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EWMA Historical Volatility Estimators

LUCIA JAREŠOVÁ

Praha

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In this paper different types of historical volatility estimators based on open-high-low-close (OHLC) values are studied. The estimators are broken down to the main building blocks and the correlation structure of these building blocks with time dependent variance (volatility squared) is investigated. The building blocks are estimated from the equity index (SPX in USA and DAX in Germany) and compared with a volatility index (VIX in USA and VDAX in Germany) which stands as a proxy for volatility, because the values of the volatility process are in general not available. In an empirical study it is observed that both the autocorrelation function of variance and the cross-correlation functions of building blocks with the variance decrease exponentially with the same degree. This dependence can be explained as exponentially decreasing "amount of information" and it naturally leads to use of exponentially decreasing weights in historical estimators. The proposed EWMA style estimators have higher predicting power over the commonly used estimators and in prediction beat the very popular GARCH(1,1).

Introduction

The word volatility is used most often in finance as a measure of variability in the changes of asset prices. Higher volatility means in general higher probability of bigger losses, so volatility is directly linked to the risk of the asset.

The recent development in financial markets increased the importance of time dependent volatility modeling and new and new more "sophisticated" models are proposed to better represent the "reality". The volatility process is in general assumed to be directly unobservable and consequently the estimation of these models is very difficult.

Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic

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E-mail address: jaresova@karlin.mff.cuni.cz

In the last decades the continuous volatility models attracted much attention because of the theoretically nice and elegant stochastic calculus. For an overview about the volatility modeling and measurement see [1]. Unfortunately the practical part of these continuous time models is developing much more slowly and it is mostly necessary to discretize these models for estimation, since the observed values are available for an analysis only as daily time series. Of course more detailed data exists, for example in the form of high-frequency tick-by-tick data, but this data is not available to everybody and its analysis requires more computer power as it is very technologically demanding.

Daily Financial Data

The asset price process is denoted by S_t, $t \geq 0$. A widely used model for S_t is the Generalized geometric Brownian motion given by the following equation

$$dS_t = \mu_t S_t dt + \sigma_t S_t dW_t, \tag{1}$$

where the drift μ_t is the expected return on the asset, the volatility σ_t measures the variability around μ_t and W_t is the standard Brownian motion.

In the standard Geometric Brownian motion (GBM) it is assumed that $\mu_t = \mu$ and $\sigma_t = \sigma$ (the parameters are constant) and it holds for $T > t$ that

$$\ln(S_T) = \ln(S_t) + (\mu - \sigma^2/2)(T - t) + \sigma\sqrt{T - t}\epsilon \tag{2}$$

where ϵ is the standard normal random variable. From equation (2) it follows that the logarithmic returns $r(t, T) = \ln(S_T/S_t)$ are normally distributed.

The asset price S_t is only observed during the time when the market is open and the daily financial data are often available in the form of OHLC (open-high-low-close) values. For the trading days $i = 0, \ldots, N$ we denote the morning opening price by O_i, the evening closing price by C_i, the daily lowest price by L_i and the daily highest price by H_i.

These values are for many assets and indices available to everybody, since they can be downloaded for example from Yahoo Finance[1] by using free software environment R [8] and a Rmetrics[2] package `fImport`.

The daily log-returns $r_1, \ldots, r_N$ are computed from the closing prices C_i as

$$r_i = \ln(C_i/C_{i-1}). \tag{3}$$

The time dependent yearly volatility will further be denoted by σ_i and it is mostly estimated by the standard deviation of daily log-returns multiplied by the scaling factor[3] $\sqrt{260}$. The yearly variance is defined as $h_i = \sigma_i^2$.

[1] `http://finance.yahoo.com/`
[2] `https://www.rmetrics.org/`
[3] It is common to consider the annualized daily volatility, so the standard deviation has to be scaled by the square root of the number of trading days in a year. We assume that a year has 260 trading days.

A more sophisticated widely used approach for time-dependent volatility modeling in discrete time is the GARCH (Generalized AutoRegressive Conditional Heteroskedasticity) model. This model uses only daily closing data. The description of these models can be found in almost every financial time series literature, for example in [10]. A lot of generalizations of the GARCH were derived, summary of them is in [2] (more than 100 models).

