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FINITE-TIME TRACKING CONTROL OF MULTIPLE NONHOLONOMIC MOBILE ROBOTS WITH EXTERNAL DISTURBANCES

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This paper investigates finite-time tracking control problem of multiple nonholonomic mobile robots in dynamic model with external disturbances, where a kind of finite-time disturbance observer (FTDO) is introduced to estimate the external disturbances for each mobile robot. First of all, the resulting tracking error dynamic is transformed into two subsystems, i. e., a third-order subsystem and a second-order subsystem for each mobile robot. Then, the two subsystem are discussed respectively, continuous finite-time disturbance observers and finite-time tracking control laws are designed for each mobile robot. Rigorous proof shows that each mobile robot can track the desired trajectory in finite time. Simulation example illustrates the effectiveness of our method.

Keywords: finite-time tracking control, finite-time disturbance observer, external disturbances, nonholonomic mobile robot, dynamic model

Classification: 93A14, 93D15, 93D21

1. INTRODUCTION

Coordination control of multiple autonomous agents has received considerable attention recently because of its challenging features and many applications in rescue mission,troop hunting, formation control, cluster of satellites, and so on. A group of autonomous agents can coordinate with each other via communication or sensing networks to perform certain challenging tasks, which can not be well accomplished by a single agent. Multiple agents have been widely used in many fields, such as cooperative control of unmanned air vehicles, flocking of birds, schooling for underwater vehicles, distributed sensor networks, attitude alignment for cluster of satellites, collective motion of particles, and distributed computations [17, 21, 22, 26, 27, 29]. In the area of cooperative control of multiple autonomous agents, consensus is an important and fundamental problem. The consensus means that a group of dynamic agents can reach an agreement on certain quantities of interest by implementing an appropriate control strategy or protocol on the basis of local state information. Obviously, the consensus state and the convergence rate are crucial for the study of consensus problem.

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In recent years, on the basis of the algebraic graph theory, many consensus-based results have been applied to cooperative control of multi-robot systems [17, 22, 26, 27]. However, the robot system models in these papers are all linear. In practical cooperative control applications, many robot system dynamics are nonlinear and have nonholonomic constraints. Therefore, it is necessary to discuss cooperative control of multiple nonholonomic mobile robots. Cooperative control of wheeled mobile robots has been studied in [5, 14, 16]. For cooperative control of multiple nonholonomic mobile robots in kinematic model, consensus-based control laws were proposed in [7]. However, in practice, most of nonholonomic mechanical systems are of dynamic model which require generalized forces as their inputs. Hence, the control inputs are generalized velocities which are designed based on the kinematic models can not be directly used to control the practical dynamic systems. For this reason, to study the cooperative control of multiple nonholonomic mobile robots with dynamic systems is not only theoretically challenging but also practically imperative. [6, 8] investigated the cooperative control of multiple nonholonomic systems with dynamics and uncertainty. In both papers, the backstepping design schemes are employed to solve the cooperative control problem.

In almost all engineering control systems, the presence of disturbances, model uncertainties and nonlinear model parts is inevitable. If no adequate control method is used to deal with disturbances, the existence of disturbances may influence system performance, cause oscillation, and lead to instability. Thus, in recent years, the problem of controlling uncertain dynamical systems subject to external disturbances has been a interesting topic. Various robust control methods have been proposed, e. g., internal model control, sliding mode control and H_∞ control, these methods are well known for their performance and robustness. However, these conventional feedback control methods usually can not react directly and promptly to reject these disturbances. This may result in a degradation of system performance when meeting severe disturbances.

Disturbance-observer-based control (DOBC) approach is from practice and depends on the idea of feed-forward compensation. That is, the controller design is the composite of two parts. On the one hand, observer or filter can be designed to estimate disturbances based on the outputs or states. Thus, the estimations are to reject the disturbances. On the other hand, stabilizer for nominal system can be designed. Recently DOBC approach as a robust control approach has widely found its applications in mechanical and electrical, dynamics and structure control areas [2, 3, 4, 10, 32]. A characteristic of DOBC is having simple structure, combining with different control laws according to different desire of control performance and very easily setting on line and engineering realization.

