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DYNAMIC APPROACH TO OPTIMUM SYNTHESIS OF A FOUR-BAR MECHANISM USING A SWARM INTELLIGENCE ALGORITHM

Edgar A. Portilla-Flores, Maria B. Calva-Yáñez, Miguel G. Villarreal-Cervantes, Paola A. Niño Suárez and Gabriel Sepúlveda-Cervantes

This paper presents a dynamic approach to the synthesis of a crank-rocker four-bar mechanism, that is obtained by an optimization problem and its solution using the swarm intelligence algorithm called Modified-Artificial Bee Colony (M-ABC). The proposed dynamic approach states a mono-objective dynamic optimization problem (MODOP), in order to obtain a set of optimal parameters of the system. In this MODOP, the kinematic and dynamic models of the whole system are consider as well as a set of constraints including a dynamic constraint. The M-ABC algorithm is adapted to solve the optimization problem by adding a suitable constraint-handling mechanism that is able to incorporate the kinematic and dynamic constraints of the system. A set of independent computational runs were carried out in order to validate the dynamic approach. An analysis from the mechanical and computational point of view is presented, based on the obtained results. From the analysis of the simulation and its results, it is shown that the solutions for the proposed algorithm lead to a more suitable design based on the dynamic approach.

Keywords: synthesis, four-bar mechanism, M-ABC algorithm

Classification: 93E12, 62A10

1. INTRODUCTION

Mechanisms are used continuously in a wide variety of machines and electromechanical devices. Several working processes require a continuous input motion that provides a non-symmetrical or complex output motion. It is difficult to design a mechanism which achieves an adequate output motion as it is specified, so that the mechanism design requires an improvement. The synthesis of the mechanism could be achieved by using graphical, analytical and numerical methods [6, 17, 20]. The computational cost increases as the number of precision points increases [17]. The formulation of the mechanism synthesis as an optimization problem is an alternative approach to find the dimension of the links by using heuristic or gradient optimization techniques [11, 12].

In [1], the path synthesis of a four-bar mechanism (FBM) to track more than five points in the coupler link is solved by using three different evolutionary algorithms

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with a new refinement technique. In that work, the DE shows faster convergence to the optimal result and a smaller error of adjustment to target points, than the genetic algorithm (GA) and the particle swarm optimization (PSO). The work presented in [13] proposed an evolutionary algorithm to solve the path synthesis problem of a four-bar linkage. In [10], another design approach of a four-bar mechanism for path generation purposes is formulated as a constrained multi-objective optimization problem. The tracking error, the transmission angle's deviation and the maximum angular velocity ratio are introduced as the mechanical performance indexes. It proposes a hybridization of the traditional NSGA-II algorithm with an adaptive local search mechanism which presents a superior mechanical design in terms of energy efficiency and practical viability. In [16], an optimum synthesis of a FBM that meets the necessary input motion for a continuously variable transmission was carried out. A mechanical synthesis of a FBM is established as an optimization problem. The kinematic analysis of the FBM is presented and also objective functions and constraints are proposed, and the solutions are obtained with a swarm intelligent algorithm called Modified Bacterial Foraging Algorithm. The goal of the mechanical design is to obtain a set of dimensions of the mechanical elements of the FBM, which allows a large amplitude on the motion of the rocker as well as ensures a smooth transmission of force and speed on the joint of the connecting rod and the rocker of the FBM.

On the other hand, there are several researches focused on finding better heuristic algorithms to tackle several complex problems of the real world. Scientists have observed the nature for years in order to improve their heuristic algorithms. Natural selection eliminates species with poor foraging behaviour and favours species with high foraging behaviour which is essential for maximization of species fitness. Hence, the real world optimization problems can be solved by using heuristic algorithms based on the natural selection called bio-inspired algorithms.

Many bio-inspired algorithms use the concept of swarm intelligence (SI) such as PSO [9], artificial fish swarm algorithm [14], ant colony [4] and bacterial foraging algorithm [18]. They have been studied and used in several optimization problems. In the SI models the population of interacting agents or swarms is able to organize itself. In recent years, a new swarm intelligence has been used: the artificial bee colony algorithm [7], developed by Prof. Karaboga in 2005. The artificial bee colony (ABC) algorithm simulates the intelligent foraging behaviour of honey bee swarms. The ABC algorithm has had a rapid developed and it has been used in machining processes [19], in filter design [8] and in chaos control and synchronization of nonlinear systems [5].

In this paper, the synthesis of a four-bar mechanism that provides a symmetric motion in its rocker link is formulated as a dynamic optimization problem. In this dynamic optimization problem the kinematic and dynamic behaviours of the mechanism are merged and included as a dynamic constraint. This can be achieved because the dynamic synthesis is developed in the framework of the mechatronic design approach, where structural and control aspects must be integrated in the design of systems. On the other hand, due to the usual complexity of the dynamic problem, an heuristic algorithm called the M-ABC algorithm was used in order to solve it.

The rest of this paper is structured as follows: Section 2 describes the dynamic model of the FBM mechanism and the driving motor, Section 3 establishes the design variable vector, the design objective and the constraints of the dynamic optimization problem. The dynamic approach of the mechanism synthesis is established in Section 4. In Section 5 the ABC algorithm is explained as well as its modification for accelerating the search. The discussion of the algorithm, the optimum design and the results are presented in Section 6. Conclusions are drawn in Section 7.