Historical Volatility Estimators

In this section the main historical volatility estimators widely used by practitioners are introduced. The formulas for historical estimators are taken from the Quant Equation Archive `http://www.sitmo.com/eqcat/4`.

Historical Close-to-Close Volatility. This simplest estimator is equal to the standard deviation of log-returns scaled to one year given by the formula

$$\sigma_{cc} = \sqrt{\frac{260}{N-1}\sum_{i=1}^{N}(r_i - \bar{r})^2} \quad \text{or} \quad \sigma_{cc} = \sqrt{\frac{260}{N}\sum_{i=1}^{N}(r_i - \bar{r})^2}, \tag{4}$$

where $\bar{r} = \frac{1}{N}\sum_{i=1}^{N} r_i$. The drift of asset prices estimated by $\bar{r}$ is usually very small, so sometimes the following formulas are used

$$\sigma_{cc} = \sqrt{\frac{260}{N-1}\sum_{i=1}^{N} r_i^2} \quad \text{or} \quad \sigma_{cc} = \sqrt{\frac{260}{N}\sum_{i=1}^{N} r_i^2}. \tag{5}$$

Historical High-Low Volatility (Parkinson). This estimator uses only the highest and lowest daily values and is given by

$$\sigma_p = \sqrt{\frac{260}{4N\ln(2)}\sum_{i=1}^{N}\left(\ln\frac{H_i}{L_i}\right)^2}. \tag{6}$$

Historical Open-High-Low-Close Volatility (Garman and Klass). This estimator uses all OHLC values and is given by

$$\sigma_{gk} = \sqrt{\frac{260}{N}\sum_{i=1}^{N}\left[\frac{1}{2}\left(\ln\frac{H_i}{L_i}\right)^2 - (2\ln 2 - 1)\left(\ln\frac{C_i}{O_i}\right)^2\right]} \tag{7}$$

Historical Open-High-Low-Close Volatility (Garman and Klass, Yang Zhang extension). This estimator is currently the preferred version of OHLC volatility estimator and it differs from the previous estimator only by the term $(\ln(O_i/C_{i-1}))^2$ which takes into account the opening jump (the change from the closing price yesterday C_{i-1} to the opening price today O_i).

$$\sigma_{gkyz} = \sqrt{\frac{260}{N}\sum_{i=1}^{N}\left[\left(\ln\frac{O_i}{C_{i-1}}\right)^2 + \frac{1}{2}\left(\ln\frac{H_i}{L_i}\right)^2 - (2\ln 2 - 1)\left(\ln\frac{C_i}{O_i}\right)^2\right]} \tag{8}$$

Historical Open-High-Low-Close Volatility (Rogers Satchell). The last estimator uses all OHLC values and is given by

$$\sigma_{rs} = \sqrt{\frac{260}{N}\sum_{i=1}^{N}\left[\ln\frac{H_i}{C_i}\ln\frac{H_i}{O_i} + \ln\frac{L_i}{C_i}\ln\frac{L_i}{O_i}\right]} \tag{9}$$

Building Blocks Historical Volatility Estimators. The mentioned historical volatility estimators have the following building blocks:
$A = \{\ln(C_i/C_{i-1})^2\} = \{r_i^2\}$ represents the daily squared close-to-close changes.
$B = \{\ln(H_i/L_i)^2\}$ represents the daily squared extreme changes.
$C = \{\ln(O_i/C_{i-1})^2\}$ represents the squared opening jumps.
$D = \{\ln(C_i/O_i)^2\}$ represents the squared trading daily changes.
$E = \{\ln(H_i/C_i)\ln(H_i/O_i)\}$ is based on the first term of Rogers Satchell estimator.
$F = \{\ln(L_i/C_i)\ln(L_i/O_i)\}$ is based on the second term of Rogers Satchell estimator.
The building blocks A, B, C, D, E and F will be used as an input in the correlation empirical study.

