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GENERALIZED TANAKA-WEBSTER AND LEVI-CIVITA CONNECTIONS FOR NORMAL JACOBI OPERATOR IN COMPLEX TWO-PLANE GRASSMANNIANS

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Abstract. We study classifying problems of real hypersurfaces in a complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$. In relation to the generalized Tanaka-Webster connection, we consider that the generalized Tanaka-Webster derivative of the normal Jacobi operator coincides with the covariant derivative. In this case, we prove complete classifications for real hypersurfaces in $G_2(\mathbb{C}^{m+2})$ satisfying such conditions.

Keywords: real hypersurface; complex two-plane Grassmannian; Hopf hypersurface; Levi-Civita connection; generalized Tanaka-Webster connection; normal Jacobi operator

MSC 2010: 53C40, 53C15

1. Introduction

In complex projective spaces or in quaternionic projective spaces, many differential geometers studied real hypersurfaces with parallel curvature tensor ([7], [13]). From a new perspective, it is investigated to classify real hypersurfaces in complex two-plane Grassmannians with parallel normal Jacobi operator, that is, $\nabla \overline{R}_N = 0$ (see [5], [6], [12]).

As a prevailing notion, in a Riemannian manifold $(\overline{M}, \overline{g})$, a vector field X along a geodesic γ of $\widetilde{M}$ is called a *Jacobi field* if it satisfies the following second order Jacobi equation

$$\overline{\nabla}^2_{\dot{\gamma}} X + \overline{R}(X, \dot{\gamma})\dot{\gamma} = 0,$$

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where $\dot{\gamma}$ is the vector tangent to γ. For any tangent vector field X at $x \in \overline{M}$, the Jacobi operator $\overline{R}_X$ is defined by

$$(\overline{R}_X Y)(x) = (\overline{R}(Y, X)X)(x),$$

for any vector field $Y \in T_x\overline{M}$.

On the other hand, let us put a unit normal vector field N to a hypersurface M into the curvature tensor $\overline{R}$ of the ambient space $\overline{M}$. Then for any tangent vector field X on M, the *normal Jacobi operator* $\overline{R}_N$ is defined by

$$\overline{R}_N(X) = \overline{R}(X, N)N.$$

Our ambient space, a complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$ consists of all complex two-dimensional linear subspaces in $\mathbb{C}^{m+2}$. This Riemannian symmetric space is the unique compact irreducible Riemannian manifold being equipped with both a Kähler structure J and a quaternionic Kähler structure $\mathfrak{J}$ not containing J. Then, naturally we could consider two geometric conditions for hypersurfaces M in $G_2(\mathbb{C}^{m+2})$ that the 1-dimensional distribution $[\xi] = \operatorname{span}\{\xi\}$ and the 3-dimensional distribution $\mathfrak{D}^{\perp} = \operatorname{span}\{\xi_1, \xi_2, \xi_3\}$ are both invariant under the shape operator A of M (see [3]), where the *Reeb vector field* ξ is defined by $\xi = -JN$, N denotes a local unit normal vector field of M in $G_2(\mathbb{C}^{m+2})$ and the *almost contact* 3*-structure* vector fields ξ_ν are defined by $\xi_\nu = -J_\nu N$ $(\nu = 1, 2, 3)$.

By using the result in Alekseevskii [1], Berndt and Suh [3] proved the following:

Theorem A. *Let M be a connected orientable real hypersurface in $G_2(\mathbb{C}^{m+2})$, $m \geqslant 3$. Then both $[\xi]$ and $\mathfrak{D}^{\perp}$ are invariant under the shape operator of M if and only if*

(A) *M is an open part of a tube around a totally geodesic $G_2(\mathbb{C}^{m+1})$ in $G_2(\mathbb{C}^{m+2})$, or*

(B) *m is even, say $m = 2n$, and M is an open part of a tube around a totally geodesic $\mathbb{H}P^n$ in $G_2(\mathbb{C}^{m+2})$.*

Besides, the Reeb *vector field* ξ is said to be *Hopf vector field* if it is invariant under the shape operator A. The one dimensional foliation of M by the integral manifolds of the Reeb vector field ξ is said to be a *Hopf foliation* of M. We say that M is a *Hopf hypersurface* in $G_2(\mathbb{C}^{m+2})$ if and only if the Hopf foliation of M is totally geodesic. Using the formulas in ([5], Section 3) it can be easily checked that M is Hopf if and only if the Reeb vector field ξ is Hopf.

