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BOUNDEDNESS OF SOLUTIONS TO PARABOLIC-ELLIPTIC CHEMOTAXIS-GROWTH SYSTEMS WITH SIGNAL-DEPENDENT SENSITIVITY

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Abstract. This paper deals with parabolic-elliptic chemotaxis systems with the sensitivity function $\chi(v)$ and the growth term $f(u)$ under homogeneous Neumann boundary conditions in a smooth bounded domain. Here it is assumed that $0 < \chi(v) \leqslant \chi_0/v^k$ ($k \geqslant 1$, $\chi_0 > 0$) and $\lambda_1 - \mu_1 u \leqslant f(u) \leqslant \lambda_2 - \mu_2 u$ ($\lambda_1, \lambda_2, \mu_1, \mu_2 > 0$). It is shown that if χ_0 is sufficiently small, then the system has a unique global-in-time classical solution that is uniformly bounded. This boundedness result is a generalization of a recent result by K. Fujie, M. Winkler, T. Yokota.

Keywords: chemotaxis; global existence; boundedness

MSC 2010: 35B40, 35K60

1. Introduction and main result

In this paper we consider the global existence and boundedness in the parabolic-elliptic chemotaxis-growth system

$$\begin{cases} u_t = \Delta u - \nabla\cdot(u\chi(v)\nabla v) + f(u), & x\in\Omega,\ t>0,\\ 0 = \Delta v - v + u, & x\in\Omega,\ t>0,\\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0) = u_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ ($n\in\mathbb{N}$) with smooth boundary $\partial\Omega$. We assume

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that the initial data u_0 satisfies

$$u_0 \in C^0(\overline{\Omega}), \quad u_0 \geqslant 0 \quad \text{and} \quad \int_\Omega u_0 > 0. \tag{1.2}$$

As for the chemotactic sensitivity function, we assume that

$$\chi \in C^1((0,\infty)) \quad \text{with } \chi > 0. \tag{1.3}$$

Also we assume that $f \in C^1([0,\infty))$ and there exist λ_1, λ_2, μ_1, $\mu_2 > 0$ such that

$$\lambda_1 - \mu_1 s \leqslant f(s) \leqslant \lambda_2 - \mu_2 s \quad \text{for all } s \in [0,\infty). \tag{1.4}$$

This system was introduced by Keller and Segel [6], [7] (see also [4], [14], [15]), and the mathematical study of this system has developed extensively. In this paper we especially focus on the signal-sensitivity function and the growth term. There are some known results related to this system in [1], [2], [8]–[13], [16]–[19]. The present work is devoted to the global existence and boundedness. We remark that the existence of classical solutions to (1.1) is shown by a similar way as in [3]. Since $f(0) \geqslant \lambda_1 > 0$ by (1.4), the solution to (1.1) is nonnegative.

In order to formulate our main result, given a nonnegative $0 \not\equiv u_0 \in C^0(\overline{\Omega})$, let us define a constant $\gamma > 0$ as

$$\gamma := \min\left\{\|u_0\|_{L^1(\Omega)}, \frac{\lambda_1}{\mu_1}|\Omega|\right\} \int_0^\infty \frac{1}{(4\pi t)^{n/2}} \,\mathrm{e}^{-(t+(\operatorname{diam}\Omega)^2/(4t))}\,\mathrm{d}t < \infty, \tag{1.5}$$

where $\operatorname{diam}\Omega := \max_{x,y\in\overline{\Omega}} |x-y|$. We remark that the integrand in (1.5) decays exponentially not only as $t \to \infty$ but also as $t \to 0$, and so $\gamma < \infty$ for all $n \in \mathbb{N}$. The constant γ marks an a priori pointwise lower bound on the solution component v, as we shall see below. In what follows, when $k = 1$ we regard the value of $k^k/(k-1)^{k-1}$ as 1.

Theorem 1.1. *Let $n \in \mathbb{N}$, and suppose that u_0, χ and f satisfy* (1.2), (1.3) *and* (1.4), *respectively. Moreover, assume that χ satisfies*

$$\chi(s) \leqslant \frac{\chi_0}{s^k} \quad \textit{for all } s \in [\gamma,\infty),$$

with some $k \geqslant 1$ and some $\chi_0 > 0$ fulfilling

$$\chi_0 < \frac{2}{n}\frac{k^k}{(k-1)^{k-1}}\gamma^{k-1}.$$

Then (1.1) *possesses a unique global classical solution* (u, v) *which satisfies*

$$\|u(\cdot,t)\|_{L^\infty} \leqslant M_\infty \quad \textit{for all } t \in [0,\infty)$$

with some constant $M_\infty > 0$.