Proxy for the Unobserved Volatility

All variables except for volatility are directly observable in the market. Volatility can be indirectly observed in the market through the option prices, since the higher volatility the higher prices of plain vanilla options. We can use the observed option market price and the Black-Scholes (BS) option pricing formula (for more information about option pricing see [5] or [11]) to calculate the volatility that yields a theoretical value of the option equal to the observed market price. This volatility is called the *implied volatility* and it in general depends (in contrast to the BS model where the volatility is assumed to be constant) on the strike price K and the expiration of the option $T - t$ (time to maturity). The collection of all such implied volatilities with respect to the strike price and time to maturity is known as the *volatility surface*. For more understanding of volatility surfaces look in [4] or [9].

Implied volatility is a very good financial indicator of the “fear” in the market since it often signifies financial turmoil. Some exchanges have transformed this information into *volatility indices*. The most known index is the VIX index of the CBOE (Chicago Board Options Exchange) launched in 2003 (data begins 1990). The CBOE utilizes a wide variety of strike prices of options on the S&P 500 index (SPX index, the core index for U.S. equities) in order to obtain the estimator of 30-day expected volatility.

More details about the calculation are described in the methodology [3]. VDAX-NEW index (launched in 2005 as a successor for VDAX launched in 1996, data begins 1992) is an analogous index of the Deutsche Börse in Germany based on the prices of options on the German DAX index.

The volatility indices have become very popular and their number is increasing. The volatility index can nowadays even be traded through exchange traded futures (since 2004) and exchange trade options (since 2006).

The volatility indices were used in the empirical study [6], where the implied volatility, GARCH volatility and historical volatility were compared. In this paper we will use the volatility index value as a proxy for an unobserved volatility process σ_i. Further we define $Y = \{\sigma_i^2/260\} = \{h_i/260\}$ as the actual one day variance[4].

Auto- and Cross-Correlation Study

Now we will take the data A, B, C, D, E, F and Y and investigate their correlation relationships. The used data are from 3. 1. 2000 to 2. 4. 2010. To make the results more relevant, the analysis is made on two different markets. In the US market the equity index SPX and the volatility index VIX are used. In the Germany market the equity index DAX and the volatility index VDAX are used.

Table 1. Price index SPX, volatility index VIX

	Y	A	B	C	D	E	F
Y	1.00	**0.53**	**0.71**	**0.13**	**0.60**	**0.52**	**0.45**
A	0.53	1.00	0.81	0.04	0.67	0.19	0.20
B	0.71	0.81	1.00	0.11	0.70	0.49	0.66
C	0.13	0.04	0.11	1.00	0.08	0.01	0.05
D	0.60	0.67	0.70	0.08	1.00	0.47	0.24
E	0.52	0.19	0.49	0.01	0.47	1.00	0.19
F	0.45	0.20	0.66	0.05	0.24	0.19	1.00

The estimated correlation matrices of the data are in the tables 1 and 2. It is very interesting, that the correlations are in both tables very similar.

[4] Volatility is in the market quoted as a value per annum similarly as interest rates. We need to divide the squared value by the number of working days per annum to obtain the scaled daily values.

Table 2. Price index DAX, volatility index VDAX

	Y	A	B	C	D	E	F
Y	1.00	**0.48**	**0.68**	**0.23**	**0.55**	**0.51**	**0.45**
A	0.48	1.00	0.74	0.17	0.66	0.22	0.24
B	0.68	0.74	1.00	0.26	0.66	0.61	0.66
C	0.23	0.17	0.26	1.00	0.31	0.24	0.19
D	0.55	0.66	0.66	0.31	1.00	0.40	0.28
E	0.51	0.22	0.61	0.24	0.40	1.00	0.23
F	0.45	0.24	0.66	0.19	0.28	0.23	1.00

