(n)x^p(n-\omega) - c(n)x^q(n-\omega)\}, \tag{1.2}$$

where $a(n), b(n)$ and $c(n)$ are positive sequences of period ω and p and q are positive integers. By using Mawhin's continuation theorem, they established a sufficient condition for the existence of a positive periodic solution to (1.2). However, few papers have been published on the existence of positive almost periodic solutions to discrete time population models which are done by using the Mawhin's continuation theorem.

Motivated by the above, in this paper we will study the discrete system of (1.1),

$$y(n+1) = y(n)\exp\{a(n) - b(n)y^p(n-\sigma(n)) - c(n)y^q(n-\tau(n))\}, \tag{1.3}$$

where $n \in \mathbb{Z}$, p and q are constants with $0 < p \leqslant q$, $a(n)$, $b(n)$, $\sigma(n)$, $\tau(n)$ are all almost periodic sequences defined on $\mathbb{Z}$ with $b(n) \geqslant 0$, $c(n) \geqslant 0$ for all $n \in \mathbb{Z}$.

Applying the coincidence degree theory, we will study the existence of positive almost periodic solutions of (1.3). To the best of our knowledge, this is the first paper to study the existence of almost periodic solutions to (1.3) by using the method of coincidence degree theory.

The organization of the paper is as follows. In Section 2, we introduce some preliminary results which are needed later. In Section 3, we establish our main results for the existence of positive almost periodic solutions of (1.3).

2. Preliminaries

In this section we state some preliminary results.

Definition 2.1 ([3]). A sequence $x\colon \mathbb{Z} \to \mathbb{R}$ is called an almost periodic sequence, if the ε-translation number set of x

$$E(\varepsilon, x) = \{\tau \in \mathbb{Z}\colon |x(n+\tau) - x(n)| < \varepsilon, \ \forall n \in \mathbb{Z}\}$$

is relatively dense, that is, for any $\varepsilon > 0$ there exists an integer $l(\varepsilon) > 0$ such that each discrete interval of length $l(\varepsilon)$ contains an integer $\tau \in E(\varepsilon, x)$ such that $|x(n+\delta) - x(n)| < \varepsilon$ for any $n \in \mathbb{Z}$. τ is called the ε-translating number of ε-almost period.

Definition 2.2 ([3]). Let $f\colon \mathbb{Z} \times D \to \mathbb{R}$, where D is an open set in $\mathbb{R}$, $f(n,x)$ is said to be almost periodic in n uniformly for $x \in D$, or uniformly almost periodic for short, if for any $\varepsilon > 0$ and any compact $S \in D$ there exists a positive integer $l(\varepsilon, S)$ such that any interval of length $l(\varepsilon, S)$ contains an integer τ for which

$$|f(n+\tau, x) - f(n,x)| < \varepsilon$$

for any $n \in \mathbb{Z}$ and $x \in S$. τ is called the ε-translating number of $f(n,x)$.

Lemma 2.1 ([26]). *The following statements are true.*

(i) *If $x(n)$ is an almost periodic sequence, then $x(n)$ is bounded.*
(i) *If $f(n,x)$ is almost periodic uniformly in n, then $f(n,x)$ is bounded in n.*

For convenience, we denote by $\mathrm{AP}(\mathbb{Z})$ the set of all real valued, almost periodic functions on $\mathbb{Z}$. Suppose $f(n,\varphi)$ is almost periodic in n, uniformly with respect to $\varphi \in C([-r,0]_Z, \mathbb{R})$. Further $T(f,\varepsilon,S)$ denotes the set of ε-almost periods with respect to $\varphi \in C([-r,0]_Z, \mathbb{R})$ and $l(\varepsilon, S)$ is the inclusion interval.