2. Preliminaries

We begin with the following lemma shown in [3]. This lemma is key to deriving a *uniform-in-time* estimate for v.

Lemma 2.1. *Let* $w \in C^0(\overline{\Omega})$ *be a nonnegative function such that* $\int_\Omega w > 0$. *If* z *is a weak solution to*

$$\begin{cases} -\Delta z + z = w, & x \in \Omega, \\ \dfrac{\partial z}{\partial \nu} = 0, & x \in \partial\Omega, \end{cases}$$

then

$$z \geqslant \left(\int_0^\infty \frac{1}{(4\pi t)^{n/2}} \mathrm{e}^{-(t+(\operatorname{diam}\Omega)^2/(4t))}\,\mathrm{d}t\right)\int_\Omega w > 0 \quad \textit{in } \Omega.$$

Here we give an a priori pointwise lower bound on the solution component v. The first equation in (1.1) and the condition (1.4) imply

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega u = \int_\Omega f(u) \geqslant \lambda_1|\Omega| - \mu_1\int_\Omega u.$$

Integrating this inequality, we have

$$\int_\Omega u \geqslant \frac{\lambda_1}{\mu_1}|\Omega| + \mathrm{e}^{-\mu_1 t}\Big(\|u_0\|_{L^1(\Omega)} - \frac{\lambda_1}{\mu_1}|\Omega|\Big) \quad \text{for all } t \in (0,\infty),$$

and then

$$\int_\Omega u \geqslant \min\Big\{\|u_0\|_{L^1(\Omega)}, \frac{\lambda_1}{\mu_1}|\Omega|\Big\}.$$

By virtue of Lemma 2.1 we can thereby estimate v from below as follows:

$$v(x,t) \geqslant \gamma \tag{2.1}$$

for all $x \in \Omega$ and $t \in (0,T)$, whenever (u,v) solves (1.1) in $\Omega \times (0,T)$ for some $T > 0$. Here $\gamma > 0$ is a constant defined as (1.5).

Remark 2.1. The maximum principle yields the lower *pointwise* estimate for $v(\cdot,t)$ for fixed $t > 0$. On the other hand, Lemma 2.1 and the uniform-in-time estimate for mass imply the *uniform* estimate (2.1).

We next collect some known facts concerning the Neumann Laplacian in Ω. For the proof of (iii) see [5], Lemma 2.1.

Lemma 2.2. *For $r \in (1, \infty)$, let Δ denote the realization of the Laplacian in $L^r(\Omega)$ with domain $\{w \in W^{2,r}(\Omega);\ \partial w/\partial\nu = 0 \text{ on } \partial\Omega\}$. Then the operator $-\Delta + 1$ is sectorial and possesses closed fractional powers $(-\Delta+1)^\theta$, $\theta \in (0,1)$, with dense domain $D((-\Delta+1)^\theta)$. Moreover, the following statements hold:*

(i) *If $m \in \{0,1\}$, $p \in [1,\infty]$ and $q \in (1,\infty)$, then there exists a constant $c_{m,p} > 0$ such that for all $w \in D((-\Delta+1)^\theta)$,*

$$\|w\|_{W^{m,p}(\Omega)} \leqslant c_{m,p} \|(-\Delta+1)^\theta w\|_{L^q(\Omega)},$$

provided that $m < 2\theta$ and $m - n/p < 2\theta - n/q$.

(ii) *Let $p \in (1,\infty)$. Then there exist $c > 0$ and $\nu_1 > 0$ such that for all $u \in L^p(\Omega)$ and any $t > 0$,*

$$\|(-\Delta+1)^\theta \mathrm{e}^{t(\Delta-1)} u\|_{L^p(\Omega)} \leqslant c t^{-\theta} \mathrm{e}^{-\nu_1 t} \|u\|_{L^p(\Omega)}.$$

(iii) *Let $p \in (1,\infty)$. Then there exists $\nu_2 > 1$ such that for $\varepsilon > 0$ there exists $c_\varepsilon > 0$ such that for all $\mathbb{R}^n$-valued $z \in C_0^\infty(\Omega)$,*

$$\|(-\Delta+1)^\theta \mathrm{e}^{t(\Delta-1)} \nabla \cdot z\|_{L^p(\Omega)} \leqslant c_\varepsilon t^{-\theta-1/2-\varepsilon} \mathrm{e}^{-\nu_2 t} \|z\|_{L^p(\Omega)}, \quad t > 0.$$

Accordingly, for all $t > 0$ the operator $(-\Delta+1)^\theta \mathrm{e}^{t\Delta}\nabla\cdot$ admits a unique extension to all of $L^p(\Omega)$ which, again denoted by $(-\Delta+1)^\theta \mathrm{e}^{t\Delta}\nabla\cdot$, satisfies the above estimate for all $\mathbb{R}^n$-valued $z \in L^p(\Omega)$.