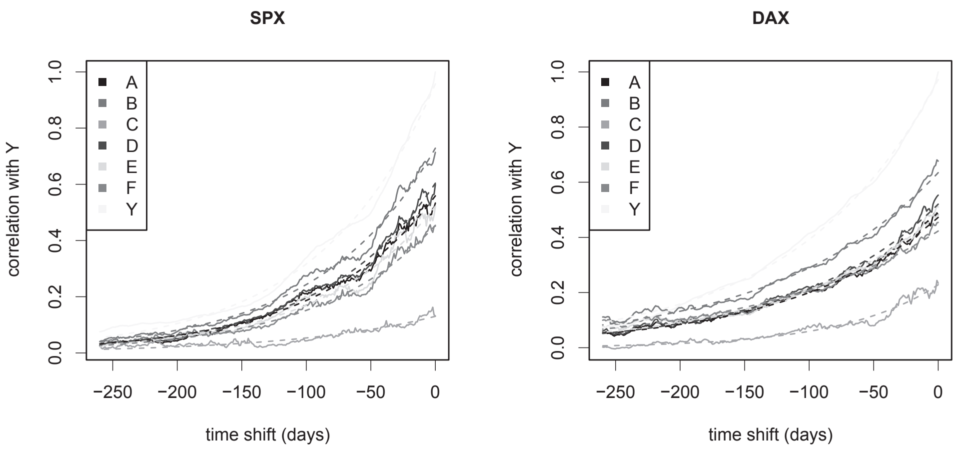

Figure 1: Cross-Correlation Functions

The autocorrelation function of Y and the cross-correlation function of A, B, C, D, E, F with Y are graphed by solid lines in the figure 1. The dashed lines in the figure represent the fitted exponential functions. It can be seen, that the exponential functions fit the auto- and cross-correlation functions very well. It means that the linear dependence on the past values decreases exponentially as the time goes on.

It is very obvious, that these functions look very similar in both markets. It points to the hypothesis, that these patterns are properties of the real time dependent variance process and every good volatility model should reflect these properties as well.

GARCH model and EWMA model

GARCH models are nowadays very popular. Process $\{r_n\}, n \in \mathbb{Z}$ is a *(strong) GARCH(p, q)*, if $\mathrm{E}[r_n|\mathscr{F}_{n-1}] = 0$ (the conditional mean is unpredictable) and $\mathrm{Var}[r_n|\mathscr{F}_{n-1}] = \sigma_t^2$ (the conditional variance is time dependent), where

$$\sigma_n^2 = \omega + \sum_{i=1}^{q} \alpha_i r_{n-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{n-j}^2 \tag{10}$$

and $Z_n = r_n/\sigma_n$ are i.i.d. random variables. $\mathscr{F}_{n-1}$ denotes the σ-algebra generated by the historical returns, α_i and β_j are real coefficients. Sufficient condition for $\sigma_n^2 \geq 0$ is $\omega, \alpha_i, \beta_j \geq 0$. In the GARCH(1, 1) model the conditional variance has the form

$$\sigma_n^2 = \omega + \alpha r_{n-1}^2 + \beta \sigma_{n-1}^2. \tag{11}$$

The EWMA (Exponentially Weighted Moving Average) model of volatility is known very well thanks to the RiskMetrics[5] technical report [7] and has been used in many practical applications. The EWMA volatility is defined as

$$\sigma_n^2 = (1-\lambda) \sum_{j=1}^{\infty} \lambda^{j-1} (r_{n-j} - \bar{r})^2 = (1-\lambda)(r_{n-1} - \bar{r})^2 + \lambda \sigma_{n-1}^2 \tag{12}$$

or, when $\bar{r}$ is small, as

$$\sigma_n^2 = (1-\lambda) \sum_{j=1}^{\infty} \lambda^{j-1} r_{n-j}^2 = (1-\lambda) r_{n-1}^2 + \lambda \sigma_{n-1}^2, \tag{13}$$

where λ is a *decay factor*. Typical values of the decay factor are close to one (values 0.94 and 0.97 are recommended in [7]).