For $f \in \mathrm{AP}(\mathbb{Z})$, denote

$$m[f] = \lim_{N\to+\infty} \frac{1}{N} \sum_{n=0}^{N} f(n).$$

Lemma 2.2 ([26]). *Let $f(n,\varphi)$ and $g(n,\varphi)$ be almost periodic in n uniformly for $\varphi \in D$. For any sequence $\{\tau_k\}$, which has a limit (including infinite one), if $f(n+\tau_k,\varphi)$ uniformly converges on $\mathbb{Z}\times S$ as $k\to\infty$ implies that $g(n+\tau_k,\varphi)$ uniformly converges on $\mathbb{Z}\times S$ as $k\to\infty$, where S is any compact set of D, then* $\operatorname{mod}(g) \subset \operatorname{mod}(f)$.

Lemma 2.3 ([26]). *Let $f\colon \mathbb{Z}\times D \to \mathbb{R}$ be almost periodic in n uniformly for $\varphi \in D$ and continuous in $\varphi \in D$. If $p(n)$ is an almost periodic sequence such that $p_n \in S$ for all $n \in \mathbb{Z}$, where S is a compact set in D and $p_n(s) = p(n+s)$ for $s \in [-r,0]_{\mathbb{Z}}$, then $f(n,p_n)$ is almost periodic in n.*

Similarly to the case of a periodic sequence, we have the following lemma which plays an important role in our results.

Lemma 2.4. *Let $x \in \mathrm{AP}(\mathbb{Z})$ and $k_0 \in \mathbb{Z}$. Then for any $\varepsilon > 0$ with inclusion length $l(\varepsilon)$ and any $k_1, k_2 \in [k_0, k_0 + l(\varepsilon)]_{\mathbb{Z}}$ we have*

$$x(k) \leqslant x(k_1) + \sum_{s=k_0}^{k_0+l(\varepsilon)} |x(s+1) - x(s)| + \varepsilon \quad \forall k \in \mathbb{Z}$$

and

$$x(k) \geqslant x(k_2) - \sum_{s=k_0}^{k_0+l(\varepsilon)} |x(s+1) - x(s)| - \varepsilon \quad \forall k \in \mathbb{Z}.$$

P r o o f. For any $k \in \mathbb{Z}$ and $\varepsilon > 0$, since $x(n)$ is almost periodic, there exists an integer $\tau \in E(\varepsilon, x)$ such that $k \in [k_0 - \tau, k_0 - \tau + l(\varepsilon)]_{\mathbb{Z}}$, that is $k + \tau \in [k_0, k_0 + l(\varepsilon)]_{\mathbb{Z}}$. Thus, we have

$$\begin{aligned} x(k) - x(k_1) &= \sum_{s=k_1}^{k-1} (x(s+1) - x(s)) \\ &= \sum_{s=k_1}^{k-1+\tau} (x(s+1) - x(s)) + \sum_{s=k-1+\tau+1}^{k-1} (x(s+1) - x(s)) \\ &\leqslant \sum_{s=k_0}^{k_0+l(\varepsilon)} |x(s+1) - x(s)| + \varepsilon. \end{aligned}$$

Hence, we have

$$x(k) \leqslant x(k_1) + \sum_{s=k_0}^{k_0+l(\varepsilon)} |x(s+1) - x(s)| + \varepsilon \quad \forall k \in \mathbb{Z}.$$

Similarly, we can obtain

$$\begin{aligned} x(k) - x(k_2) &= \sum_{s=k_2}^{k-1} (x(s+1) - x(s)) \\ &= \sum_{s=k_2}^{k-1+\tau} (x(s+1) - x(s)) + \sum_{s=k-1+\tau+1}^{k-1} (x(s+1) - x(s)) \\ &\geqslant - \sum_{s=k_0}^{k_0+l(\varepsilon)} |x(s+1) - x(s)| - \varepsilon, \end{aligned}$$

that is,

$$x(k) \geqslant x(k_2) - \sum_{s=k_0}^{k_0+l(\varepsilon)} |x(s+1) - x(s)| - \varepsilon \quad \forall k \in \mathbb{Z}.$$

The proof of Lemma 2.4 is complete. □

Definition 2.3. A set Ω of functions $x\colon \mathbb{Z} \to \mathbb{R}$ is uniformly Cauchy (or equi-Cauchy) if for every $\varepsilon > 0$ there exists an integer N such that $|x(i) - x(j)| < \varepsilon$ whenever $i, j > N$ or $i, j < -N$ for any $x = x(n) \in \Omega$.