Finite-time control method is another an efficient feedback control scheme to improve disturbance rejection performance. Apart from this advantage, finite-time control system has other good point: faster convergence, better robustness [1, 9, 15, 24, 33, 30]. For the tracking control problem of nonholonomic mobile robot systems, some finite-time control laws have been proposed [12, 23, 25, 31]. The authors of [31] proposed finite-time tracking controller for the nonholonomic systems with extended chained form, where the relay switching technique and the terminal sliding mode control scheme with finite time convergence were used to design the controller. In [23], finite-time tracking control problem of multiple nonholonomic wheeled mobile robots in dynamic model was investigated, and finite-time tracking control laws were designed for each mobile robot. The authors of [12] discussed finite-time formation control problem for a group of nonholonomic mobile robots, where the desired formation trajectory was represented by a virtual dynamic leader, finite-time observer was proposed for each follower to estimate the leader's states and finite-time tracking control law was designed for each mobile robot. However, all these above papers don't consider external disturbances in robot system models. The authors of [25] studied finite-time tracking control problem of a nonholonomic wheeled mobile robot in dynamic model with external disturbances, finite-time disturbance observers was introduced to estimate the external disturbances and finite-time tracking control laws were proposed for the mobile robot . However, the paper [25] only considered single mobile robot system model.

This paper will discuss finite-time tracking control problem of multiple nonholonomic mobile robots in dynamic model with external disturbances. On one hand, finite-time disturbance observers are derived for each mobile robot to estimate the external disturbances. On the other hand, continuous finite-time tracking control laws are proposed such that the states of each mobile robot converges to a desired value, in which the estimated values are used as a feed-forward disturbance compensation part. In the first stage, the unified tracking error system consists of two subsystems for the mobile robot is introduced. In the second stage, the two subsystems are discussed respectively, continuous finite-time disturbance observers and finite-time tracking control laws are proposed for each mobile robot. The second stage is divided into two subproblems, each with its own design objectives. The first subproblem is that finite-time disturbance observers will be designed for each mobile robot to estimate the external disturbances. The second subproblem is to design finite-time tracking control laws for each robot, where finite-time disturbance observers are introduced to compensate for the influence of the disturbances using proper feedback. It is shown that the proposed finite-time tracking control laws make the states of each mobile robot converge to the desired value.

This paper is organized as follows. In Section 2, related preliminary results and the problem formulation are first presented. The main results are presented in Section 3. Section 4 gives numerical simulations. Some conclusions are given in Section 5.

2. PRELIMINARIES

Notations. For convenience, in the sequel, set

$$\mathrm{sig}^{\alpha}(y) = |y|^{\alpha}\mathrm{sgn}(y), \quad \alpha > 0, \tag{1}$$

where $\mathrm{sgn}(\cdot)$ denotes the standard signum function.

It can be verified that

$$\frac{\mathrm{d}}{\mathrm{d}y}|y|^{1+\alpha} = (1+\alpha)\mathrm{sig}^{\alpha}(y) \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}y}\mathrm{sig}^{1+\alpha}(y) = (1+\alpha)|y|^{\alpha}. \tag{2}$$

2.1. Related lemma

Lemma 2.1. (Hardy, Littlewood and Polya [11]) For $x_i \in R,\ i = 1, 2, \dots, n$, $0 < p \leq 1$ is a real number, then the following inequality holds:

$$(|x_1| + \cdots + |x_n|)^p \leq |x_1|^p + \cdots + |x_n|^p. \tag{3}$$

2.2. Graph theory

This paper considers n noholonomic mobile robots. If we consider each robot as a node, the communication between robots can be described by a directed graph $G = (V, \mathcal{E}, A)$, where $V = \{1, 2, \ldots, n\}$ is a node set, $\mathcal{E} \subseteq V \times V$ is an edge set with element (i, j) that describes the communication from node i to node j. The node indexes belong to a finite index set $\Gamma = \{1, 2, \ldots, n\}$. If the state of robot i is available to robot j, there will be an edge $(i, j) \in \mathcal{E}$, and we say robot i is a neighbor of robot j. The set of neighbors of robot i is denoted by $N_i = \{j \in V : (i, j) \in \mathcal{E}\}$. A directed path is a sequence of ordered edges of the form (i_1, i_2),(i_3, i_4),..., where $(i_k, i_{k+1}) \in \mathcal{E}$ in a directed graph. The weighted adjacency matrix is defined as $A = [a_{ij}]$, the element a_{ij} associated with the arc of the digraph is positive, i. e., $a_{ij} > 0 \Leftrightarrow (i, j) \in \mathcal{E}$. Moreover, it is usually assumed that $a_{ii} = 0$ for all $i \in V$. A diagonal matrix $D = diag\{d_1, d_2, \ldots, d_n\} \in R^{n\times n}$ is a degree matrix of G, whose diagonal elements $d_i = \sum_{j\in N_i} a_{ij}$ for $i = 1, 2, \ldots, n$. Then the Laplacian of the weighted digraph G is defined as $L = D - A \in R^{n\times n}$.

In contrast to a directed graph, the pairs of nodes in an undirected graph are unordered, where the edge (i, j) denotes that robot i and j can obtain information from each other. An undirected graph is connected if there is an undirected path between every pair of distinct nodes. In this paper, we assume that the graph G is undirected.

2.3. Problem formulation

Consider a group of n nonholonomic mobile robots indexed with $1, 2, \ldots, n$, which are moving on a plane. It is assumed that each member of the group has the same mechanical structure and each mobile robot has two-degrees-of-freedom. A simplified dynamic model of each mobile robot is give by [13]

$$\begin{aligned}
\dot{x}_i &= v_i \cos\theta_i, && (4a)\\
\dot{y}_i &= v_i \sin\theta_i, && (4b)\\
\dot{\theta}_i &= \omega_i, && (4c)\\
\dot{v}_i &= u_{1i} + d_{1i}, && (4d)\\
\dot{\omega}_i &= u_{2i} + d_{2i}, \qquad i \in \Gamma. && (4e)
\end{aligned}$$

The problem we consider here is the tracking control problem. The reference trajectory $\mathcal{T}$ of the group of robots is described by the following equations:

$$\begin{aligned}
\dot{x}_r &= v_r \cos\theta_r, && (5a)\\
\dot{y}_r &= v_r \sin\theta_r, && (5b)\\
\dot{\theta}_r &= \omega_r. && (5c)
\end{aligned}$$

The definitions of variables in (4) and (5) are given in the following Table 1. In this paper, the communication topology among the group of nonholonomic mobile robots is denoted by the graph G and satisfies the following assumption.

Assumption 2.2. The communication topology graph G is undirected and connected.

x_i, y_i	Cartesian coordinates of the center of mass of the robot i
θ_i	Angle between the heading direction of the ith robot and the x_i axis
v_i	Translational velocity of the ith robot
ω_i	Angular velocity of the ith robot
u_{1i}, u_{2i}	Torques of robot i
d_{1i}, d_{2i}	Disturbances of robot i
x_r, y_r	Cartesian coordinates of the center of mass of the reference robot
θ_r	Angle between the heading direction of the reference robot and the x_r axis
v_r	Reference translational velocity
ω_r	Desired angular velocity

Tab. 1.

2.4. Error system

In this section, let us make some transformation for systems (4)−(5), and obtain the tracking error system. Firstly, we have the following assumptions which will be used in this paper.

Assumption 2.3. Suppose $\omega_r, \dot{\omega}_r, \ddot{\omega}_r$ are bounded with $0 < \omega_r^{\min} \leq |\omega_r(t)| \leq \omega_r^{\max}$, $|\dot{\omega}_r(t)| < |\omega_1^{\max}|$ and $|\ddot{\omega}_r(t)| < |\omega_2^{\max}|$ for each $t \geq t_0 \geq 0$, where $\omega_r^{\min}, \omega_r^{\max}, \omega_1^{\max}$ and $\omega_2^{\max}$ are appropriate constants.