In the case when we have N historical observations, the EWMA estimator of volatility can be computed as

$$\sigma_{E,n} = \sqrt{\frac{260(1-\lambda)}{1-\lambda^N} \sum_{i=1}^{N} \lambda^{i-1} (r_{n-i} - \bar{r})^2} \tag{14}$$

or

$$\sigma_{E,n} = \sqrt{\frac{260(1-\lambda)}{1-\lambda^N} \sum_{i=1}^{N} \lambda^{i-1} r_{n-i}^2}, \tag{15}$$

where n is a time index. This estimator can be expressed in the following way

$$\sigma_{E,n} = \sqrt{260 \sum_{i=1}^{N} w_i(\lambda) r_{n-i}^2}, \tag{16}$$

where the weights $w_i(\lambda) = \frac{(1-\lambda)\lambda^{i-1}}{1-\lambda^N} = \frac{\lambda^{i-1}}{\sum_{i=1}^{N} \lambda^{i-1}}$ sum to 1. Since the weights are equal to $1/N$ for $\lambda = 1$, the EWMA estimator is equal to the historical close-to-close volatility

[5] http://www.riskmetrics.com/

estimator for $\lambda = 1$. The main advantage of this estimator is, that it is very simple and that it does not need any additional software, because it can be computed even in Excel.

EWMA Style Estimators

The observed pattern of exponentially declining amount of information with the time left can be used to improve all historical volatility estimators by introducing exponential weights in the same way as in the classical close-to-close EWMA estimator. It means that an estimator of the form

$$\sigma^2 = \frac{260}{N} \sum_{i=1}^{N} (building\ blocks)_{n-i} \tag{17}$$

is changed to an EWMA-style estimator by weights w_i

$$\sigma^2_{EWMA} = 260 \sum_{i=1}^{N} w_i(\lambda) \cdot (building\ blocks)_{n-i}, \tag{18}$$

where $\lambda \in [0, 1]$ and

$$w_i(\lambda) = \frac{\lambda^{i-1}}{\sum_{i=1}^{N} \lambda^{i-1}} \overset{\lambda \neq 1}{=} \frac{(1-\lambda)\lambda^{i-1}}{1-\lambda^N}. \tag{19}$$

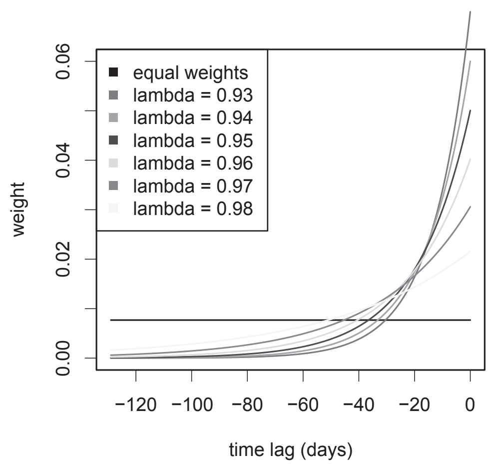

Figure 2: Comparison of EWMA weights with the equal weights in classical historical estimators

It is clear that $\sum_{i=1}^{N} w_i(\lambda) = 1$ for all λ and that EWMA style estimator is equal to the classical estimator for $\lambda = 1$. The EWMA weights $w_i(\lambda)$ give more importance on the more recent observations. The time structure of weights can be seen in the

Figure 2, where the horizontal line represents the equal weights in the classical estimators corresponding to $\lambda = 1$ and the other lines represent the EWMA weights for different values of decay factor λ.

Empirical Study

In the empirical study [6] the actual volatility was estimated for t equal to dates from 3.5.2002 to 24.4.2009. For each t the three year history till t was taken to estimate the volatility. The main goal of the performance study was to evaluate the performance of GARCH(1,1) in forecasting volatility. The results of this study are shown in the Table 3. The GARCH(1,1) was estimated by using the environment R with the Rmetrics library `fGarch`.

Table 3. Coefficient of determination in the linear model where the actual values of volatility index are regressed on the estimated values and constant term

Index	previous index value	GARCH(1,1)	GARCH(1,1) forecast	3M hist. volatility
SPX	97.47%	88.43%	89.21%	84.24%
DAX	98.23%	86.71%	87.44%	83.36%

In this section a similar empirical study is performed, where the classical historical volatility estimators are compared to the EWMA style estimators with the decay factor $\lambda = 0.96$. Recall that an EWMA style estimator with decay factor 1 is equal to classical historical estimator.