Similarly to the proof of the Discrete Arzela-Ascoli Theorem in [8], one can easily show

Lemma 2.5. *A bounded, uniformly Cauchy subset Ω of functions $x\colon \mathbb{Z} \to \mathbb{R}$ is relatively compact.*

In order to explore the existence of almost periodic solutions of (1.3), and for the reader's convenience, we shall first summarize below a few concepts and results without proof, borrowing the notation from [12].

Let X, Y be normed vector spaces, $L\colon \operatorname{Dom} L \subset X \to Y$ a linear mapping, and $N\colon X \to Y$ a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projectors $P\colon X \to X$ and $Q\colon Y \to Y$ such that $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I-Q)$, it follows that the mapping $L|_{\operatorname{Dom} L \cap \operatorname{Ker} P}\colon (I-P)X \to \operatorname{Im} L$ is invertible. We denote the inverse of that mapping by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I-Q)N\colon\ \overline{\Omega}\to X$ is compact. Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists an isomorphism $J\colon\ \operatorname{Im} Q\to\operatorname{Ker} L$.

Lemma 2.6 ([12]). *Let $\Omega\subset X$ be an open bounded set and let $N\colon\ X\to Y$ be a continuous operator which is L-compact on $\overline{\Omega}$. Assume*

(a) *for each $\lambda\in(0,1), x\in\partial\Omega\cap\operatorname{Dom} L, Lx\neq\lambda Nx$;*
(b) *for each $x\in\partial\Omega\cap\operatorname{Ker} L, QNx\neq 0$;*
(c) $\deg(JNQ,\Omega\cap\operatorname{Ker} L,0)\neq 0$.

Then $Lx=Nx$ has at least one solution in $\overline{\Omega}\cap\operatorname{Dom} L$.

3. Main results

In this section we will state and prove the main results of this paper.

By making the substitution

$$y(n)=\exp\{x(n)\},$$

(1.3) can be reformulated as

$$x(n+1)-x(n)=a(n)-b(n)\exp\{px(n-\sigma(n))\}-c(n)\exp\{qx(n-\tau(n))\}. \tag{3.1}$$

Set

$$X=Y=\{x(n)\in\mathrm{AP}(\mathbb{Z})\colon\ \operatorname{mod}(x)\subset\operatorname{mod}(F)\},$$

where

$$F=F(n,\varphi)=a(n)-b(n)\exp\{p\varphi(-\sigma(n))\}-c(n)\exp\{q\varphi(-\tau(n))\},\qquad \varphi\in C([-r,0]_{\mathbb{Z}},\mathbb{R}),$$

$$r=\max\{\sup_{n\in\mathbb{N}}|\sigma(n)|,\sup_{n\in\mathbb{N}}|\tau(n)|\}.$$

For $x\in X$ or Y, define $\|x\|=\sup\limits_{n\in\mathbb{Z}}|x(n)|$.