2. MECHANISM SYNTHESIS PROBLEM

One of the most used mechanism in the industrial machinery is the Four-Bar mechanism (FBM). That is because the operational principle of this mechanism enables a coupling with a continuous rotational power supply in order to obtain a desired output motion.

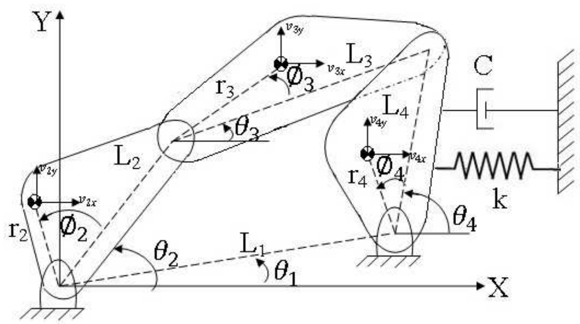


Fig. 1. A four-bar mechanism in a crank-rocker configuration.

2.1. Kinematic analysis of the FBM

The kinematics of this mechanism has been widely described [20]. The crank-rocker schematic representation is shown in Figure 1. This FBM configuration is composed by a reference bar (L_1), a crank bar (L_2), a connecting rod bar (L_3) and a rocker bar (L_4), where θ_i with $i = 1, 2, 3, 4$, is the ith angle between the horizontal axis and the ith bar in the counterclockwise direction. Also, the center of mass of each bar is denoted by a black-white circle and its location are described by r_i and ϕ_i where $i = 2, 3, 4$. Finally, $\dot{\theta}_i$ represents the angular velocity, and υ_{ix} and υ_{iy} are the x and y velocity components of the center of mass of the ith bar i.

From the kinematic analysis it can be probed that θ_3 and θ_4 can be expressed as a function of θ_2. Therefore, the equations of motion for each bar of the mechanism are established as follows:

$$\dot{\theta}_i = \gamma_i \dot{\theta}_2 \qquad i = 2, 3, 4 \tag{1}$$

$$\upsilon_{ix} = \alpha_i \dot{\theta}_2 \qquad i = 2, 3, 4 \tag{2}$$

$$\upsilon_{iy} = \beta_i \dot{\theta}_2 \qquad i = 2, 3, 4 \tag{3}$$

where:

$$\begin{aligned}
\alpha_2 &= -r_2 \sin(\theta_2 + \phi_2) && (4)\\
\alpha_3 &= -L_2 \sin\theta_2 - r_3\gamma_3 \sin(\theta_3 + \phi_3) && (5)\\
\alpha_4 &= -r_4\gamma_4 \sin(\theta_4 + \phi_4) && (6)\\
\beta_2 &= r_2 \cos(\theta_2 + \phi_2) && (7)\\
\beta_3 &= L_2 \cos\theta_2 + r_3\gamma_3 \cos(\theta_3 + \phi_3) && (8)\\
\beta_4 &= r_4\gamma_4 \cos(\theta_4 + \phi_4) && (9)\\
\gamma_2 &= 1 && (10)\\
\gamma_3 &= \frac{L_2 \sin(\theta_4 - \theta_2)}{L_3 \sin(\theta_3 - \theta_4)} && (11)\\
\gamma_4 &= \frac{L_2 \sin(\theta_3 - \theta_2)}{L_3 \sin(\theta_3 - \theta_4)}. && (12)
\end{aligned}$$

The angles θ_3 and θ_4 can be computed as follows:

$$\theta_3 = 2\text{arctan}\left[\frac{-b_1 \pm \sqrt{b_1^2 + a_1^2 - c_1^2}}{c_1 - a_1}\right] \tag{13}$$

$$\theta_4 = 2\text{arctan}\left[\frac{-e_1 \pm \sqrt{d_1^2 + e_1^2 - f_1^2}}{f_1 - d_1}\right] \tag{14}$$

where:

$$\begin{aligned}
a_1 &= 2L_3\left(L_2 \cos\theta_2 - L_1 \cos\theta_1\right) && (15)\\
b_1 &= 2L_3\left(L_2 \sin\theta_2 - L_1 \sin\theta_1\right) && (16)\\
c_1 &= L_1^2 + L_2^2 + L_3^2 - L_4^2 - 2L_1L_2 \cos\left(\theta_1 - \theta_2\right) && (17)\\
d_1 &= 2L_4\left(L_1 \cos\theta_1 - L_2 \cos\theta_2\right) && (18)\\
e_1 &= 2L_4\left(L_1 \sin\theta_1 - L_2 \sin\theta_2\right) && (19)\\
f_1 &= L_1^2 + L_2^2 + L_4^2 - L_3^2 - 2L_1L_2 \cos\left(\theta_1 - \theta_2\right). && (20)
\end{aligned}$$

In order to obtain the appropriate values of the angles θ_3 and θ_4, the sign of the radicals in (13) and (14) are $(+\sqrt{\ })$ and $(-\sqrt{\ })$, respectively; this is due to the open configuration in the FBM considered.

2.2. Dynamic analysis of the FBM

The four-bar mechanism has one degree of freedom (*dof*) in the crank (bar L_2). This *dof* is actuated by a DC motor. From the schematic representation of the mechanism in Figure 1, the mass, the inertia, the length, the mass center length and the mass center angle of the ith bar are represented by $m_i, J_i, L_i, r_i, \phi_i$, respectively. The stiffness constant of the spring and the damping coefficient of the damper are represented by k and C, respectively.