Assumption 2.4. Suppose v_r and $\dot{v}_r$ are bounded with $|v_r(t)| \leq v_r^{\max}$ and $|\dot{v}_r(t)| \leq v_1^{\max}$ for each $t \geq t_0 \geq 0$, where $v_r^{\max}$ and $v_1^{\max}$ are appropriate constants.

Assumption 2.5. It is assumed that the disturbances d_{ji} $(j = 1, 2, i \in \Gamma)$ are unknown, time-varying but with bounded variation. That is

$$|\dot{d}_{1i}| \leq \delta_{1i}, \quad |\dot{d}_{2i}| \leq \delta_{2i}, \quad \forall t \geq 0, \quad i \in \Gamma, \tag{6}$$

where δ_{1i} and δ_{2i} are two known positive constants for robot i.

For simplicity, we convert the coordinates representation to the Cartesian coordinates fixed on each member of the group where the following global transformation is used [18]:

$$\begin{pmatrix} x_{ie} \\ y_{ie} \\ \theta_{ie} \end{pmatrix} = \begin{pmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_r - x_i \\ y_r - y_i \\ \theta_r - \theta_i \end{pmatrix}, i \in \Gamma, \tag{7}$$

which implies that

$$\begin{aligned}
\dot{x}_{ie} &= \omega_i y_{ie} - v_i + v_r \cos\theta_{ie}, && \text{(8a)}\\
\dot{y}_{ie} &= -\omega_i x_{ie} + v_r \sin\theta_{ie}, && \text{(8b)}\\
\dot{\theta}_{ie} &= \omega_r - \omega_i, && \text{(8c)}\\
\dot{v}_i &= u_{1i} + d_{1i}, && \text{(8d)}\\
\dot{\omega}_i &= u_{2i} + d_{2i}. && \text{(8e)}
\end{aligned}$$

If we denote $\omega_i = \omega_i - \omega_r + \omega_r$, $\bar{\omega}_i = \omega_r - \omega_i$, (8) can be rephrased as follows

$$\begin{aligned}
\dot{x}_{ie} &= \omega_r y_{ie} - v_i + v_r - \bar{\omega}_i y_{ie} + v_r(\cos\theta_{ie} - 1), && \text{(9a)}\\
\dot{y}_{ie} &= -\omega_r x_{ie} + \bar{\omega}_i x_{ie} + v_r \sin\theta_{ie}, && \text{(9b)}\\
\dot{\theta}_{ie} &= \bar{\omega}_i, && \text{(9c)}\\
\dot{v}_i &= u_{1i} + d_{1i}, && \text{(9d)}\\
\dot{\bar{\omega}}_i &= \dot{\omega}_r - u_{2i} - d_{2i}. && \text{(9e)}
\end{aligned}$$

Consider a state transformation defined by

$$X_{1i} = \begin{pmatrix} x_{1i} \\ x_{2i} \\ x_{3i} \end{pmatrix} = \begin{pmatrix} y_{ie} \\ -\omega_r x_{ie} \\ -\omega_r^2 y_{ie} + \omega_r(v_i - v_r) + \dot{\omega}_r \frac{x_{2i}}{\omega_r} \end{pmatrix}, \; X_{2i} = \begin{pmatrix} x_{4i} \\ x_{5i} \end{pmatrix} = \begin{pmatrix} \theta_{ie} \\ \bar{\omega}_i \end{pmatrix}. \tag{10}$$