In the following *GARCH* denotes the GARCH forecast from the last year empirical study (was the best performing), *EWMA(λ)* denotes the EWMA volatility estimator, *PARK(λ)* denotes the EWMA-style Parkinson estimator, *GK(λ)* denotes the EWMA-style Garman Klass estimator, *GKYZ(λ)* denotes the EWMA-style Garman Klass (Yang Zhang) estimator, *RS(λ)* denotes the EWMA-style Rogers Satchell estimator, *index yesterday* denotes the previous value of the volatility index and λ is the decay factor.

Models were estimated for t equal to dates from 12. 4. 2002 to 2. 4. 2010. Estimators with $\lambda = 0.96$ are based on two year history and estimators with $\lambda = 1$ are based on three months history, since using of longer history makes the estimation worse.

To evaluate the performance of these estimators we again consider a linear model $y_i = a + bx_i + e_i$, where a and b are constant and e_i is an error term. The estimated volatility values are the explanatory data $\boldsymbol{x} = \{x_1, \ldots, x_n\}$ and the values of volatility index on the corresponding days are the dependent data $\boldsymbol{y} = \{y_1, \ldots, y_n\}$. The coefficient of determination R^2 is in this situation equal to $\text{Cor}(\boldsymbol{x}, \boldsymbol{y})^2$ and it will be used as an overall performance measure (the higher value of R^2 the better estimator).

Further the following performance measures are considered[6]

$$bias_A = \text{mean}(\boldsymbol{x} - \boldsymbol{y}) \qquad bias_R = \text{mean}(\boldsymbol{x}/\boldsymbol{y} - 1)$$
$$sd_A = \sqrt{\text{Var}(\boldsymbol{x} - \boldsymbol{y})} \qquad sd_R = \sqrt{\text{Var}(\boldsymbol{x}/\boldsymbol{y} - 1)}$$
$$MSE_A = \sqrt{\text{mean}((\boldsymbol{x} - \boldsymbol{y})^2)} \qquad MSE_R = \sqrt{\text{mean}((\boldsymbol{x}/\boldsymbol{y} - 1)^2)}.$$

Absolute measures are denoted with the subscript A and the relative measures are denoted with the subscript R. The bias is measured by $bias_A$ and $bias_R$ and the standard deviation is measured by sd_A and sd_R. The measures MSE_A and MSE_R (mean squared error) take into account both bias and variance in the same time. The lower absolute value of these measures the better estimator.

The performance results are shown in the Table 4. The best volatility predictor is the previous value of volatility index. The problem with this predictor is, that we do not have volatility index for many assets.

Comparing the prediction properties of GARCH model with classical historical estimators, we find out, that based on the coefficient of determination is the GARCH model in all cases better. When we compare the coefficients of determination of the classical estimators with the EWMA style estimators, we find out, that the EWMA style estimators are not just in all cases better than the classical estimators, but they performed even better than the GARCH estimators.

The absolute and relative measures give us additional information about the properties of the estimators. For example in the case of SPX index we can see that the GK(0.96) performed better than PARK(0.96) even if the coefficient of determination is the same.

Conclusion

In this paper the auto- and cross- correlation structure of variance with the building blocks of some open-high-low-close historical volatility estimators was investigated. It was empirically shown through the correlation structure, that the linear dependence decreases exponentially and as a consequence new EWMA style historical estimators based on the open-high-low-close values were proposed.

The EWMA style estimators of volatility were in an empirically study compared with the GARCH prediction and with the classical estimators. The EWMA style estimators were in all cases better than the classical estimators. A very interesting result is, that the EWMA estimators were in all cases better than the GARCH prediction,