Let θ_2 and $\dot{\theta}_2$ be the generalized coordinate and velocity of the FBM, respectively, then the Lagrange's equation is formulated in (21). The variables K, P and D are the kinetic energy (22), potential energy (23) and the Rayleigh's dissipation function (24), respectively. Finally, the variable T represents the external input torque applied on crank bar. The angular position when the spring is in equilibrium is named as $\theta_{4,0}$.

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta_2}}\right)-\frac{\partial \mathcal{L}}{\partial \theta_2}+\frac{\partial D}{\partial \dot{\theta_2}}=T \tag{21}$$

where:

$$K=\sum_{i=2}^{4}\left(\frac{1}{2}m_i\left(\upsilon_{ix}^2+\upsilon_{iy}^2\right)+\frac{1}{2}J_i\dot{\theta}_i^2\right)=\frac{1}{2}A\left(\theta_2\right)\dot{\theta}_2^2 \tag{22}$$

$$P=\frac{1}{2}k\left(\theta_4-\theta_{4,0}\right)^2 \tag{23}$$

$$D=\frac{1}{2}C\dot{\theta_4}^2=\frac{1}{2}C(\gamma_4\dot{\theta_2})^2 \tag{24}$$

$$A\left(\theta_2\right)=\sum_{i=2}^{4}\left(m_i\left(\alpha_i^2+\beta_i^2\right)+\gamma_i^2 J_i\right). \tag{25}$$

It is important to remark that in the potential energy function (23), the angle θ_4 can be expressed as a function of θ_2 using (14).

Computing the total and partial derivative of (21), the motion equation of the FBM results in (26).

$$T=A\left(\theta_2\right)\ddot{\theta}_2+\frac{1}{2}\frac{\mathrm{d}A\left(\theta_2\right)}{\mathrm{d}\theta_2}\dot{\theta}_2^2+k\gamma_4\left(\theta_4-\theta_{4,0}\right)+C\gamma_4^2\dot{\theta}_2 \tag{26}$$

where:

$$A\left(\theta_2\right) = C_0+C_1\gamma_3^2+C_2\gamma_4^2+C_3\gamma_3\cos\left(\theta_2-\theta_3-\phi_3\right) \tag{27}$$

$$\begin{aligned}\frac{\mathrm{d}A\left(\theta_2\right)}{\mathrm{d}\theta_2} &= 2C_1\gamma_3\frac{\mathrm{d}\gamma_3}{\mathrm{d}\theta_2}+2C_2\gamma_4\frac{\mathrm{d}\gamma_4}{\mathrm{d}\theta_2}\\ &\quad +C_3\frac{\mathrm{d}\gamma_3}{\mathrm{d}\theta_2}\cos\left(\theta_2-\theta_3-\phi_3\right)\\ &\quad -C_3\gamma_3\left(1-\gamma_3\right)\sin\left(\theta_2-\theta_3-\phi_3\right)\end{aligned} \tag{28}$$

$$C_0 = J_2+m_2r_2^2+m_3L_2^2 \tag{29}$$

$$C_1 = J_3+m_3r_3^2 \tag{30}$$

$$C_2 = J_4+m_4r_4^2 \tag{31}$$

$$C_4 = 2m_3L_2r_3 \tag{32}$$

$$\frac{\mathrm{d}\gamma_3}{\mathrm{d}\theta_2} = \frac{L_2\left(D_1+D_2\right)}{L_3\sin^2\left(\theta_3-\theta_4\right)} \tag{33}$$

$$\frac{\mathrm{d}\gamma_4}{\mathrm{d}\theta_2} = \frac{L_2\left(D_3+D_4\right)}{L_4\sin^2\left(\theta_3-\theta_4\right)} \tag{34}$$

$$D_1 = \left(\gamma_4-1\right)\sin\left(\theta_3-\theta_4\right)\cos\left(\theta_4-\theta_2\right) \tag{35}$$

$$D_2 = \left(\gamma_4-\gamma_3\right)\sin\left(\theta_4-\theta_2\right)\cos\left(\theta_3-\theta_4\right) \tag{36}$$

$$D_3 = \left(\gamma_3-1\right)\sin\left(\theta_3-\theta_4\right)\cos\left(\theta_3-\theta_2\right) \tag{37}$$

$$D_4 = \left(\gamma_4-\gamma_3\right)\sin\left(\theta_3-\theta_2\right)\cos\left(\theta_3-\theta_4\right). \tag{38}$$

In order to model the whole system, the dynamic of the actuator must be included into the dynamics of the FBM (26). A schematic diagram of the DC motor is represented in Figure 2, where L and R represent the inductance and the armature resistance, and $i(t)$ and $u(t)$ are the input current and the input voltage, respectively. J and B are the inertia moment and the friction coefficient of the output shaft, respectively. T_L, T_m and V_b are the load torque, the magnetic motor torque and the back electromotive force of the motor, respectively. The motor constant is represented by K_f and the constant of the back electromotive force is represented by K_b.