Lemma 3.1. *X and Y are Banach spaces equipped with the norm $\|\cdot\|$.*

Proof. If $\{x^{\{k\}}(n)\}\subset X$ and $x^{\{k\}}(n)$ converges to $\bar{x}(n)$, then it is easy to show that $\overline{y}(n)\in\mathrm{AP}(\mathbb{Z})$ and $\operatorname{mod}(\bar{x})\subset\operatorname{mod}(F)$. Thus, X and Y are Banach spaces equipped with the norm $\|\cdot\|$. The proof of Lemma 3.1 is complete. $\square$

Lemma 3.2. *Let*

$$L\colon\ X\to Y,\quad Lx(n)=\Delta x(n),$$

where $\Delta x(n)=x(n+1)-x(n)$. *Then* L *is a Fredholm mapping of index zero.*

P r o o f. It is easy to see that L is a linear operator,

$$\operatorname{Ker} L=\{x(n)=h\in\mathbb{R}\}$$

and

$$\operatorname{Im} L=\{y\in Y\colon\ m[y]=0\}.$$

Furthermore, one can easily show that $\operatorname{Im} L$ is closed in Y and

$$\dim\operatorname{Ker} L=1=\operatorname{codim}\operatorname{Im} L,$$

therefore, L is a Fredholm mapping of index zero. The proof of Lemma 3.2 is complete. □

Lemma 3.3. *Let*

$$N\colon\ X\to Y,\quad Nx=G_x,$$

where

$$G_x(n)=a(n)-b(n)\exp\{px(n-\sigma(n))\}-c(n)\exp\{qx(n-\tau(n))\}$$

and

$$P\colon\ X\to X,\quad Px=m[x],\quad Q\colon\ Y\to Y,\quad Qy=m[y].$$

Then N *is* L*-compact on* $\overline{\Omega}$*, where* Ω *is an open bounded subset of* X*.*

P r o o f. Obviously, P and Q are continuous projectors such that

$$\operatorname{Im} P=\operatorname{Ker} L,\quad \operatorname{Im} L=\operatorname{Ker} Q.$$

Hence

$$\operatorname{Im}(I-Q)=\operatorname{Im} L.$$

Then in view of

$$\operatorname{Im} P=\operatorname{Ker} L,\quad \operatorname{Im} L=\operatorname{Ker} Q=\operatorname{Im}(I-Q),$$

we obtain that the inverse $K_P\colon\ \operatorname{Im} L\to\operatorname{Ker} P\cap\operatorname{Dom} L$ of L_P exists and is given by

$$K_P(y)=\sum_{s=0}^{n}y(s)-m\Bigg[\sum_{s=0}^{n}y(s)\Bigg].$$

Thus, we have

$$QNx = m(G_x)$$

and

$$K_P(I-Q)Nx = f(x(n)) - Qf(x(n)),$$

where

$$f(x(n)) = \sum_{s=0}^{n}(Nx(s) - QNx(s)).$$

Clearly, QN and $(I-Q)N$ are continuous. Now we will show that K_P is also continuous.

By assumptions, for any $0<\varepsilon<1$ and any compact set $S\subset C([-r,0]_{\mathbb{Z}},\mathbb{R})$, let $l(\varepsilon)$ be the length of the inclusion interval of $T(F,\varepsilon,S)$. Suppose that $\{y^{\{k\}}(s)\}\subset \operatorname{Im} L$ and $y^{\{k\}}(s)$ uniformly converges to $\overline{y}(s)$. Because of $\sum\limits_{s=0}^{n} y^{\{k\}} \in Y$, there exists $\delta(0<\delta<\varepsilon)$ such that $K(F,\delta,S)\subset T\Bigl(\sum\limits_{s=0}^{n} y^{\{k\}},\varepsilon,S\Bigr)$. Let $l(\delta,S)$ be the length of the inclusion interval of $T(F,\delta,S)$ and

$$l = \max\{l(\delta,S), l(\varepsilon,S)\}.$$

It is easy to see that l is the length of the inclusion interval of $T(F,\varepsilon,S)$ and $T(F,\delta,S)$. Hence, for any $n\notin[0,l]_{\mathbb{Z}}$, there exists $\xi_n\in T(F,\delta,S)\subset T\Bigl(\sum\limits_{s=0}^{n} y^{\{k\}},\varepsilon,S\Bigr)$ such that $n+\xi_n\in[0,l]_{\mathbb{Z}}$. Hence, by the definition of the almost periodic sequence we have