3. Proof of main result

We first deduce L^p-boundedness of solutions to (1.1). Next let us show that L^p-boundedness with sufficiently large p implies L^∞-boundedness. Combining these results will prove our main theorem.

Lemma 3.1. *Let $p > 1$, and suppose that (u, v) is a classical solution to (1.1) in $\Omega \times (0,T)$ for some $T > 0$. Then there exist $C_1, C_2 > 0$ such that*

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega u^p \leqslant & -\frac{p(p-1)}{2}\int_\Omega u^{p-2}|\nabla u|^2 + \frac{p(p-1)}{2}\int_\Omega u^p \chi^2(v)|\nabla v|^2 \\ & + C_1 \int_\Omega u^p + C_2 \quad \textit{for all } t \in (0,T).\end{aligned}$$

P r o o f. By virtue of the first equation in (1.1) and Young's inequality, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega u^p \leqslant -\frac{p(p-1)}{2}\int_\Omega u^{p-2}|\nabla u|^2 + \frac{p(p-1)}{2}\int_\Omega u^p\chi^2(v)|\nabla v|^2 + \int_\Omega u^{p-1}f(u).$$

The condition (1.4) yields $\int_\Omega u^{p-1}f(u) \leqslant \lambda_2\int_\Omega u^{p-1} - \mu_2\int_\Omega u^p \leqslant C_1\int_\Omega u^p + C_2$ for some constants $C_1, C_2 > 0$, and hence we obtain the desired inequality. $\square$

The next lemma is obtained in [3]. For convenience we give the sketch of the proof.

Lemma 3.2. *Let $p > 1$, and suppose that (u,v) is a classical solution to* (1.1) *in $\Omega\times(0,T)$ for some $T > 0$. Moreover, for $\gamma > 0$ given by* (1.5) *(see also* (2.1)*), let $\varphi \in C^1([\gamma,\infty))$ such that $\varphi \geqslant 0$ and there exists a constant $M > 0$ satisfying $s\varphi(s) \leqslant M$ for all $s \geqslant \gamma$. Let A and B be positive constants such that $AB = p$. Then*

$$\int_\Omega u^p\Big(-\varphi'(v) - \frac{B^2}{2}\varphi^2(v)\Big)|\nabla v|^2 \leqslant \frac{A^2}{2}\int_\Omega u^{p-2}|\nabla u|^2 + M\int_\Omega u^p \quad \textit{for all } t\in(0,T).$$

Sketch of the p r o o f. Multiplying the second equation in (1.1) by $u^p\varphi(v)$ and using integration by parts, we see that

$$-\int_\Omega u^p\varphi'(v)|\nabla v|^2 = p\int_\Omega u^{p-1}\varphi(v)\nabla u\cdot\nabla v + \int_\Omega u^p\varphi(v)v - \int_\Omega u^{p+1}\varphi(v).$$

Applying Young's inequality completes the proof. $\square$

Now we give L^p-boundedness of solutions to (1.1).

Proposition 3.3. *Suppose that $n\in\mathbb{N}$, and that u_0, χ and f satisfy* (1.2), (1.3) *and* (1.4)*, respectively. Let (u,v) be a classical solution to* (1.1) *in $\Omega\times(0,T)$ for some $T > 0$. Moreover, let $\gamma > 0$ be as in* (1.5) *and* (2.1)*. Suppose that there exist $k \geqslant 1$ and $\chi_0 > 0$ such that $\chi(s) \leqslant \chi_0/s^k$ for all $s \geqslant \gamma$. Then for any $p \in \big[1, \chi_0^{-1}\big[k^k/(k-1)^{k-1}\big]\gamma^{k-1}\big)$ there exists a constant $M_p > 0$ fulfilling*

$$\|u(\cdot,t)\|_{L^p} \leqslant M_p \quad \textit{for all } t\in[0,T).$$

P r o o f. Taking any $p \in \big[1, \chi_0^{-1}\big[k^k/(k-1)^{k-1}\big]\gamma^{k-1}\big)$, we have $\chi_0 < p^{-1}\big[k^k/(k-1)^{k-1}\big]\gamma^{k-1}$. Now we take $\varepsilon > 0$ and $L > 0$ such that

$$\varepsilon < p(p-1), \quad L < \gamma < \frac{k}{k-1}L \quad \text{and} \quad \chi_0 \leqslant \frac{1}{p}\sqrt{\frac{p(p-1)-\varepsilon}{p(p-1)}}\,\frac{k^k}{(k-1)^{k-1}}L^{k-1}.$$