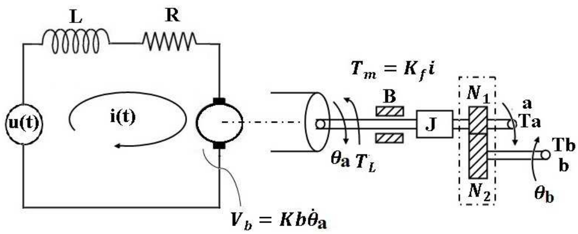


Fig. 2. Schematic diagram of a DC Motor.

The dynamic model of the DC motor [2], takes into account the electrical and mechanical subsystems. Using Kirchhoff's second law, the closed loop circuit of Figure 2 can be written as (39).

$$L\frac{\mathrm{d}i\left(t\right)}{\mathrm{d}t} + Ri\left(t\right) = u\left(t\right) - K_b\dot{\theta}_a \tag{39}$$

The equation (40) is obtained by applying the Newton's second law to the mechanical part of the DC motor, where T_a and T_b are the output torques of the shafts a and b, respectively (see Figure 2).

$$T_m - B\dot{\theta}_a - T_a - T_L = J\ddot{\theta}_a. \tag{40}$$

The mechanical transmission among the two gears in the shafts, that is the gear ratio n of the output gear box of the DC motor is expressed in (41), where r_i and N_i, $\forall\ i = 1, 2$ are the radius and the number of teeth of the gears, respectively. Moreover, $\dot{\theta}_a$ and $\dot{\theta}_b$ are the angular velocities of shafts a and b, respectively.

$$\frac{T_b}{T_a} = \frac{\dot{\theta}_a}{\dot{\theta}_b} = \frac{r_2}{r_1} = \frac{N_2}{N_1} = n. \tag{41}$$

Substituting from (41) the value of T_a into (40), the torque applied to the mechanical system is written as (42).

$$T_b = n\left(T_m - T_L - B\dot{\theta}_a - J\ddot{\theta}_a\right). \tag{42}$$

Finally, by using in (42) the mathematical relationships $\dot{\theta}_a = n\dot{\theta}_b$ from (41), $T_m = K_f i$ and $T_L = 0$, the dynamic equations of the DC motor are as follows:

$$T_b = nK_f i\,(t) - n^2 B\dot{\theta}_b - n^2 J\ddot{\theta}_b \tag{43}$$

$$L\frac{\mathrm{d}i\,(t)}{\mathrm{d}t} + Ri\,(t) = u\,(t) - nK_b\dot{\theta}_b. \tag{44}$$

Considering that the shaft "b" of the DC motor is jointed to the crank bar of the FBM, the torque and the angular displacement are related as follows: $T = T_b$, $\dot{\theta}_b = \dot{\theta}_2$. So, the next step is to couple the dynamics of the DC motor (43) – (44) with the dynamics of the FBM (26). Hence, the coupled dynamic representation of the DC Motor with the FBM in the state variable vector $\vec{x} = [x_1, x_2, x_3]^T = [\theta_2, \dot{\theta}_2, i]^T$ is given by (45).

$$\begin{aligned}\dot{\vec{x}} &= f(\vec{x}, u(t), t)\\ &= \begin{bmatrix} x_2 \\ A_0\left[A_1 x_2^2 + A_2 x_2 + nK_f x_3 + A_3\right] \\ \frac{1}{L}\left(u\,(t) - nK_b x_2 - Rx_3\right)\end{bmatrix}\end{aligned} \tag{45}$$

where:

$$A_0 = \frac{1}{A\,(x_1) + n^2 J1} \tag{46}$$

$$A_1 = -\frac{1}{2}\frac{\mathrm{d}A\,(x_1)}{\mathrm{d}x_1} \tag{47}$$

$$A_2 = -\left(C\gamma_2^4 + n^2 B\right) \tag{48}$$

$$A_3 = -k\gamma_4\left(\theta_4 - \theta_{4,0}\right). \tag{49}$$

3. OPTIMAL STRATEGY

As it was previously mentioned, the goal of the dynamic approach to optimum synthesis of the FBM is not only to consider the kinematics of the mechanism, but to consider both the dynamics and the kinematics. This work represents a first step in the development of this approach.

3.1. Design variables

Once the dynamic and kinematic analysis of the FBM are carried out, it should be clear that the vector of design variables is composed by the dimensions of the bars and the θ_1 angle of the reference bar. Let the vector of design variables $\vec{p}$ defined as follows:

$$\vec{p} = (p_1, p_2, p_3, p_4, p_5)^T = (L_1, L_2, L_3, L_4, \theta_1)^T. \tag{50}$$

3.2. Objective function

It is important to mention that in this synthesis case, the motion amplitude of the rocker is the output of the system. A maximum value for the objective function expressed in

(51), implies the maximization of the output of the FBM. From the kinematic analysis [20], the amplitude of the motion of this mechanical element is given by:

$$\Phi_1 = \theta_{4\,\max} - \theta_{4\,\min} \tag{51}$$

where $\theta_{4\,\max}$ and $\theta_{4\,\min}$ are computed according with the four-bar configuration as follows:

- Case a) $\theta_1 < 0$:

$$\begin{aligned} \theta_{4\,\max} &= \pi - \left[abs(\theta_1) + \arccos\left(\frac{L_1^2 + L_4^2 - (L_3 - L_2)^2}{2L_1L_4}\right)\right] & (52)\\ \theta_{4\,\min} &= \pi - \left[abs(\theta_1) + \arccos\left(\frac{L_1^2 + L_4^2 - (L_3 + L_2)^2}{2L_1L_4}\right)\right]. & (53) \end{aligned}$$

