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THE WEIGHTS OF THE RESULTS OF PARTIAL TESTS FOR DETERMINING THE TOTAL RESULT OF THE TEST.

By Anton Kotzig.

Summary: The author draws attention to a simple method of deriving weights of the results of partial tests for assessing the result of the whole test. During their derivation, he starts out from the demand that the coefficient of correlation between the total result of the test and the factor (attribute being tested) should be a maximum. For the case where only one factor is involved, he derives from the general formula a simple relation. Attention is drawn to practical applications in psychometry and an example is given.

Introduction.

In examinations conducted by means of tests, we try to measure some property or quality. At the same time we assume that the coefficient of correlation between the values of the measured attribute and the numbers by which we characterise the result of a test is fairly high for any population of examined persons. In other words, if this coefficient of correlation were small, the uncertainty in assessing the results would render impossible (or at least very difficult) the formation of any conclusions. It therefore seems a legitimate demand to establish a test and to find a correct assessment of the test, such that the above-mentioned correlation coefficient will be a maximum.

Usually, however, this task is by no means an easy one. Much experience and study is often necessary in order to approach the desired aim. In the following paragraphs, the author describes one of the means by which the process can be facilitated. The method described is suitable for tests where the problem falls into a number of partial problems to which the examination has to give the answers. In such a test, we can consider each of the problems (tasks, examples) as an independent test. Only seldom does it happen that the individual problems are equally difficult or of the same nature. When assessing the result of the total test, we are confronted with the question of how to assess the timely and correct solution of any one particular problem. In what follows, we shall show that this assessment can decide whether the coefficient of correlation between the result of the whole test and the measured attribute will be very small, or whether it will have an acceptably high value. We shall further show how it is possible to calculate suitable „weights" which enable us to assess the tests so that the above-mentioned correlation coefficient will be a maximum for the given conditions.

It should be mentioned that this work does not deal with the analysis of the correlations between the results of the individual tests (factor analysis);[1]) it is assumed, however, that factor analysis has already been carried out and that the correlation coefficients between measured attributes (factors) are already known.

1. *Outline of the problem.*

Suppose we have a population S of persons $A_1, A_2, \dots, A_m$. Let every person of the population S be subjected to the test T, which consists of the partial tests $T_1, T_2, \dots, T_n$. Let us denote the number of points obtained by the person A_i in test T_k by the mark $\alpha_{i,k}$ ($i = 1, 2, \dots, m$; $k = 1, 2, \dots, n$). If the partial test is, for instance, a single problem, the result can only be one of two possible: either the examinee solves the problem correctly, or he solves it incorrectly (in a questionnaire, he either replies positively or negatively). Here it is the custom to award a correct (positive) reply with one point, and an incorrect (negative) reply with no point. Suppose the value of a certain attribute (let us denote it by X) of the person A_i to be ξ_i. In order to simplify the calculations in what follows, let us introduce for the assessment of the results of any particular test as well as for the assessment of the attribute X a new standard variable, i. e. let us write

$$a_{j,k} = \frac{\alpha_{j,k} - \frac{1}{m}\sum_{i=1}^{m} \alpha_{i,k}}{\sqrt{\frac{1}{m}\sum_{i=1}^{m} \alpha_{i,k}^2 - \left(\frac{1}{m}\sum_{i=1}^{m} \alpha_{i,k}\right)^2}}, \quad (j = 1, 2, \dots, m;\ k = 1, 2, \dots, n) \qquad (1)$$

$$x_j = \frac{\xi_j - \frac{1}{m}\sum_{i=1}^{m} \xi_i}{\sqrt{\frac{1}{m}\sum_{i=1}^{m} \xi_i^2 - \left(\frac{1}{m}\sum_{i=1}^{m} \xi_i\right)^2}}, \quad (j = 1, 2, \dots, m). \qquad (2)$$

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[1]) See, for instance, Dr V. ČERVINKA: „Factor Analysis", Statistický obzor, Vol. XXVIII, No 2 and the literature mentioned there.

Let ϱ_j ($j = 1, 2, \ldots, n$) be the correlation coefficient between the values of the attribute X and the numbers of points obtained in the test T_j for the population of persons S; i. e. in view of the fact that $a_{i,j}$, x_j are the standard variables

$$\varrho_j = \sum_{i=1}^{m} a_{ij} x_i \quad (j = 1, 2, \ldots, n). \tag{3}$$

Further, let us write

$$y_j = a_{j,1}\eta_1 + a_{j,2}\eta_2 + \ldots + a_{jn}\eta_n \quad (j = 1, 2, \ldots, m) \tag{4}$$

where $\eta_1, \eta_2, \ldots, \eta_n$ are constants.