$$\begin{aligned}
(3.2)\qquad \Bigl\|\sum_{s=0}^{n} y^{\{k\}}(s)\Bigr\| &= \sup_{n\in\mathbb{Z}}\Bigl|\sum_{s=0}^{n} y^{\{k\}}(s)\Bigr| \leqslant \sup_{n\in[0,l]_{\mathbb{Z}}}\Bigl|\sum_{s=0}^{n} y^{\{k\}}(s)\Bigr| \\
&\quad + \sup_{n\notin[0,l]_{\mathbb{Z}}}\Bigl|\sum_{s=0}^{n} y^{\{k\}}(s) - \sum_{s=0}^{n+\xi_n} y^{\{k\}}(s) + \sum_{s=0}^{n+\xi_n} y^{\{k\}}(s)\Bigr| \\
&\leqslant 2\sup_{n\in[0,l]_{\mathbb{Z}}}\Bigl|\sum_{s=0}^{n} y^{\{k\}}(s)\Bigr| + \sup_{n\notin[0,l]_{\mathbb{Z}}}\Bigl|\sum_{s=0}^{n} y^{\{k\}}(s) - \sum_{s=0}^{n+\xi_n} y^{\{k\}}(s)\Bigr| \\
&\leqslant 2\sum_{s=0}^{l}|y^{\{k\}}(s)| + \varepsilon.
\end{aligned}$$

By (3.2), we conclude that $\sum\limits_{s=0}^{n} y(s)$ is continuous, where $y\in\operatorname{Im} L$. Consequently, K_P and $K_P(I-Q)Ny$ are continuous.

From (3.2), we also have that $\sum\limits_{s=0}^{n} y(s)$ and $K_P(I-Q)N$ are also uniformly bounded on $\overline{\Omega}$. Further, it is not difficult to verify that $QN(\overline{\Omega})$ is bounded and $K_P(I-Q)N$ is equicontinuous on $\overline{\Omega}$. By the proof of Lemma 3.3 in [28], we can immediately conclude that $\overline{K_P(I-Q)N(\overline{\Omega})}$ is compact on any compact subset of $\mathbb{Z}$. From this, (3.2) and the expression of $K_P(I-Q)Nx$ it follows that if $\{x^k(n)\} \subset \overline{K_P(I-Q)N(\overline{\Omega})}$, then the sequence $\{x^k(n)\}$ contains a subsequence $\{x^{k_1}(n)\}$ that is uniformly Cauchy. Thus, by Lemma 2.5, N is L-compact on $\overline{\Omega}$. The proof of Lemma 3.3 is complete.

$\square$

Theorem 3.1. *Assume that*

(H) $m[a] > 0, m[b+c] \neq 0,$

then (1.3) *has at least one positive almost periodic solution.*

P r o o f. In order to use the continuation theorem of coincidence degree theory to establish the existence of a solution of (3.1), we consider the same Banach spaces X and Y as those in Lemma 3.1 and the same mappings L, N, P, Q as those in Lemma 3.2 and Lemma 3.3, respectively. Then we can obtain that L is a Fredholm mapping of index zero and N is a continuous operator which is L-compact on $\overline{\Omega}$.

Now, we are in the position to search for an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation

$$Lx = \lambda Nx, \quad \lambda \in (0,1),$$

we obtain

$$x(n+1)-x(n) = \lambda(a(n)-b(n)\exp\{px(n-\sigma(n))\}-c(n)\exp\{qx(n-\tau(n))\}). \tag{3.3}$$