- Case b) $\theta_1 = 0$:

$$\begin{aligned} \theta_{4\,\max} &= \pi - \arccos\left(\frac{L_1^2 + L_4^2 - (L_3 - L_2)^2}{2L_1L_4}\right) & (54)\\ \theta_{4\,\min} &= \pi - \arccos\left(\frac{L_1^2 + L_4^2 - (L_3 + L_2)^2}{2L_1L_4}\right). & (55) \end{aligned}$$

- Case c) $\theta_1 > 0$:

$$\begin{aligned} \theta_{4\,\max} &= \pi + \left[abs(\theta_1) - \arccos\left(\frac{L_1^2 + L_4^2 - (L_3 - L_2)^2}{2L_1L_4}\right)\right] & (56)\\ \theta_{4\,\min} &= \pi + \left[abs(\theta_1) - \arccos\left(\frac{L_1^2 + L_4^2 - (L_3 + L_2)^2}{2L_1L_4}\right)\right]. & (57) \end{aligned}$$

3.3. Design constraints

In order to obtain a set of values which produce a suitable FBM, a set of design constraints is established.

3.3.1. Grashof's law

In order to conduct a continuous motion, the mechanical elements of the FBM must fulfill the Grashof's law: *for a plane four-bar linkage, the sum of the length for the shortest and largest links can not be greater than the sum of the length of the two remaining links, if a continuous relative rotation between any two elements is desired* [17]. Denoting as s and l the shortest and the largest links of the four-bar mechanism, and as p and q the other two links, Grashof's law is established as detailed in (58).

$$s + l \le p + q. \tag{58}$$

In the problem tackled in this work, Grashof's law is given by:

$$L_2 + L_3 \le L_1 + L_4. \tag{59}$$

Furthermore, in order to ensure that the solution method produces Grashof-type mechanisms, the solutions must fulfills the conditions established in (60) and (61).

$$L_1 \leq L_3 \tag{60}$$
$$L_4 \leq L_3. \tag{61}$$

3.3.2. Transmission angle

One of the most used characteristics to evaluate the quality of a linked mechanism is the measure of its transmission angle. The transmission angle is defined as [17]:*the angle between the output link and the coupler link. Generally taken as the absolute value of the pair of acute angle corners formed at the intersection of the two links and varies continuously from a maximum value to a minimal, as the linkage passes through its range of motion.*

The transmission angle in a FBM is defined as follows:

$$\mu = \begin{cases} |\theta_3 - \theta_4|, & \text{if } \mu \leq \frac{\pi}{2} \\ \pi - \mu, & \text{if } \mu > \frac{\pi}{2} \end{cases} \tag{62}$$

where the angles θ_3 and θ_4 are computed from (13) and (14).

A recommended design constraint is that μ must be greater than 45° along the crank cycle, so that it fulfills (63).

$$\mu \geq 45^\circ. \tag{63}$$

3.3.3. Motion Symmetry

A necessary characteristic of the rocker motion is a symmetrical motion around the vertical axis. Hence, in order to guarantee the symmetrical motion, an equality constraint (64) is established,

$$180^\circ - \theta_{4\,\max} = \theta_{4\,\min}. \tag{64}$$

3.3.4. System size

Due to the available space, the mechanical elements of the FBM must fulfill dimensional constraints. For this reason, the length of each bar is determined between 0.05m and 0.5m, and these constraints are presented in (65) to (68).

$$0.05 \leq L_1 \leq 0.5 \tag{65}$$
$$0.05 \leq L_2 \leq 0.5 \tag{66}$$
$$0.05 \leq L_3 \leq 0.5 \tag{67}$$
$$0.05 \leq L_4 \leq 0.5\,. \tag{68}$$

On the other hand, the angle between the horizontal axis and the reference bar (L_1) is limited between 45° and −45°, as pointed out in (69).

$$-45^\circ \leq \theta_1 \leq 45^\circ. \tag{69}$$

4. DYNAMIC APPROACH STATEMENT FOR THE DIMENSIONAL SYNTHESIS

The dynamic approach statement for the dimensional synthesis of the FBM is formulated as a mono-objective dynamic optimization problem (MODOP). This MODOP consists on finding the optimal design variables $\vec{p}^* \in \mathbb{R}^5$ which maximize the performance function (70) subject to the dynamic behaviour of the FBM represented in state variables (71) and the inequality and equality constraints in the design (72)–(76); it is important to remark that the inequality constraint 75 is a dynamic constraint which is evaluated using the profile of the state vector.

$$\max_{\vec{p} \in \mathbb{R}^5} \quad \Phi_1(\vec{p}) = (\theta_{4\,\max} - \theta_{4\,\min})^2 \tag{70}$$

subject to:

$$\begin{aligned}
\dot{\vec{x}} &= f(\vec{x}, \vec{p}, t) && (71)\\
g_1(\vec{p}) &= p_2 + p_3 - p_1 - p_4 \leq 0 && (72)\\
g_2(\vec{p}) &= p_1 - p_3 \leq 0 && (73)\\
g_3(\vec{p}) &= p_4 - p_3 \leq 0 && (74)\\
g_4(\vec{p}, t) &= \frac{\pi}{4} - \mu(\vec{p}, t) \leq 0 && (75)\\
h_1(\vec{p}) &= \pi - \theta_{4\,\max} - \theta_{4\,\min} = 0 && (76)
\end{aligned}$$

$$0.05 \leq p_i \leq 0.5, \quad i = 1, \ldots, 4 \quad \text{and} \ -\tfrac{\pi}{4} \leq p_5 \leq \tfrac{\pi}{4}. \tag{77}$$