The number y_j expressing the total result (attribute Y) of the person A_j in the population of tests $T_1, T_2, \ldots, T_n$ depends on how we choose the constants $\eta_1, \eta_2, \ldots, \eta_n$. We require to find the values $\eta_1, \eta_2, \ldots, \eta_n$ such that the coefficient r_{xy} of correlation between the values of the attributes X, Y for the population S will be a maximum.

2. *Solution.*

Let us consider the relations [see Eq. (1), (4)]:

$$\frac{1}{m}\sum_{i=1}^{m} y_i = \frac{1}{m}\left[\eta_1 \sum_{i=1}^{m} a_{i,1} + \eta_2 \sum_{i=1}^{m} a_{i,2} + \ldots + \eta_n \sum_{i=1}^{m} a_{i,n}\right] = 0 \tag{5}$$

$$\left.\begin{aligned} \sigma_y^2 = \frac{1}{m}\sum_{i=1}^{m} y_1^2 = \eta_1^2 \frac{1}{m}\sum_{i=1}^{m} a_{i,1}^2 + \eta_2^2 \frac{1}{m}\sum_{i=1}^{m} a_{i,2}^2 + \ldots + \\ + \eta_n^2 \frac{1}{m}\sum_{i=1}^{m} a_{i,n}^2 + 2\eta_1\eta_2 \frac{1}{m}\sum_{i=1}^{m} a_{i,1}a_{i,2} + 2\eta_1\eta_3 \frac{1}{m}\sum_{i=1}^{m} a_{i,1}a_{i,3} + \ldots \\ \ldots 2\eta_{n-1}\eta_n \frac{1}{m}\sum_{i=1}^{m} a_{i,n-1}a_{i,n}. \end{aligned}\right\} \tag{6}$$

We know, however (in view of the fact that $a_{i,k}$ are the values of the standard variable) that

$$\frac{1}{m}\sum_{i=1}^{m} a_{i,k}^2 = 1 \tag{7}$$

$$\frac{1}{m}\sum_{i=1}^{m} a_{i,j}a_{i,k} = r_{j,k} \tag{8}$$

where $r_{j,k}$ is the correlation coefficient between the number of points obtained in the tests T_j and T_k.

We can, therefore, simplify relation (6) to the form

$$\sigma_y^2 = \eta_1^2 + \eta_2^2 + \ldots + \eta_n^2 + 2r_{1.2}\eta_1\eta_2 + 2r_{1.3}\eta_1\eta_3 + \ldots + \\ + 2r_{n-1,n}\eta_{n-1}\eta_n. \tag{9}$$

Further, we have from Eq. (2)

$$\frac{1}{m}\sum_{i=1}^{m} x_i = 0 \tag{10}$$

$$\frac{1}{m}\sum_{i=1}^{m} x_i^2 = 1. \tag{11}$$

For the sum of products $\sum_{i=1}^{m} x_i y_i$, we also have the relation

$$\left.\begin{aligned} \frac{1}{m}\sum_{i=1}^{m} x_i y_i &= \frac{1}{m}\sum_{i=1}^{m} x_i(a_{i,1}\eta_1 + a_{i,2}\eta_2 + \ldots + a_{i,n}\eta_n) = \\ = \eta_1 \frac{1}{m}\sum_{i=1}^{m} a_{i,1}x_i &+ \eta_2 \frac{1}{m}\sum_{i=1}^{m} a_{i,2}x_i + \ldots + \eta_n \frac{1}{m}\sum_{i=1}^{m} a_{i,n}x_i = \\ &= \eta_1\varrho_1 + \eta_2\varrho_2 + \ldots + \eta_n\varrho_n. \end{aligned}\right\} \tag{12}$$

Hence, for the correlation coefficient $r_{x,y}$ we have

$$r_{x,y} = \frac{\eta_1\varrho_i + \eta_2\varrho_2 + \ldots + \eta_n\varrho_n}{\sqrt{\eta_1^2 + \eta_2^2 + \ldots + \eta_n^2 + 2r_{1,2}\eta_1\eta_2 + 2r_{1.3}\eta_1\eta_3 + \ldots + 2r_{n-1,n}\eta_{n-1}\eta_n}}. \tag{13}$$

It is obvious that the values $r_{x,y}$ remain unchanged when, in place of the values $\eta_1, \eta_2, \ldots, \eta_n$ we substitute the values $\varkappa_1, \varkappa_2, \ldots, \varkappa_n$, on the assumption that the relation

$$\varkappa_i = \omega\eta_i \quad (i = 1, 2, \ldots, n) \tag{14}$$

hold, where ω is any coefficient other than zero.