Applying Lemma 3.2 to $\varphi(s) := 1/(B^2(s-L))$, $A := \sqrt{p(p-1)-\varepsilon}$ and $B := p/\sqrt{p(p-1)-\varepsilon}$, we infer that

$$\int_\Omega u^p\Big(-\varphi'(v)-\frac{B^2}{2}\varphi^2(v)\Big)|\nabla v|^2 \leqslant \frac{p(p-1)-\varepsilon}{2}\int_\Omega u^{p-2}|\nabla u|^2 + M\int_\Omega u^p \tag{3.1}$$

and

$$\frac{p(p-1)}{2}\chi^2(s) \leqslant -\varphi'(s)-\frac{B^2}{2}\varphi^2(s) \quad \text{for all } s \geqslant \gamma. \tag{3.2}$$

Now by (3.2), we can combine (3.1) with Lemma 3.1 to see that

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega u^p \leqslant & -\frac{p(p-1)}{2}\int_\Omega u^{p-2}|\nabla u|^2 + \frac{p(p-1)-\varepsilon}{2}\int_\Omega u^{p-2}|\nabla u|^2 \\ & + (M+C_1)\int_\Omega u^p + C_2 \\ = & -\frac{\varepsilon}{2}\int_\Omega u^{p-2}|\nabla u|^2 + (M+C_1)\int_\Omega u^p + C_2\end{aligned} \tag{3.3}$$

for all $t \in (0,T)$. Since the first equation in (1.1) and the condition (1.4) yield

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega u = \int_\Omega f(u) \leqslant \lambda_2|\Omega| - \mu_2\int_\Omega u,$$

we see that for all $t \in (0,\infty)$,

$$\int_\Omega u \leqslant \frac{\lambda_2}{\mu_2}|\Omega| + \mathrm{e}^{-\mu_2 t}\Big(\|u_0\|_{L^1(\Omega)} - \frac{\lambda_2}{\mu_2}|\Omega|\Big) \leqslant \max\Big\{\|u_0\|_{L^1(\Omega)}, \frac{\lambda_2}{\mu_2}|\Omega|\Big\}.$$

By virtue of this estimate, proceeding similarly as in [3], Proposition 4.3, we can complete the proof from (3.3). □

Next, assuming L^p-boundedness, we derive L^∞-boundedness.

Proposition 3.4. *Let $n \in \mathbb{N}$, and assume that u_0, χ and f satisfy* (1.2), (1.3) *and* (1.4)*, respectively. Let (u,v) be the classical solution to* (1.1) *in $\Omega \times (0,T)$, and assume further that $\chi \in L^\infty((\gamma,\infty))$ with $\gamma > 0$ given by* (1.5) (see also (2.1)). *Then if there exist $p > n/2$ and a constant $M_p > 0$ such that $\|u(\cdot,t)\|_{L^p} \leqslant M_p$ for all $t \in (0,T)$, then there exists a constant $M_\infty > 0$ independent of T such that*

$$\|u(\cdot,t)\|_{L^\infty} \leqslant M_\infty \quad \textit{for all } t \in (0,T).$$

P r o o f. Let $p > n/2$. We may assume that $p < n$. We see from (1.4) that $f(s)+s \leqslant C(1+s)$ for some $C > 0$. We can take $q > n$ so that $q > p$. Then we have

$$
\begin{aligned}
\|f(u)+u\|_{L^q(\Omega)} &\leqslant C\|1+u\|_{L^p(\Omega)}^{p/q}\|1+u\|_{L^\infty(\Omega)}^{1-p/q} \\
&\leqslant C_p'\|1+u\|_{L^\infty(\Omega)}^{1-p/q} \\
&\leqslant C_p''+C_p''\|u\|_{L^\infty(\Omega)}^{1-p/q},
\end{aligned}
\tag{3.4}
$$

where C_p', C_p'' are some positive constants. Recalling the choice of q, we see that $1-p/q \in (0,1)$. Moreover, we choose $q > n$ satisfying further that $1-(n-p)q/(np) > 0$, which enables us to pick $\lambda \in (1,\infty)$ fulfilling $1/\lambda < 1-(n-p)q/(np)$. The elliptic regularity $\bigl(\|\nabla v\|_{L^{np/(n-p)}(\Omega)} \leqslant k_p\|u\|_{L^p(\Omega)}\bigr)$ and Hölder's inequality yield