5. SWARM INTELLIGENCE STRATEGY

Currently, Swarm Intelligence Algorithms (SIA's) are a suitable option in order to solve optimization problems. One of the most popular algorithms is the Artificial Bee Colony (ABC), which is an algorithm based on the foraging behaviour of the honey bee [7]. Originally, this algorithm deals with unconstrained non-linear optimization problems. However, taking into account that the engineering problems usually include a set of constrains, a Modified Artificial Bee Colony (M-ABC) for constrained numerical optimization version [15] was used in this work. In this section, a brief explanation of the main aspects of the ABC algorithm is presented. Then, the M-ABC algorithm is presented, remarking the computational implementation that was carried out in the present work.

5.1. Artificial Bee Colony

In [7], the process of the search of nectar in the flowers by the honey bees has been seen as an optimization process. The way that this kind of social insects manages to focus efforts on areas with high amounts of food sources has been modelled as a heuristic for optimization. Two behaviours are used in order to do this: the recruitment of bees into a food source and the abandonment of a source. It is important to remark that in the ABC algorithm, the solutions of the problem are represented by the food sources, not by the bees. The bees act as variation operators, so when one of them comes to a food source, the bee calculates a new candidate solution based on this source.

In the ABC algorithm, the colony of artificial bees consists of three types of bees: employed, onlooker and scout bees. Usually, the number of employed bees is equal to the number of food sources and each employed bee will be assigned to each one of the sources. When the bee arrives to the food source, it will calculate a new solution (the bee will fly to another nearby food source) from it and retain the best solution based on a greedy selection. The number of onlooker bees is usually the same as the number of employed bees. This type of bees is assigned to a food source according to the profitability of such source. In the same way that the employed bees, the onlooker bees will calculate a new solution based on their assigned food source. When a food source profitability does not improve after a certain number of iterations, this source is abandoned and is replaced by a new one randomly assigned. The user-defined parameters required by the ABC algorithm are: the number of food sources or solutions SN, the total number of iterations or cycles MCN, and the number of cycles that a non improved food source will be kept before being replaced by a new source, *limit*. It is important to remark that an advantage of the ABC algorithm is that the solutions are real-encoding. Therefore, it is widely used in engineering design problems.

5.2. Modified Artificial Bee Colony

The M-ABC algortihm is shown in Figure 3. In this algorithm, the variation operator used by both employed and onlooker bees in order to generate a new candidate solution ν_i^g includes a recombination mechanism. The variation operator is given by:

$$\nu_{i,j}^g = \begin{cases} x_{i,j}^g + \phi_j \cdot (x_{i,j}^g - x_{k,j}^g), & \text{if rand(0,1)} < \text{MR} \\ x_{i,j}^g, & \text{otherwise} \end{cases} \tag{78}$$

where subscript j indicates the corresponding variable of the ith candidate solution at iteration g, x_i^g represents the solution in which the bee is located at that moment, x_k^g is a randomly chosen food source (which must be different to x_i^g) and ϕ is a real number within $[-1, 1]$ generated randomly for every variable. Finally, the user-defined recombination mechanism is established as $0 \leq MR \leq 1$. On the other hand, in order to select the best food source, a tournament selection is carried out based on the set of rules defined in [3]. Such set of rules is stated as follows:

- Between two feasible food sources, the one with the best objective function value is preferred.
- Between a feasible food source and an infeasible food source, the feasible one is preferred.
- Between two infeasible food sources, the one with the lowest value of the sum of constraint violations is preferred.

In the M-ABC algorithm, a dynamic tolerance for equality constraints is proposed. Such mechanism is established as follows:

$$\epsilon(g+1) = \frac{\epsilon(g)}{dec} \tag{79}$$

where g is the current iteration and dec is the decreasing rate value of each iteration ($dec > 1$). The aim of this is to start with a feasible region larger than the original one, in order to meet in an easier way the equality constraints at the beginning of the iterations. A simple way to compute the decreasing rate value is given by:

$$dec = e^{\left(\frac{ln(\epsilon_0) - ln(\epsilon_f)}{MCN}\right)} \tag{80}$$

where ϵ_0 is the initial tolerance value and ϵ_f is the final tolerance value.

In the M-ABC algorithm, a smart flight operator is included. This operator combines three elements: (1) the information of the solution to be replaced x_i^g, the solution that is used as a reference point in order to generate a new solution, after this, the original solution is eliminated, (2) the best solution x_B^g, that will bias the location of the new solution, the aim of including this food source is to find a feasible solution or, at least, an infeasible solution closer to the feasible region, and (3) a solution which is randomly selected x_k^g in order to avoid a full attraction by the best solution so far. The new solution taking into account the smart flight operator is computed as follows:

$$\nu_{i,j}^g = x_{i,j}^g + \phi \cdot (x_{k,j}^g - x_{i,j}^g) + (1 - \phi) \cdot (x_{B,j}^g - x_{i,j}^g). \tag{81}$$

Finally, the boundary constraint-handling used for the design variables is implemented as follows:

$$\nu_{i,j}^g = \begin{cases} 2 * L_j - \nu_{i,j}^g, & \text{if } \nu_{i,j}^g < L_j \\ 2 * U_j - \nu_{i,j}^g, & \text{if } \nu_{i,j}^g > U_j \\ \nu_{i,j}^g, & \text{otherwise} \end{cases} \tag{82}$$

where, as previously mentioned $\nu_{i,j}^g$ is the jth variable of the ith candidate solution at iteration g. Finally, L_j is the lower limit and U_j is the upper limit of the jth variable, respectively.