Assume that $x(n) \in X$ is a solution of (3.3) for a certain $\lambda \in (0,1)$. Denote

$$x^M = \sup_{n\in\mathbb{Z}} x(n), \quad x^m = \inf_{n\in\mathbb{Z}} x(n).$$

Summing on both sides of (3.3) from 0 to $N-1$ with respect to n, we get

$$\sum_{n=0}^{N-1}(x(n+1)-x(n)) = \lambda\sum_{n=0}^{N-1}(a(n)-b(n)\exp\{px(n-\sigma(n))\}-c(n)\exp\{qx(n-\tau(n))\}),$$

thus

$$m[a] = m[b(n)\exp\{px(n-\sigma(n))\}+c(n)\exp\{qx(n-\tau(n))\}]$$

and consequently

$$m[a] \geqslant \exp\{px^m\}m[b+c],$$

that is,

$$x^m \leqslant \frac{1}{p} \ln \frac{m[a]}{m[b+c]}. \tag{3.4}$$

Similarly, we can get

$$x^M \geqslant \frac{1}{q} \ln \frac{m[a]}{m[b+c]}. \tag{3.5}$$

For $n_0 \in \mathbb{Z}$ and $\forall \varepsilon > 0$ we can choose a point $\tilde{n} - n_0 \in [l, 2l]_{\mathbb{Z}} \cap T(F, \delta, S)$, where δ $(0 < \delta < \varepsilon)$ satisfies $T(F, \delta, S) \subset T(x, \varepsilon, S)$. Summing on both sides of (3.3) from n_0 to $\tilde{n} - 1$ with respect to n, we get

$$\begin{aligned} &\lambda \sum_{s=n_0}^{\tilde{n}-1} (b(s) \exp\{px(s-\sigma(s))\} + c(s) \exp\{qx(s-\tau(s))\}) \\ &\quad = \lambda \sum_{s=n_0}^{\tilde{n}-1} a(s) + \sum_{s=n_0}^{\tilde{n}-1} (x(s+1) - x(s)) \leqslant \lambda \sum_{s=n_0}^{\tilde{n}-1} |a(s)| + \varepsilon. \end{aligned} \tag{3.6}$$

Hence, from (3.3) and (3.6) we have

$$\begin{aligned} \sum_{s=n_0}^{\tilde{n}-1} (x(s+1) - x(s)) &\leqslant \lambda \sum_{s=n_0}^{\tilde{n}-1} a(s) + \lambda \sum_{s=n_0}^{\tilde{n}-1} \big(b(s) \exp\{px(s-\sigma(s))\} \\ &\qquad + c(s) \exp\{qx(s-\tau(s))\}\big) \\ &\leqslant 2\lambda \sum_{s=n_0}^{\tilde{n}-1} |a(s)| + \varepsilon < 2\lambda \sum_{s=n_0}^{\tilde{n}-1} |a(s)| + 1. \end{aligned} \tag{3.7}$$

Therefore, in view of (3.4) and (3.7), by Lemma 2.4, for any $\varepsilon > 0$ with inclusion length $l(\varepsilon)$ there exist n_1, n_2 such that we have

$$\begin{aligned} x(n) &\leqslant x(n_1) + \sum_{s=n_0}^{n_0+l(\varepsilon)} |x(s+1) - x(s)| + \varepsilon \\ &\leqslant x^m + \varepsilon + 2\lambda \sum_{s=n_0}^{\tilde{n}-1} |a(s)| + 1 + \varepsilon \\ &\leqslant \frac{1}{p} \ln \frac{m[a]}{m[b+c]} + 2\lambda \sum_{s=n_0}^{\tilde{n}-1} |a(s)| + 3 \quad \forall n \in \mathbb{Z} \end{aligned} \tag{3.8}$$

and

$$
\begin{aligned}
(3.9) \qquad x(n) &\geqslant x(n_2) - \sum_{s=n_0}^{n_0+l(\varepsilon)} |x(s+1) - x(s)| - \varepsilon \\
&\geqslant x^M - 2\varepsilon - 2\lambda \sum_{s=n_0}^{\tilde{n}-1} |a(s)| - 1 \\
&\geqslant \frac{1}{q} \ln \frac{m[a]}{m[b+c]} - 2\lambda \sum_{s=n_0}^{\tilde{n}-1} |a(s)| - 3 \quad \forall n \in \mathbb{Z}.
\end{aligned}
$$