Then, the derivatives of X_{1i} and X_{2i} can be written as

$$\begin{aligned}
\dot{X}_{1i} &= f_1(X_{1i}, \bar{u}_{1i}) + H_1 u_{1i} + H_1 d_{1i} + g(X_{1i}, X_{2i}), && \text{(11a)}\\
\dot{X}_{2i} &= f_2(X_{2i}) + H_2 u_{2i} + H_2 d_{2i}, \quad i \in \Gamma, && \text{(11b)}
\end{aligned}$$

where

$$\bar{u}_{1i} = -\omega_r \dot{\omega}_r x_{1i} + \left(\frac{\ddot{\omega}_r}{\omega_r} - \omega_r^2 - \frac{2\dot{\omega}_r^2}{\omega_r^2}\right) x_{2i} + \frac{2\dot{\omega}_r}{\omega_r} x_{3i} - \omega_r \dot{v}_r, \tag{12}$$

$$f_1(X_{1i}, \bar{u}_{1i}) = \begin{pmatrix} x_{2i} \\ x_{3i} \\ \bar{u}_{1i} \end{pmatrix}, \quad f_2(X_{2i}) = \begin{pmatrix} x_{5i} \\ \dot{\omega}_r \end{pmatrix}, \tag{13a}$$

$$H_1 = \begin{pmatrix} 0 \\ 0 \\ \omega_r \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \tag{13b}$$

$$\begin{aligned}
g(X_{1i}, X_{2i}) &= \begin{pmatrix} g_{11} \\ g_{21} \\ g_{31} \end{pmatrix}\\
&= \begin{pmatrix} -\frac{1}{\omega_r} x_{2i} x_{5i} + v_r \sin x_{4i} \\ x_{1i} x_{5i} \omega_r - \omega_r v_r(\cos x_{4i} - 1) \\ x_{2i} x_{5i} \omega_r - \omega_r^2 v_r \sin x_{4i} + \dot{\omega}_r x_{1i} x_{5i} - \dot{\omega}_r v_r(\cos x_{4i} - 1) \end{pmatrix}. && \text{(13c)}
\end{aligned}$$

Thus, the tracking error model (8) is transformed into two subsystems, i. e., third-order subsystem (11a) and second-order subsystem (11b).

Our aim here is to design appropriate control laws u_{1i} and u_{2i} such that system (4) can track the reference system (5) in finite time, i. e., the error system (8) is finite-time stable.

Variables x_{ji} $(j = 1,2,3,4,5, i \in \Gamma)$ are unified tracking errors between each robot and the desired trajectory. By considering the relation between systems (7) and (10), we can see that $x_{ji} = 0$ $(j = 1,2,3,4,5, i \in \Gamma)$ implies that $x_i = x_r, y_i = y_r, v_i = v_r$ and $\theta_i = \theta_r, \omega_i = \omega_r$. Therefore, our idea in this paper is to prove that states x_{ji} $(j = 1,2,3,4,5, i \in \Gamma)$ can converge to zero in finite time. In the following part, we will give two steps to design controllers u_{1i} and u_{2i} . In the first step, we design u_{2i} such that x_{4i} and $x_{5i}(i \in \Gamma))$ are forced to converge to zero in finite time. In the second step, we design u_{1i} such that x_{1i}, x_{2i} and $x_{3i}(i \in \Gamma)$ are driven to converge to zero in finite time.

Remark 2.6. The authors of [23] discussed finite-time tracking control problem of system (4) without external disturbances, and finite-time control laws were designed as follows

$$\begin{aligned} u_{1i} &= -\frac{k_3}{\omega_r}\left(x_{3i}^{\frac{p}{2-p}} + k_2^{\frac{p}{2-p}}\left(x_{2i}^p + k_1^p \sum_{j\in N_i} a_{ij}(x_{1i} - x_{1j}) + k_1^p x_{1i}\right)\right)^{\frac{3}{p}-2} + \dot{\omega}_r x_{1i} \\ &\quad -(\frac{\ddot{\omega}_r}{\omega_r^2} - \omega_r - \frac{2\dot{\omega}_r^2}{\omega_r^3})x_{2i} - \frac{2\dot{\omega}_r}{\omega_r^2}x_{3i} + \dot{v}_r \qquad (14a) \\ u_{2i} &= \dot{\omega}_r + k_4 \mathrm{sig}^{\alpha_1} x_{5i} + k_5\ \mathrm{sig}^{\alpha_2} x_{4i} + \sum_{j=1}^{n} a_{ij}\mathrm{sig}^{\alpha_1}(x_{5i} - x_{5j}) \\ &\quad + \sum_{j=1}^{n} a_{ij}\mathrm{sig}^{\alpha_2}(x_{4i} - x_{4j}), \qquad (14b) \end{aligned}$$