5.3. Implementation issues

As it is mentioned in this section, a M-ABC algorithm was used. However, due to the type of optimization problem some adaptations were carried out:

- Since the solution of the optimization problem must meet the Grashof's criteria, the set of rules was not applied directly, in a step prior to the tournament it was verified that the food sources participants meet the Grashof's criteria. In fact, in the lines 15, 21 and 24 of the M-ABC algorithm, the selection between the contenders is based on the next rules:

  1. When both food sources meet the Grashof's criteria, the set of rules is applied.
  2. When a food source meets the Grashof's criteria and the other one does not, the Grashof's one is preferred.
  3. When both food sources do not meet the Grashof's criteria, the one with the lowest value of the sum of constraint violations is preferred.

```
1   BEGIN
2     Initialize the set of food sources $x_i^0$, $i = 1, \ldots, SN$
3     Evaluate each $x_i^0$, $i = 1, \ldots, SN$
4     $g = 1$
5     IF There are equality constraints
6       Initialize $\epsilon(g)$
7     END IF
8     REPEAT
9       IF There are equality constraints
10        Evaluate each $x_i^0$, $i = 1, \ldots, SN$ with $\epsilon(g)$
11      END IF
12      FOR $i = 1$ TO $SN$
13        Generate $\nu_i^g$ with $x_i^{g-1}$ by using Eq. 78
14        Evaluate $\nu_i^g$
15        IF $\nu_i^g$ is better than $x_i^{g-1}$(based on feasibility criteria in Section 5.2)
16          $x_i^g = \nu_i^g$
17        ELSE
18          $x_i^g = x_i^{g-1}$
19      END FOR
20      FOR $i = 1$ TO $SN$
21        Select food source $x_i^g$ based on binary tournament selection (Section 5.2)
22        Generate $\nu_i^g$ with $x_i^g$ by using Eq. 78
23        Evaluate $\nu_i^g$
24        IF $\nu_i^g$ is better than $x_i^g$(based on feasibility criteria in Section 5.2)
25          $x_i^g = \nu_i^g$
26        END IF
27      END FOR
28      Apply the smart flight by the scout bees (Eq. 81) for those
        solutions whose limit to be improved has been reached
29      Keep the best solution so far
30      $g = g + 1$
31      IF There are equality constraints
32        Update $\epsilon(g)$ by using Eq. FF
33      END IF
34    UNTIL $g = MCN$
35  END
```

Fig. 3. Modified Artificial Bee Colony Algortihm (M-ABC).

- In order to apply the smart flight operator (line 28 of the M-ABC algorithm), the best source is needed, therefore, the search of such source is carried out as follows:

    1. In the current distribution of food sources, a search by a food source that meets the Grashof's law is carried out. Once that a food source is found, a search based on the three rules above mentioned continues for this type of food sources, in order to compare and select the best one of the whole distribution

2. In case that the current distribution of food sources does not include Grashof's sources, a search for the food source with the lowest value of the sum of constraint violations is carried out.

- Since the inequality constraint g_4 of the optimization problem is a dynamic constraint, it is evaluated only when the food source meets the Grashof's criteria.

It is important to remark that the constraints applied to the M-ABC algorithm are originated by the type of optimization problem that is solved in the present work, but the mean idea of the algorithm is preserved.

6. RESULTS AND DISCUSSION

In the present work, a set of 10 independent runs was carried out. A fixed set of values for the M-ABC parameters was used in all runs as follows: number of solutions $SN = 20$, maximum cycle number $MCN = 10000$, $Limit = MCN/(2 * SN) = 250$, and modification rate $MR = 0.8$. On the other hand, in order to evaluate the equality constraint, the initial and final desired values of ϵ_0 and ϵ_f were taken as 1.0 and 0.01, respectively. The M-ABC algorithm was coded in Matlab® R2008a and was run in a Laptop computer with 6 GB RAM, Intel® Core i5 processor @ 2.5 GHz, and Microsoft Windows® 7 OS.

The results of these computational experiments are shown in Table 1. The statistical analysis of the independent runs can be observed in Table 2. Finally, the time required per run is shown in Table 3.

Run	**Vector of design variables**	**Objective function**
1	0.434183161 0.111987151 0.434193070 0.200010277 -0.198555316	1.474881068
2	0.436286674 0.112199817 0.436288484 0.200024842 -0.197834785	1.480289545
3	0.462774558 0.113985089 0.462909518 0.200010509 -0.183396807	1.527252312
4	0.499960734 0.116714229 0.499963500 0.200009037 -0.169495104	1.602442026
5	0.499314875 0.116673002 0.499315293 0.200004459 -0.169621422	1.601355458
6	0.496355846 0.116497626 0.496356328 0.200014750 -0.170800563	1.596298246
7	0.499756750 0.116698292 0.499759307 0.200000141 -0.169562651	1.602141700
8	0.499882643 0.116710523 0.499883948 0.200010631 -0.169479315	1.602308234
9	0.496954796 0.116528730 0.496956381 0.200001937 -0.170573303	1.597376571
10	0.496757589 0.116506497 0.496765845 0.200040121 -0.170134742	1.596069930

Tab. 1. Details of the solutions obtained by the M-ABC algorithm.