Now, we consider another one instead of Levi-Civita connection for real hypersurfaces in Kähler manifolds, namely, the *generalized Tanaka-Webster connection* (in short, the g-Tanaka-Webster connection) $\widehat{\nabla}^{(k)}$ for a non-zero real number k ([4], [8]). The *Tanaka-Webster connection* ([14], [16]) is a unique affine connection on a non-degenerate CR-manifold. Tanno [15] introduced the notion of generalized Tanaka-Webster connection $\widehat{\nabla}$ for contact metric manifolds by the canonical connection which coincides with the Tanaka-Webster connection if the associated CR-structure is integrable. In particular, if the shape operator of a real hypersurface in Kähler manifolds satisfies $\varphi A + A\varphi = 2k\varphi$, $k \neq 0$, then the g-Tanaka-Webster connection $\widehat{\nabla}^{(k)}$ coincides with the Tanaka-Webster connection. Cho [4] defined the g-Tanaka-Webster connection by

$$\widehat{\nabla}^{(k)}_X Y = \nabla_X Y + F^{(k)}_X Y,$$

where the operator $F^{(k)}$ is given by

$$F^{(k)}_X Y = g(\varphi AX, Y)\xi - \eta(Y)\varphi AX - k\eta(X)\varphi Y, \quad k \in \mathbb{R} \setminus \{0\}$$

and this is said to be *g-Tanaka-Webster operator.*

Using this g-Tanaka-Webster connection $\widehat{\nabla}^{(k)}$, we have proved a non-existence theorem about parallelism of the normal Jacobi operator $\overline{R}_N$ (see [11]). In this paper, let us consider a new notion between the g-Tanaka-Webster connection $\widehat{\nabla}^{(k)}$and the Levi-Civita connection ∇ for the normal Jacobi operator $\overline{R}_N$ as follows:

$$(\widehat{\nabla}^{(k)}_X \overline{R}_N)Y = (\nabla_X \overline{R}_N)Y, \tag{1.1}$$

$$(\widehat{\nabla}^{(k)}_{\mathfrak{D}^\perp} \overline{R}_N)Y = (\nabla_{\mathfrak{D}^\perp} \overline{R}_N)Y, \tag{1.2}$$

and

$$(\widehat{\nabla}^{(k)}_{\mathfrak{D}} \overline{R}_N)Y = (\nabla_{\mathfrak{D}} \overline{R}_N)Y, \tag{1.3}$$

for any vector field $Y \in TM$, where $\mathfrak{D}$ and $\mathfrak{D}^\perp$ denote the distributions defined by $T_x\overline{M} = \mathfrak{D} \oplus \mathfrak{D}^\perp$, $x \in M$, and $\mathfrak{D}^\perp = \text{span}\{\xi_1, \xi_2, \xi_3\}$. The condition (1.1) means that the g-Tanaka-Webster covariant derivative and the Levi derivative of the normal Jacobi operator $\overline{R}_N$ coincide with each other on the tangent bundle TM. As a further generalization, the condition (1.2) (or (1.3)) has a weakened meaning that the two derivatives coincide on the distribution $\mathfrak{D}^\perp$ (or $\mathfrak{D}$, respectively) of the tangent bundle TM in $G_2(\mathbb{C}^{m+2})$.

In this paper, related to the conditions (1.1), (1.2), and (1.3) mentioned above, we want to study some non-existence properties of the normal Jacobi operator. First we give the following

Theorem 1.1. *There does not exist any Hopf hypersurface in a complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$, $m \geqslant 3$, if the g-Tanaka-Webster connection of the normal Jacobi operator coincides with the Levi-Civita connection.*

As a generalization of Theorem 1.1, we give two theorems on the distributions $\mathfrak{D}$ and $\mathfrak{D}^\perp$ for the bundle TM of real hypersuraces M in $G_2(\mathbb{C}^{m+2})$ as follows:

Theorem 1.2. *There does not exist any Hopf hypersurface in a complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$, $m \geqslant 3$, if the g-Tanaka-Webster connection of the normal Jacobi operator coincides with the Levi-Civita connection on the distribution $\mathfrak{D}^\perp$.*