For a suitable choice of the number ω, we can ensure that

$$1 = \varkappa_1^2 + \varkappa_2^2 + \ldots + \varkappa_n^2 + 2r_{1,2}\varkappa_1\varkappa_2 + 2r_{1,3}\varkappa_1\varkappa_3 + \ldots + \\ + 2r_{n-1,n}\varkappa_{n-1}\varkappa_n \tag{15}$$

will hold. Then, of course, we have for the correlation coefficient r_{xy} the equation

$$r_{x,y} = \varkappa_1\varrho_1 + \varkappa_2\varrho_2 + \ldots + \varkappa_n\varrho_n. \tag{16}$$

Let us now form the function F such that

$$F = \varkappa_1\varrho_1 + \varkappa_2\varrho_2 + \ldots + \varkappa_n\varrho_n + \tfrac{1}{2}\lambda[1 - (\varkappa_1^2 + \varkappa_2^2 + \ldots + \varkappa_n^2 + \\ + 2r_{1,2}\varkappa_1\varkappa_2 + 2r_{1,3}\varkappa_1\varkappa_3 + \ldots + 2r_{n-1,n}\varkappa_{n-1}\varkappa_n)] \tag{17}$$

and let us write

$$\frac{\partial F}{\partial \varkappa_i} = 0 \quad (i = 1, 2, \ldots, n). \tag{18}$$

We then obtain the equations

$$\begin{aligned}
\varrho_1 &= \lambda(r_{1,1}\varkappa_1 + r_{1.2}\varkappa_2 + \ldots + r_{1,n}\varkappa_n) \\
\varrho_2 &= \lambda(r_{2,1}\varkappa_1 + r_{2,2}\varkappa_2 + \ldots + r_{2,n}\varkappa_n) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\varrho_i &= \lambda(r_{i.1}\varkappa_1 + r_{i,2}\varkappa_2 + \ldots + r_{i,n}\varkappa_n) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\varrho_n &= \lambda(r_{n,1}\varkappa_1 + r_{n.2}\varkappa_2 + \ldots + r_{n.n}\varkappa)
\end{aligned} \tag{19}$$

where $r_{i,i} = 1$ for all $i = 1, 2, \ldots, r.$[2])

The relation

$$\eta_1 : \eta_2 : \ldots : \eta_n = \varkappa_1 : \varkappa_2 : \ldots : \varkappa_n =$$

$$= \begin{vmatrix} \varrho_1, & r_{1,2}, & r_{1.2}, & \ldots, & r_{1,n} \\ \varrho_2, & r_{2,2}, & r_{2,3}, & \ldots, & r_{2.n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \varrho_n, & r_{n.2}, & r_{n.3}, & \ldots, & r_{n.n} \end{vmatrix} : \begin{vmatrix} r_{1,1}, & \varrho_1, & r_{1,3}, & \ldots, & r_{1,n} \\ r_{2.1}, & \varrho_2, & r_{2.3}, & \ldots, & r_{2,n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ r_{n,1}, & \varrho_n, & r_{n,3}, & \ldots, & r_{n,n} \end{vmatrix} : \ldots :$$

$$: \ldots : \begin{vmatrix} r_{1.1}, & r_{1.2}, & r_{1,3}, & \ldots, & \varrho_1 \\ r_{2.1}, & r_{2,2}, & r_{2,3}, & \ldots, & \varrho_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ r_{n,1}, & r_{n,2}, & r_{n,3}, & \ldots, & \varrho_n \end{vmatrix} \tag{20}$$