[6] $\boldsymbol{x} - \boldsymbol{y} := \{x_1 - y_1, \ldots, x_n - y_n\}$, $\quad \boldsymbol{x}/\boldsymbol{y} - 1 := \{x_1/y_1 - 1, \ldots, x_n/y_n - 1\}$, $\quad \boldsymbol{x}^2 := \{x_1^2, \ldots, x_n^2\}$, $\text{mean}(\boldsymbol{x}) = \bar{\boldsymbol{x}} = \frac{1}{n}\sum_{i=1}^n x_i$, $\quad \text{Var}(\boldsymbol{x}) = \frac{1}{n-1}\sum_{i=1}^n (x_i - \bar{\boldsymbol{x}})^2$,
$\text{Cor}(\boldsymbol{x}, \boldsymbol{y}) = \frac{\frac{1}{n-1}\sum_{i=1}^n (x_i - \bar{\boldsymbol{x}})(y_i - \bar{\boldsymbol{y}})}{\sqrt{\text{Var}(\boldsymbol{x}) \cdot \text{Var}(\boldsymbol{y})}}$.

TABLE 4. Performance of the estimators. The absolute measures are in the same units as the volatility (in percent), the sign % is omitted in absolute measures (i.e. 1 is for example the change from volatility 20% to 21%), whereas the relative measures are stated as percent of volatility (i.e. 1% is for example the change from volatility 20% to 20.2%)

SPX index	R^2	$bias_A$	sd_A	MSE_A	$bias_R$	sd_R	MSE_R
GARCH	87.8%	–3.0	4.3	5.2	–16.5%	13.7%	21.4%
EWMA(1)	83.6%	–2.8	5.0	5.7	–15.8%	17.5%	23.6%
EWMA(0.96)	91.0%	–2.8	3.8	4.7	–16.6%	14.7%	22.2%
PARK(1)	83.3%	–5.6	4.4	7.1	–27.4 %	13.4%	30.5%
PARK(0.96)	91.4%	–5.6	3.2	6.5	–28.0%	10.7%	29.9%
GK(1)	83.4%	0.4	5.7	5.7	0.1%	18.8%	18.8%
GK(0.96)	91.4%	0.4	4.5	4.5	-0.8%	15.1%	15.1%
GKYZ(1)	83.5%	0.7	5.8	5.8	1.0%	19.0%	19.1%
GKYZ(0.96)	91.5%	0.6	4.6	4.7	0.2%	15.3%	15.3%
RS(1)	82.5%	–7.0	4.7	8.4	–33.2%	12.1%	35.3%
RS(0.96)	90.7%	–7.0	3.7	7.9	–33.7%	9.8%	35.1%
index yesterday	97.4%	0.0	1.8	1.8	0.2%	5.9%	5.9%
DAX index	R^2	$bias_A$	sd_A	MSE_A	$bias_R$	sd_R	MSE_R
GARCH	86.8%	–0.7	5.1	5.1	–5.3%	16.0%	16.8%
EWMA(1)	83.4%	–0.5	5.3	5.3	–4.1%	18.2%	18.7%
EWMA(0.96)	91.0%	–0.5	4.2	4.2	–5.0%	14.7%	15.5%
PARK(1)	84.4%	–3.7	4.5	5.8	–18.2%	15.8%	24.1%
PARK(0.96)	91.4%	–3.8	3.4	5.0	–18.9%	13.2%	23.0%
GK(1)	84.1%	3.4	7.1	7.8	10.4%	21.5%	23.9%
GK(0.96)	91.2%	3.3	6.2	7.0	9.5%	18.1%	20.4%
GKYZ(1)	84.0%	4.4	7.4	8.6	15.1%	21.7%	26.4%
GKYZ(0.96)	91.3%	4.4	6.5	7.8	14.2%	17.8%	22.7%
RS(1)	84.9%	–4.4	4.3	6.2	–21.0%	15.2%	25.9%
RS(0.96)	91.2%	–4.5	3.3	5.5	–21.6%	13.0%	25.2%
index yesterday	98.2%	0.0	1.5	1.5	0.1%	4.7%	4.7%

even in the case of the simplest EWMA close-to-close volatility estimator, which is actually a special case of GARCH model with fixed parameters. This points to some possible inefficiencies in the estimation of the GARCH model or to some possible misspecifications in the GARCH model. The positive thing is, that it is probably not necessary to buy software to estimate the GARCH model in order to predict the volatility, since the simple EWMA volatility estimator works even better.

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