Theorem 1.3. *There does not exist any Hopf hypersurface in a complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$, $m \geqslant 3$, if the g-Tanaka-Webster connection of the normal Jacobi operator coincides with the Levi-Civita connection on the distribution $\mathfrak{D}$.*

On the other hand, this condition (1.1) has a geometric meaning that the g-Tanaka-Webster operator $F_X^{(k)}$ and the normal Jacobi operator $\overline{R}_N$ commute with each other, that is, $F_X^{(k)}(\overline{R}_N Y) = \overline{R}_N(F_X^{(k)} Y)$. Then the conditions (1.2) and (1.3) also have the meaning that $F_X^{(k)}(\overline{R}_N Y) = \overline{R}_N(F_X^{(k)} Y)$ holds for any $X \in \mathfrak{D}^\perp$ and $X \in \mathfrak{D}$, respectively.

In Section 2 we introduce a key lemma being used to solve theorems. In Section 3 we will give a complete proof of the theorems. In this paper, we refer to [1], [2], [3], [5], [9] for Riemannian geometric structures of $G_2(\mathbb{C}^{m+2})$ and its geometric quantities, respectively.

2. Key lemma

Let us denote by $\overline{R}(X,Y)Z$ the curvature tensor in $G_2(\mathbb{C}^{m+2})$. Then the normal Jacobi operator $\overline{R}_N$ of a real hypersurface M in a complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$ can be defined by $\overline{R}_N X = \overline{R}(X,N)N$ for any vector field $X \in T_xM = \mathfrak{D} \oplus \mathfrak{D}^\perp$, $x \in M$ (see [5]).

In [5], [6], the normal Jacobi operator is obtained as

$$\begin{aligned}\overline{R}_N X = X + 3\eta(X)\xi + 3\sum_{\nu=1}^{3} \eta_\nu(X)\xi_\nu \\ - \sum_{\nu=1}^{3}\{\varphi_\nu(\xi)\varphi_\nu\varphi X - \eta_\nu(\xi)\eta(X)\xi_\nu - \eta_\nu(\varphi X)\varphi_\nu\xi\}\end{aligned} \tag{2.1}$$

for any tangent vector field X on M.

Recall that the g-Tanaka-Webster operator

$$F_X^{(k)}Y = g(\varphi AX, Y)\xi - \eta(Y)\varphi AX - k\eta(X)\varphi Y, \tag{2.2}$$

where $k \in \mathbb{R} \setminus \{0\}$ and a geometric meaning in the introduction

$$F_X^{(k)}(\overline{R}_N Y) = \overline{R}_N(F_X^{(k)}Y). \tag{2.3}$$

From now on, unless otherwise stated in the present section, we may write the Reeb vector field ξ as follows:

$$\xi = \eta(X_0)X_0 + \eta(\xi_1)\xi_1, \quad \eta(X_0)\eta(\xi_1) \neq 0 \tag{$*$}$$

for some unit vector fields $X_0 \in \mathfrak{D}$ and $\xi_1 \in \mathfrak{D}^{\perp}$.

Now, using this fact, we prove the following:

Lemma 2.1. *Let M be a Hopf hypersurface in a complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$, $m \geqslant 3$. If the g-Tanaka-Webster connection of the normal Jacobi operator coincides with the Levi-Civita connection along any vector field X, the distribution $\mathfrak{D}^{\perp}$, or the distribution $\mathfrak{D}$, then ξ belongs to either the distribution $\mathfrak{D}$ or the distribution $\mathfrak{D}^{\perp}$, respectively.*

Proof. By taking $X = \xi$, $X = \xi_1$ and $X = \varphi\xi_1$ in (2.1) and using the condition $(*)$, we have

$$\overline{R}_N\xi = 4\xi + 4\eta_1(\xi)\xi_1, \tag{2.4}$$

$$\overline{R}_N\xi_1 = 4\xi_1 + 4\eta_1(\xi)\xi, \tag{2.5}$$

$$\overline{R}_N(\varphi\xi_1) = 0, \tag{2.6}$$

respectively.

Putting $Y = \varphi\xi_1$ in (2.2), it becomes

$$F_X^{(k)}(\varphi\xi_1) = \sigma\xi + k\eta(X)\xi_1, \tag{2.7}$$

where $\sigma = \eta_1(AX) - \alpha\eta(\xi_1)\eta(X) - k\eta(X)\eta(\xi_1)$.