will then hold. According to equation (20), we can therefore calculate the required values $\eta_1, \eta_2, \ldots, \eta_n$, which may be considered as weights of the points obtained in the individual tests. It is necessary, of course, to realise that in order to determine the total result (to assess the result of all the tests) i. e. to determine the values y_j ($j = 1, 2, \ldots, m$) the values of the standard variable must be multiplied by the weights $\eta_1, \eta_2, \ldots, \eta_n$. In practical work it is more convenient to determine other weights, namely such weights that we do not require to perform the calculation in two stages, i. e. first to express the obtained number of points in each test in terms of the standard variable, and secondly to multiply by the weights $\eta_1, \eta_2, \ldots, \eta_n$. It is more convenient to find weights $v_1, v_2, \ldots, v_n$ such that the total result can be calculated directly from the values $\alpha_{i,k}$ by means of the weights $v_1, v_2, \ldots, v_n$. We can do this if we write

$$v_k = \frac{\eta_k}{\sigma_k} \quad (k = 1, 2, \ldots, n) \tag{21}$$

[2]) It should be mentioned that $r_{i,i}$ does not mean here the correlation coefficient between two results of the same test, obtained by the same persons; we introduce the symbol $r_{i,i}$ only for easier manipulation.

where

$$\sigma_k^2 = \frac{1}{m}\sum_{i=1}^{m}\alpha_{i,k}^2 - \left(\frac{1}{m}\sum_{i=1}^{m}\alpha_{i,k}\right)^2 \tag{22}$$

We therefore have

$$y_j = a_{j,1}\eta_1 + a_{j,2}\eta_2 + \ldots + a_{j,n}\eta_n =$$

$$= \left(\alpha_{j,1} - \frac{1}{m}\sum_{i=1}^{m}\alpha_{i,1}\right)v_1 + \left(\alpha_{j,2} - \frac{1}{m}\sum_{i=1}^{m}\alpha_{i,2}\right)v_2 + \ldots +$$

$$+ \ldots + \left(\alpha_{j,n} - \frac{1}{m}\sum_{i=1}^{m}\alpha_{i,n}\right)v_n, \tag{23}$$

Let us further denote

$$\frac{1}{m}\sum_{i=1}^{m}\alpha_{i,k} = p_k\,;\ \sum_{i=1}^{m} p_i v_i = C. \tag{24}$$

We obtain, after rearrangement:

$$y_j = \alpha_{j,1}v_1 + \alpha_{j,2}v_2 + \ldots + \alpha_{j,n}v_n - C. \tag{25}$$

As is well-known, the value of the correlation coefficient remains unchanged if we alter the origin from which we measure the value of the attribute. We therefore write

$$h_j = y_j + C = \alpha_{j,1}v_1 + \alpha_{j,2}v_2 + \ldots + \alpha_{j,n}v_n. \tag{26}$$

By means of the weights $v_1, v_2, \ldots, v_n$, calculated in the described manner, we can weight the results obtained in the individual tests and determine the values h_j $(j = 1, 2, \ldots, m)$ for the persons $A_1, A_2, \ldots, A_m$ in such a way that the correlation between the values $h_j(y_j)$ and the values x_j will be a maximum.

3. *Case where the correlations between the results of individual tests are produced by one factor.*

For a large number of tests, the calculation of the values of the determinants [see Eq. (20)] is very time-consuming. We now show that, for the case where it can be assumed that the correlations between the results of individual tests are produced by one factor, the task of determining the correct weights is very easy. Let $r_{i,k}$ be the coefficient of correlation between the results in the tests T_i and T_k; let ϱ_i be the coefficient of correlation between the results in the test T_i and the values of the attribute X, which is the factor producing the correlation between the individual tests. On these assumptions, we obtain for the coefficient of correlation, after eliminating the effect of the attribute x, the relation

$$r_{i,k\,:\,x} = \frac{r_{i,k} - \varrho_i\varrho_k}{\sqrt{(1-\varrho_1^2)(1-\varrho_k^2)}} = 0 \quad (i, k = 1, 2, \ldots, n,\ i \neq k) \tag{27}$$

that is,

$$r_{i,k} = \varrho_i \varrho_k \qquad (i,k = 1,2,\ldots,n;\ i \neq k) \tag{28}$$