where k_i $(i = 1,2,3,4,5) > 0$ are appropriate constants , $1 < p = \frac{p_1}{p_2} < 2$, p_1, p_2 are positive odd integers, $0 < \alpha_1, \alpha_2 < 1$.

Lemma 2.7. (Ou, Du and Li [23]) Suppose the external disturbances $d_{ji} = 0$ $(j = 1,2, i \in \Gamma)$, if Assumptions 2.2 − 2.4 hold, then the control laws (14) can make system (8) uniformly globally finite time stable, i. e., system (4) can globally track the reference trajectory (5) in finite time without external disturbances.

3. MAIN RESULT

In this section, we will solve the finite-time tracking control problem for system (4). In other word, we will design control laws u_{ji} $(j = 1,2, i \in \Gamma)$ such that system (11) can converge to zero in finite time. Since the external disturbances exist, i. e., $d_{ji} \neq 0$ $(j = 1,2, i \in \Gamma)$, the performance of the closed-loop system will degrade if no proper method to deal with the disturbances. In order to improve the disturbance rejection performance, disturbance observers are introduced to estimate the disturbances.

This section will give two subsections to obtain finite-time control laws u_{ji} $(j = 1,2, i \in \Gamma)$. In the first subsection, we consider the second-order subsystem (11b) and design finite-time tracking control laws u_{2i} to guarantee system (11b) is finite time stable. In the second subsection, finite-time tracking control laws u_{1i} is given to guarantee third-order subsystem (11a) is finite time stable. Both subsection consists of two steps. In the first step, the finite-time disturbance observers are designed to estimate the external disturbances d_{ji} $(j = 1, 2, i \in \Gamma)$. In the second step, finite-time control laws u_{ji} $(j = 1, 2, i \in \Gamma)$ are designed, in which the estimated value of disturbances will be used for the feed-forward compensation.

3.1. Design of u_{2i} for each mobile robot

In this subsection, we will discuss second-order subsystem (11b) and design finite-time controller u_{2i} for robot $i(i \in \Gamma)$.

3.1.1. Design of finite-time disturbance observer for d_{2i}

Since there exists external disturbance d_{2i} in system (11b), the system performance will degrade if no efficient method to deal with the disturbances. In order to improve the disturbance rejection performance, a finite-time disturbance observer is introduced to estimate the disturbance d_{2i}, and the estimated value of disturbance will introduced as a feed-forward disturbance compensation part into controller (14b). Inspired by paper [19, 28], we introduce a finite-time disturbance observer (FTDO) to estimate external disturbance d_{2i}, which can be written as follows [19, 28].

$$\dot{z}_{0i} = \nu_{0i} - u_{2i} + \dot{\omega}_r, \tag{15a}$$

$$\nu_{0i} = -\lambda_{0i} L_{1i}^{\frac{1}{2}} |z_{0i} - x_{5i}|^{\frac{1}{2}} \operatorname{sign}(z_{0i} - x_{5i}) - z_{1i}, \tag{15b}$$

$$\dot{z}_{1i} = -\lambda_{1i} L_{1i} \operatorname{sign}(z_{1i} + \nu_{0i}), \quad i \in \Gamma, \tag{15c}$$

where z_{0i} and z_{1i} are the estimates of x_{5i} and d_{2i}, respectively, L_{1i} is an upper boundary of $\dot{d}_{2i}$, $\lambda_{0i}, \lambda_{1i} > 0$ being properly chosen so as to provide for the finite time convergence of the differentiator with L_{1i}.