Inserting $Y = \varphi\xi_1$ in (2.3) and using (2.6), (2.7), we have

$$\overline{R}_N(\sigma\xi + k\eta(X)\xi_1) = 0. \tag{2.8}$$

Using (2.5), (2.6) and $(*)$ in (2.8), it is written as

$$\sigma + k\eta(X)\eta_1(\xi) = 0, \tag{2.9}$$

$$\sigma\eta_1(\xi) + k\eta(X) = 0. \tag{2.10}$$

Applying $\eta_1(\xi)$ to (2.9) and subtracting (2.10), it follows that

$$k\eta(X)\eta^2(X_0) = 0. \tag{2.11}$$

Now let us check the following three cases:

Case 1: $(\widehat{\nabla}^{(k)}_X \overline{R}_N)Y = (\nabla_X \overline{R}_N)Y$ for all $X,\ Y \in TM$. Putting $X = \xi$ in (2.11), ξ belongs the distribution $\mathfrak{D}^\perp$, because $k \neq 0$ is a real number.

Case 2: $(\widehat{\nabla}^{(k)}_{\mathfrak{D}^\perp} \overline{R}_N)Y = (\nabla_{\mathfrak{D}^\perp} \overline{R}_N)Y$ for all $Y \in TM$. Replacing $X = \xi_1$ in (2.11), ξ belongs to either the distribution $\mathfrak{D}$ or the distribution $\mathfrak{D}^\perp$, because of $k \in \mathbb{R} \setminus \{0\}$.

Case 3: $(\widehat{\nabla}^{(k)}_{\mathfrak{D}} \overline{R}_N)Y = (\nabla_{\mathfrak{D}} \overline{R}_N)Y$ for all $Y \in TM$. Taking $X = X_0$ in (2.11), we have $k\eta^3(X_0) = 0$. This means that ξ belongs to the distribution $\mathfrak{D}^\perp$.

Summing up the above three cases, we can give a complete proof of our lemma. □

3. Proof of theorems

Let us consider a Hopf hypersurface M in $G_2(\mathbb{C}^{m+2})$. Then by Lemma 2.1 we shall divide our consideration in the cases that the Reeb vector field ξ belongs to either the distribution $\mathfrak{D}^\perp$ or the distribution $\mathfrak{D}$.

First of all, we consider the case $\xi \in \mathfrak{D}^\perp$. Then in this case we want to prove the following

Lemma 3.1. *Let M be a Hopf hypersurface in complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$, $m \geqslant 3$, such that the Reeb vector field ξ belongs to the distribution $\mathfrak{D}^\perp$. If the g-Tanaka-Webster connection of the normal Jacobi operator coincides with the Levi-Civita connection along any vector field X, the distribution $\mathfrak{D}^\perp$, and the distribution $\mathfrak{D}$, respectively, then the distribution $\mathfrak{D}^\perp$ is invariant under the shape operator A of M.*

Proof. Without loss of generality, we may put $\xi = \xi_1$. Now let us take $\xi = \xi_1$ in (2.1), we have

$$\overline{R}_N X = X + 4\eta(X)\xi + 3\eta_1(X)\xi_1 + 2\eta_2(X)\xi_2 + 2\eta_3(X)\xi_3 - \varphi_1\varphi X \tag{3.1}$$

for any tangent vector field X on M. Substituting X by φAX in (3.1), we get

$$\overline{R}_N(\varphi AX) = \varphi AX + 2\eta_3(AX)\xi_2 - 2\eta_2(AX)\xi_3 + \varphi_1 AX.$$

Using the equation (see [10], (1.8)), it becomes

$$\overline{R}_N(\varphi AX) = 2\varphi AX. \tag{3.2}$$

Putting $X = \xi$ in (3.1), it is written as

$$\overline{R}_N(\xi) = 8\xi. \tag{3.3}$$

On the other hand, inserting $Y = \xi$ in (2.2), we get

$$F_X^{(k)}\xi = -\varphi AX. \tag{3.4}$$

Substituting $Y = \xi$ in (2.3) and using (3.2), (3.3), (3.4), we have $\varphi AX = 0$. Applying the structure tensor field φ, it becomes

$$AX = \alpha\eta(X)\xi, \tag{3.5}$$

for any tangent vector field X in M.

Therefore, we consider the following three cases:

Case 1: $(\widehat{\nabla}_X^{(k)}\overline{R}_N)Y = (\nabla_X\overline{R}_N)Y$ for all $X,\ Y \in TM$. Since $\xi \in \mathfrak{D}^\perp$, AX of (3.5) belongs to the distribution $\mathfrak{D}^\perp$.