Let us now assume that this relation (28) holds, and let us calculate the value of the determinant,

$$D_i = \begin{vmatrix} 1, & r_{2,1}, & \ldots, r_{i-1,1}, & \varrho_1, & r_{i+1,1}, & \ldots, r_{n,1} \\ r_{1,2}, & 1, & \ldots, r_{i-1,2}, & \varrho_2, & r_{i+1,2}, & \ldots, r_{n,2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{1,i-1}, & r_{2,i-1}, & \ldots, 1, & \varrho_{i-1}, & r_{i+1,i-1}, & \ldots, r_{n,i-1} \\ r_{1,i}, & r_{2,i}, & \ldots, r_{i-1,i}, & \varrho_i, & r_{i+1,i}, & \ldots, r_{n,i} \\ r_{1,i+1}, & r_{2,i+1}, & \ldots, r_{i-1,i+1}, & \varrho_{i+1}, & 1, & \ldots, r_{n,i+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{1,n}, & r_{2,n}, & \ldots, r_{i-1,n}, & \varrho_n, & r_{i+1,n} & \ldots, 1 \end{vmatrix} =$$

$$= \begin{vmatrix} 1, & \varrho_1\varrho_2, & \ldots, \varrho_{i-1}\varrho_1, & \varrho_1, & \varrho_{i+1}\varrho_1 & \ldots, \varrho_n\varrho_1 \\ \varrho_1\varrho_2, & 1, & \ldots, \varrho_{i-1}\varrho_2, & \varrho_2, & \varrho_{i+1}\varrho_2, & \ldots, \varrho_n\varrho_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \varrho_1\varrho_{i-1}, & \varrho_2\varrho_{i-1}, & \ldots, 1, & \varrho_{i-1}, & \varrho_{i+1}\varrho_{i-1} & \ldots, \varrho_n\varrho_{i-1} \\ \varrho_1\varrho_i, & \varrho_2\varrho_i, & \ldots, \varrho_{i-1}\varrho_i, & \varrho_i, & \varrho_{i+1}\varrho_i, & \ldots, \varrho_n\varrho_i \\ \varrho_1\varrho_{i+1}, & \varrho_2\varrho_{i+1}, & \ldots, \varrho_{i-1}\varrho_{i+1}, & \varrho_{i+1}, & 1, & \ldots, \varrho_n\varrho_{i+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \varrho_1\varrho_n, & \varrho_2\varrho_n, & \ldots, \varrho_{i-1}\varrho_n, & \varrho_n, & \varrho_{i+1}\varrho_n, & \ldots, 1 \end{vmatrix} \tag{29}$$

If we multiply the i-th column by the value ϱ_1 and subtract it from the first column, then again the i-th column by the value ϱ_2 and subtract it from the second column and so on until the i-th column by the value ϱ_n and subtract it from the n-th column (i. e. we alter all the columns except the i-th column), the determinant D_i becomes

$$D_i = \begin{vmatrix} 1-\varrho_1^2, & 0, & \ldots, 0, & \varrho_1, & 0, & \ldots, 0 \\ 0, & 1-\varrho_2^2, & \ldots, 0, & \varrho_2, & 0, & \ldots, 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0, & 0, & \ldots, 1-\varrho_{i-1}^2, & \varrho_{i-1}, & 0, & \ldots, 0 \\ 0, & 0, & \ldots, 0, & \varrho_i, & 0, & \ldots, 0 \\ 0, & 0, & \ldots, 0, & \varrho_{i+1}, & 1-\varrho_{i+1}^2 & \ldots, 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0, & 0, & \ldots, 0, & \varrho_n, & 0, & \ldots, 1-\varrho_n^2 \end{vmatrix} =$$

$$= (1-\varrho_1^2)(1-\varrho_2^2)\ldots(1-\varrho_{i-1}^2)\,\varrho_i(1-\varrho_{i+1}^2)\ldots(1-\varrho_n^2)$$

$$(i = 1, 2, \ldots, n). \tag{30}$$

We know, however, that

$$\eta_1 : \eta_2 : \ldots : \eta_n = D_1 : D_2 : \ldots : D_n. \tag{31}$$

And hence [see Eq. (30)],

$$\eta_1 : \eta_2 : \ldots : \eta_n = \frac{\varrho_1}{1 - \varrho_1^2} : \frac{\varrho_2}{1 - \varrho_2^2} : \ldots : \frac{\varrho_n}{1 - \varrho_n^2} \tag{32}$$

From this we finally obtain the simple expressions for the weights $v_1, v_2, \ldots, v_n$:

$$v_i = \frac{\varrho_i}{\sigma_i(1 - \varrho_1^2)} \qquad i = 1, 2, \ldots, n.$$