It follows from (3.8) and (3.9) that

$$\|x\| \leqslant M_1,$$

where

$$M_1 = \max\left\{\frac{1}{p} \ln \frac{m[a]}{m[b+c]} + 2\lambda \sum_{s=n_0}^{\tilde{n}-1} |a(s)| + 3, \frac{1}{q} \ln \frac{m[a]}{m[b+c]} - 2\lambda \sum_{s=n_0}^{\tilde{n}-1} |a(s)| - 3\right\}.$$

Clearly, M_1 is independent of λ. Take $M = M_1 + K$, where $K > 0$ is taken sufficiently large such that the unique solution x^* of

$$m[a] - m[b] \exp\{px\} - m[c] \exp\{qx\} = 0$$

satisfies $\|x^*\| < M$. Then, take

$$\Omega = \{x \in X \colon \|x\| < M\}.$$

It is clear that Ω satisfies the requirement (a) in Lemma 2.6. When $x \in \partial\Omega \cap \operatorname{Ker} L$, x is a constant in $\mathbb{R}$ with $|x| = M$, then

$$QNx = m(a(n) - b(n) \exp\{px\} - c(n) \exp\{qx\}) \neq 0,$$

which implies that the requirement (b) in Lemma 2.6 is satisfied. Furthermore, take the isomorphism $J\colon \operatorname{Im} Q \to \operatorname{Ker} L$, $Jy \equiv y$ and let $H(\gamma, x) = -\gamma x + (1-\gamma) JQNx$, $0 \leqslant \gamma \leqslant 1$. Then for any $x \in \partial\Omega \cap \operatorname{Ker} L$, $0 \leqslant \gamma \leqslant 1$, we have $H(\gamma, x) \neq 0$ and

$$\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} = \deg\{-x, \Omega \cap \operatorname{Ker} L, 0\} \neq 0.$$

So, the requirement (c) in Lemma 2.6 is satisfied. Hence, (3.1) has at least one solution in $\overline{\Omega}$, that is, (1.3) has at least one positive almost periodic solution. The proof of Theorem 3.1 is complete. □

R e m a r k 3.1. Since any equation with positive and constant a, b, and c satisfies the assumptions of Theorem 3.1, the sufficient conditions we give are by no means restrictive.

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References

[1] *R. P. Agarwal, P. J. Y. Wong*: Advanced Topics in Difference Equations. Mathematics and its Applications 404, Kluwer Academic Publishers, Dordrecht, 1997.

[2] *J. O. Alzabut, G. T. Stamov, E. Sermutlu*: Positive almost periodic solutions for a delay logarithmic population model. Math. Comput. Modelling *53* (2011), 161–167.

[3] *D. Cheban, C. Mammana*: Invariant manifolds, global attractors and almost periodic solutions of nonautonomous difference equations. Nonlinear Anal., Theory Methods Appl. *56* (2004), 465–484.

[4] *Y. Chen*: Periodic solutions of a delayed, periodic logistic equation. Appl. Math. Lett. *16* (2003), 1047–1051.

[5] *W. Chen, B. Liu*: Positive almost periodic solution for a class of Nicholson's blowflies model with multiple time-varying delays. J. Comput. Appl. Math. *235* (2011), 2090–2097.

[6] *F. Chen, X. Xie, X. Chen*: Permanence and global attractivity of a delayed periodic logistic equation. Appl. Math. Comput. *177* (2006), 118–127.

[7] *L. Chen, H. Zhao*: Global stability of almost periodic solution of shunting inhibitory cellular neural networks with variable coefficients. Chaos Solitons Fractals *35* (2008), 351–357.

[8] *S. S. Cheng, W. T. Patula*: An existence theorem for a nonlinear difference equation. Nonlinear Anal., Theory Methods Appl. *20* (1993), 193–203.