Case 2: $(\widehat{\nabla}_{\mathfrak{D}^\perp}^{(k)}\overline{R}_N)Y = (\nabla_{\mathfrak{D}^\perp}\overline{R}_N)Y$ for all $Y \in TM$. This case has the same result as in Case 1.

Case 3: $(\widehat{\nabla}_{\mathfrak{D}}^{(k)}\overline{R}_N)Y = (\nabla_{\mathfrak{D}}\overline{R}_N)Y$ for all $Y \in TM$. Taking $X \in \mathfrak{D}$ in (3.5), we have $AX = 0$.

So, in any of Cases 1, 2 and 3, we can assert our Lemma 3.1. □

If $\xi \in \mathfrak{D}^\perp$, by Theorem A and Lemma 3.1, we can assert that M is locally congruent to a model space of type (A), that is, a tube over a totally geodesic $G_2(\mathbb{C}^{m+1})$ in $G_2(\mathbb{C}^{m+2})$. Now let us check whether a model space of type (A) satisfies one of the conditions (1.1), (1.2) and (1.3) or not. For a real hypersurface of type (A), detailed information (eigenspaces, corresponding eigenvalues, and multiplicities) was given in [3].

For Cases 1 and 2, putting $X = \xi_2$ in (3.5), we have $\beta\xi_2 = 0$. Since $\beta = \sqrt{2}\cot(\sqrt{2}r)$, $r \in (0, \pi/\sqrt{8})$, this gives $\xi_2 = 0$ which makes a contradiction. For the remaining Case 3, taking non-zero vector field $X \in T_\lambda$ in (3.5), we get $\lambda X = 0$. This gives that $\lambda = 0$. But the eigenvalue is $\lambda = -\sqrt{2}\tan(\sqrt{2}r)$, $r \in (0, \pi/\sqrt{8})$. This gives us a contradiction.

Next, if $\xi \in \mathfrak{D}$, by the assumption of Hopf and using [9], we see that M is locally congruent to a real hypersurface of type (B), which is a tube over a totally geodesic and totally real quaternionic projective space $\mathbb{H}P^n$ in $G_2(\mathbb{C}^{m+2})$. Hence it remains to check if a model space of type (B) satisfies one of the conditions (1.1), (1.2) and (1.3) or not. Now, by using the detailed information for real hypersurfaces of type (B) given in [3], we can check these cases as follows:

Taking $\xi \in \mathfrak{D}$ in (2.1), we have

$$\overline{R}_N X = X + 3\eta(X)\xi + 3\sum_{\nu=1}^{3}\eta_\nu(X)\xi_\nu + \sum_{\nu=1}^{3}\eta_\nu(\varphi X)\varphi_\nu\xi. \tag{3.6}$$

In (3.6), let us insert $X = \xi$ and $X = \varphi AX$, respectively, we get

$$\overline{R}_N \xi = 4\xi, \tag{3.7}$$

$$\overline{R}_N(\varphi AX) = \varphi AX + 3\sum_{\nu=1}^{3}\eta_\nu(\varphi AX)\xi_\nu - \sum_{\nu=1}^{3}\eta_\nu(AX)\varphi_\nu\xi. \tag{3.8}$$

Putting $Y = \xi$ in (2.3) and using (3.6), (3.7), (3.8), we obtain

$$-3\varphi AX = -3\sum_{\nu=1}^{3}\eta_\nu(\varphi AX)\xi_\nu + \sum_{\nu=1}^{3}\eta_\nu(AX)\varphi_\nu\xi. \tag{3.9}$$

For Cases 1 and 2, we can put $X = \xi_1 \in \mathfrak{D}^\perp$ in (3.9), then we have $4\beta\varphi\xi_1 = 0$. This gives $\beta = 0$, which gives us a contradiction.

For Case 3, we can put $X \in T_\lambda$ in (3.9), we get $\lambda\varphi X = 0$. Since $\beta = 2\cot(2r)$, $r \in (0, \pi/4)$ and $\lambda = \cot(r)$, $r \in (0, \pi/4)$, it gives us also a contradiction.

Hence summing up these assertions, we can give a complete proof of our Theorems 1.1, 1.2, and 1.3 in the introduction. □

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