$$
\begin{aligned}
\|u\chi(v)\nabla v\|_{L^q(\Omega)} &\leqslant \|\chi\|_{L^\infty((\gamma,\infty))}\|\nabla v\|_{L^{q\lambda'}(\Omega)}\|u\|_{L^{q\lambda}(\Omega)} \\
&\leqslant \|\chi\|_{L^\infty((\gamma,\infty))}|\Omega|^{1/(q\lambda')-(n-p)/(np)}\|\nabla v\|_{L^{np/(n-p)}(\Omega)}\|u\|_{L^{q\lambda}(\Omega)} \\
&\leqslant \|\chi\|_{L^\infty((\gamma,\infty))}|\Omega|^{1/(q\lambda')-(n-p)/(np)}k_pM_p\|u\|_{L^1(\Omega)}^{1-\beta}\|u\|_{L^\infty(\Omega)}^{\beta} \\
&\leqslant K_p\|u\|_{L^\infty(\Omega)}^{\beta},
\end{aligned}
\tag{3.5}
$$

where $\lambda' := \lambda/(\lambda-1)$, for some $\beta \in (0,1)$ and $K_p > 0$. Now let $t \in (0,T)$. Then we have

$$
u(\cdot,t) = \mathrm{e}^{t(\Delta-1)}u_0 - \int_0^t \mathrm{e}^{(t-s)(\Delta-1)}\bigl(\nabla\cdot(u(s)\chi(v(s))\nabla v(s)) + (f(u(s))+u(s))\bigr)\,\mathrm{d}s.
$$

Let $\theta \in \bigl(n/(2q),1/2\bigr)$ and $\varepsilon \in \bigl(0,1/2-\theta\bigr)$. Using Lemma 2.2, we see that

$$
\begin{aligned}
\|u(\cdot,t)\|_{L^\infty(\Omega)} \leqslant \|u_0\|_{L^\infty(\Omega)} &+ c_{0,\infty}c\int_0^t (t-s)^{-\theta}\mathrm{e}^{-\nu_1(t-s)}\|f(u(s))+u(s)\|_{L^q(\Omega)}\,\mathrm{d}s \\
&+ c_{0,\infty}c_\varepsilon\int_0^t (t-s)^{-\theta-1/2-\varepsilon}\mathrm{e}^{-\nu_2(t-s)}\|u(s)\chi(v(s))\nabla v(s)\|_{L^q(\Omega)}\,\mathrm{d}s.
\end{aligned}
$$

Combining (3.4) and (3.5) with the above inequality implies the uniform estimate:

$$
\|u(\cdot,t)\|_{L^\infty(\Omega)} \leqslant K_0 + K_1\Bigl(\sup_{t\in[0,T]}\|u(\cdot,t)\|_{L^\infty(\Omega)}\Bigr)^{\beta} + K_2\Bigl(\sup_{t\in[0,T]}\|u(\cdot,t)\|_{L^\infty(\Omega)}\Bigr)^{1-p/q}
$$

for some $K_0, K_1, K_2 > 0$. Since $\beta, 1-p/q \in (0,1)$, we obtain the desired inequality. $\square$

We are now in a position to prove the main result.

P r o o f of Theorem 1.1. As stated in Section 1, by a similar way as in [3] we can show that there exist $T_{\max} \leqslant \infty$ (depending only on $\|u_0\|_{L^\infty(\Omega)}$) and exactly one pair (u, v) of nonnegative functions $u \in C^{2,1}(\overline{\Omega} \times (0, T_{\max})) \cap C^0([0, T_{\max}); C^0(\overline{\Omega}))$, and $v \in C^{2,0}(\overline{\Omega} \times (0, T_{\max})) \cap C^0((0, T_{\max}); C^0(\overline{\Omega}))$ that solves (1.1) in the classical sense. According to the condition for k and χ_0, by Proposition 3.3 we can find some $p > n/2$ and $M_p > 0$ such that $\|u(\cdot, t)\|_{L^p} \leqslant M_p$ for all $t \in (0, T_{\max})$. Therefore Proposition 3.4 completes the proof. $\square$

R e m a r k 3.1. The local-in-time existence of classical solutions to (1.1) can be provided under the only lower condition: $\lambda_1 - \mu_1 s \leqslant f(s)$. Moreover, if the growth term f satisfies the relaxed condition: $\lambda_1 - \mu_1 s \leqslant f(s) \leqslant \lambda_2 + \mu_2 s$, then we have the upper mass estimate depending on time t similarly, and so the global existence of solutions without uniform boundedness is proved.

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