[9] *X. Ding, C. Lu*: Existence of positive periodic solution for ratio-dependent N-species difference system. Appl. Math. Modelling *33* (2009), 2748–2756.

[10] *M. Fan, K. Wang*: Periodic solutions of a discrete time nonautonomous ratio-dependent predator-prey system. Math. Comput. Modelling *35* (2002), 951–961.

[11] *H. I. Freedman*: Deterministic Mathematical Models in Population Ecology. Monographs and Textbooks in Pure and Applied Mathematics 57, Marcel Dekker, New York, 1980.

[12] *R. E. Gaines, J. L. Mawhin*: Coincidence Degree, and Nonlinear Differential Equations. Lecture Notes in Mathematics 568, Springer, Berlin, 1977.

[13] *H. Huo, W. Li*: Existence and global stability of periodic solutions of a discrete predator-prey system with delays. Appl. Math. Comput. *153* (2004), 337–351.

[14] *Y. Li, X. Fan*: Existence and globally exponential stability of almost periodic solution for Cohen-Grossberg BAM neural networks with variable coefficients. Appl. Math. Modelling *33* (2009), 2114–2120.

[15] *Y. Li, L. Lu*: Positive periodic solutions of discrete n-species food-chain systems. Appl. Math. Comput. *167* (2005), 324–344.

[16] *Y. Li, T. Zhang, Y. Ye*: On the existence and stability of a unique almost periodic sequence solution in discrete predator-prey models with time delays. Appl. Math. Modelling *35* (2011), 5448–5459.

[17] *X. Meng, L. Chen*: Almost periodic solution of non-autonomous Lotka-Volterra predator-prey dispersal system with delays. J. Theoret. Biol. *243* (2006), 562–574.

[18] *X. Meng, J. Jiao, L. Chen*: Global dynamics behaviors for a nonautonomous Lotka-Volterra almost periodic dispersal system with delays. Nonlinear Anal., Theory Methods Appl. *68* (2008), 3633–3645.
[19] *J. D. Murray*: Mathematical Biology. Biomathematics 19, Springer, Berlin, 1989.
[20] *Y. G. Sun, S. H. Saker*: Existence of positive periodic solutions of nonlinear discrete model exhibiting the Allee effect. Appl. Math. Comput. *168* (2005), 1086–1097.
[21] *Z. Teng*: Persistence and stability in general nonautonomous single-species Kolmogorov systems with delays. Nonlinear Anal., Real World Appl. *8* (2007), 230–248.
[22] *W. Wu, Y. Ye*: Existence and stability of almost periodic solutions of nonautonomous competitive systems with weak Allee effect and delays. Commun. Nonlinear Sci. Numer. Simul. *14* (2009), 3993–4002.
[23] *Y. Xie, X. Li*: Almost periodic solutions of single population model with hereditary effects. Appl. Math. Comput. *203* (2008), 690–697.
[24] *J. Yan, A. Zhao, W. Yan*: Existence and global attractivity of a periodic solution for an impulsive delay differential equation with Allee effect. J. Math. Anal. Appl. *309* (2005), 489–504.
[25] *X. Yang*: The persistence of a general nonautonomous single-species Kolmogorov system with delays. Nonlinear Anal., Theory Methods Appl. *70* (2009), 1422–1429.
[26] *S. Zhang, G. Zheng*: Almost periodic solutions of delay difference systems. Appl. Math. Comput. *131* (2002), 497–516.
[27] *W. Zhang, D. Zhu, P. Bi*: Multiple positive periodic solutions of a delayed discrete predator-prey system with type IV functional responses. Appl. Math. Lett. *20* (2007), 1031–1038.
[28] *L. Zhu, Y. Li*: Positive periodic solutions of higher-dimensional functional difference equations with a parameter. J. Math. Anal. Appl. *290* (2004), 654–664.

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