From Tables 1 and 2 it can be observed that the M-ABC algorithm has a steady behaviour from a computational point of view, as the best and worst solutions shown a similar performance. On the other hand, the computational time for the execution of the algorithm is not expensive, taking into account the kind of problem that was solved, as can be seen in Table 3.

Best	**1.602442026**
Mean	**1.568041509**
Worst	**1.474881068**
Std. Dev.	**0.050117723**

Tab. 2. Statistical results for the independent runs by the M-ABC algorithm.

Run	Time required/Hrs
1	**1.10**
2	**1.13**
3	**1.06**
4	**0.94**
5	**0.94**
6	**0.83**
7	**0.82**
8	**0.86**
9	**0.96**
10	**0.81**
Average	**0.94**

Tab. 3. Time required at each independent run by the M-ABC algorithm.

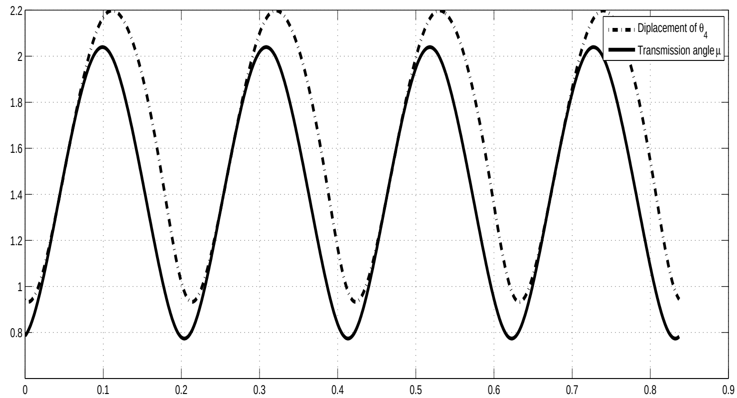
6.1. Mechanical analysis of solutions

It is worth recalling that the goal of the optimization problem is to obtain a set of values for the FBM mechanism. Therefore, the best and worst solutions obtained by the M-ABC algorithm were subject to simulation in order to obtain more information about the mechanical performance of the resulting mechanism. The simulation results for the displacement angle (θ_4) of the rocker and the transmission angle (μ) for the best and worst solutions obtained, are shown in Figure 4.

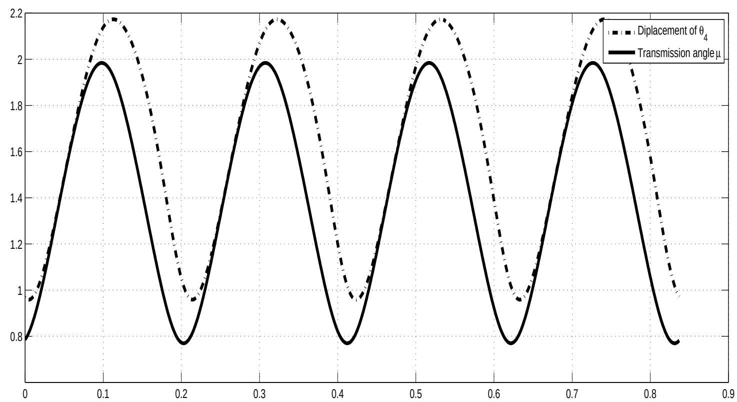
As it is mentioned in Section 3, the optimal set of values for the FBM mechanism, should allow a symmetric displacement of the rocker around the vertical axis ($\frac{\pi}{2}$ rad is the reference value), and in order to obtain a high efficiency of the mechanism, the transmission angle should be greater than $\frac{\pi}{4}rad$ and stay around of $\frac{\pi}{2}rad$, along the entire motion of the mechanism. As it can be observed in Figure 4, the best and worst solutions have values inside the requirements previously established on the optimal strategy.

7. CONCLUSIONS

In the present work a dynamic approach to obtain the optimum synthesis of a FBM mechanism using a swarm intelligence algorithm is presented. The mechanical synthesis is established as an optimization problem, where the dynamic model of the system is



a)



b)

Fig. 4. Simulation of the angular displacement of the rocker θ_4 and transmission angle μ for the best *a*) and the worst *b*) solution in Table 1.

considered besides a set of constraints and an objective function. It is important to remark that one of the constraints is a dynamic one.

A swarm intelligence algorithm called Modified-Artificial Bee Colony was implemented to obtain the solution of the optimization problem. A set of independent runs were carried out in order to test both the performance and the behaviour of the algorithm and its solutions.

The results presented in this work show that the dynamic approach proposed is suitable for the analysis and design of this type of planar mechanisms. Also, the computational implementation of the M-ABC algorithm allows its application without important changes or adaptations, in order to solve real-world problems.

Future work will include a design methodology which deals with the optimal design, considering both the mechanical structure and the controller simultaneously.

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