4. *Example.*

259 examinees (population S) are subjected to a psychotechnical test similar to that proposed by Dunajevskij. The test is composed of 24 partial tests and is used in the Czechoslovak Institute of Labour, regional institute for Slovakia in Bratislava. The examinee is permitted a certain given time for the solution of the whole test. Due to the fact that not all examinees will be able to solve all 24 problems in the alloted time, and also due to the fact that the first two problems are very simple, the analysis has been carried out only for problems nos. 3—10. The results are as follows:

1	2								3	4	5
Test no. i	Correlation coefficient Test no.								$259\,\sigma_i$	Corelat. coeffic. with factor ϱ_i	Weights η_i
	3	4	5	6	7	8	9	10			
3		0,24	0,27	0,21	0,33	0,40	0,34	0,34	129	0,61	97
4	0,24		0,31	0,13	0,22	0,22	0,15	0,25	129	0,44	55
5	0,27	0,31		0,20	0,31	0,30	0,14	0,28	110	0,51	80
6	0,21	0,13	0,20		0,25	0,20	0,16	0,17	113	0,38	44
7	0,33	0,22	0,31	0,25		0,36	0,23	0,30	129	0,57	84
8	0,40	0,22	0,30	0,20	0,36		0,34	0,33	129	0,62	101
9	0,34	0,15	0,14	0,16	0,23	0,34		0,15	107	0,45	56
10	0,34	0,25	0,28	0,17	0,30	0,33	0,15		125	0,52	71

After substituting the appropriate values into the equation for the correlation coefficient r_{xy} Eq. (13), (i. e. for the correlation coefficient between the value of the attribute and the total result of the test comprising problems 3—10), if we use the weights η_i according to the table, we obtain $r_{xy} = 0{,}87$.

If, for the total result, we used the number of correctly solved problems from among the above-mentioned 8 problems, it would mean that we used the weights $v_3 = v_4 = \ldots = v_{10} = 1$, or $\eta_1 = \sigma_1$; $\eta_2 = \sigma_2, \ldots, \eta_n = \sigma_n$; on substitution into Eq. (13), we would obtain $r_{xy}' = 0{,}45$.

Use of the correct weights, therefore, improves considerably the correlation between the total result of the eight problems and the measured attribute.

The reader will find more detailed treatment of the above matter in the works:

HOLZINGER K.: Statistical Résumé of the Spearman Two-factor Theory, 1930.
SPEARMAN G.: The abilities of Man (Appendix), 1927.

NOTE SUR UN PROBLÈME FONDAMENTAL DE LA THÉORIE DE L'ÉQUILIBRE ÉCONOMIQUE.

Par JIŘÍ SEITZ.

Au cours de la résolution du problème fondamental de la théorie de l'équilibre économique, on est amené à résoudre une question qui, en termes mathématiques, s'énonce ainsi:

A quelle condition doivent être assujettis les coefficients d'une forme quadratique à n variables $A(x, x) = \sum_{i,k=1}^{n} a_{ik}x_i x_k$ pour que cette forme ait une valeur non-négative pour tout système de nombres réels $x_1, x_2, \ldots, x_n$ vérifiant la relation linéaire $c_1x_1 + c_2x_2 + \ldots + c_nx_n = 0$. La réponse à cette question est l'affirmation suivante:

Soient respectivement $A(x, x) = \sum_{i,k=1}^{n} a_{ik}x_i x_k$ et $L(x) = \sum_{i=1}^{n} c_i x_i$ une forme quadratique de matrice **A** et une forme linéaire pour laquelle $c_1 \neq 0$ et $n \geqq 2$; supposons en outre qu'on ait

$$\begin{vmatrix} 0, & c_1, & c_2, \ldots, & c_n \\ c_1, & a_{11}, & a_{12}, \ldots, & a_{1n} \\ c_2, & a_{21}, & a_{22}, \ldots, & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ c_n, & a_{n1}, & a_{n2}, \ldots, & a_{n,n} \end{vmatrix} \neq 0 \; (a_{ik} = a_{ki}.) \qquad (1)$$

alors la condition nécessaire et suffisante pour que la forme quadratique $A(x, x)$ soit définie, positive (négative), sous l'hypothèse que les variables $x_1, x_2, \ldots, x_n$ vérifient une relation linéaire $L(x) = 0$ est que les termes de la suite