According to Assumption 2.5, the existence of L_{1i} is reasonable. Then, we have the following results.

Lemma 3.1. (Shtessel, Shkolnikov and Levant [19, 28]) For the subsystem (11b) with disturbance observer (15), there exist the observer gains λ_{0i}, λ_{1i} and L_{1i} such that the estimated states z_{0i}, z_{1i} converge to x_{5i} and d_{2i}, respectively.

3.1.2. Design of finite-time control law u_{2i}

On the basis of the disturbance observer (15), the composite controller u_{2i} for subsystem (11b) is given as follows

$$u_{2i} = \dot{\omega}_r + k_4 \operatorname{sig}^{\alpha_1} x_{5i} + k_5 \operatorname{sig}^{\alpha_2} x_{4i} + \sum_{j=1}^{n} a_{ij} \operatorname{sig}^{\alpha_1}(x_{5i} - x_{5j}) + \sum_{j=1}^{n} a_{ij} \operatorname{sig}^{\alpha_2}(x_{4i} - x_{4j}) - z_{1i}, \tag{16}$$

where α_1 and α_2 are defined as that in Remark 2.6. Then, we have the following theorem.

Theorem 3.2. If Assumptions 2.2 − 2.5 hold, control law (16) makes subsystem (11b) finite time stable in the presence of external disturbances.

Proof. Substituting control law (16) into (11b), one obtains

$$\dot{x}_{4i} = x_{5i}, \tag{17a}$$

$$\begin{aligned}\dot{x}_{5i} &= -k_4 \operatorname{sig}^{\alpha_1} x_{5i} - k_5 \operatorname{sig}^{\alpha_2} x_{4i} - \sum_{j=1}^{n} a_{ij} \operatorname{sig}^{\alpha_1}(x_{5i} - x_{5j}) \\ &\quad - \sum_{j=1}^{n} a_{ij} \operatorname{sig}^{\alpha_2}(x_{4i} - x_{4j}) + (z_{1i} - d_{2i}).\end{aligned} \tag{17b}$$

On one hand, from Lemma 3.1, we know that there exists a time $t_0 > 0$ such that $z_{1i} = d_{2i}$ for any $t > t_0$. On the other hand, according to Lemma 2.7, the closed-loop system (17) is finite time stable when $t > t_0$. Thus, in the following, we only need to prove states x_{4i} and x_{5i} are bounded during $t \leq t_0$.

Consider the following function

$$V(x_{4i}, x_{5i}) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \int_0^{x_{4i} - x_{4j}} a_{ij} \operatorname{sig}^{\alpha_2}(s)\, \mathrm{d}s + \frac{1}{2} \sum_{i=1}^{n} x_{5i}^2 + \frac{k_5}{1+\alpha_2} \sum_{i=1}^{n} |x_{4i}|^{1+\alpha_2}, \tag{18}$$

which is positive definite with respect to x_{4i}, x_{5i}. Denote $e_{1i} = z_{1i} - d_{2i}$. Based on Assumption 2.2, it can be obtained that $a_{ij} = a_{ji}$. Differentiating $V(x_{4i}, x_{5i})$ along the closed-loop system (17) and making use of Lemma 2.1, we have

$$\begin{aligned}\dot{V}(x_{4i}, x_{5i}) = &\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \operatorname{sig}^{\alpha_2}(x_{4i} - x_{4j}) \dot{x}_{4i} + \sum_{i=1}^{n} x_{5i} \dot{x}_{5i} + k_5 \sum_{i=1}^{n} \operatorname{sig}^{\alpha_2} x_{4i} \dot{x}_{4i} \\ = &-k_4 \sum_{i=1}^{n} x_{5i} \operatorname{sig}^{\alpha_1} x_{5i} - \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_{5i} \operatorname{sig}^{\alpha_1}(x_{5i} - x_{5j}) + \sum_{i=1}^{n} x_{5i}(z_{1i} - d_{2i}) \\ = &-k_4 \sum_{i=1}^{n} |x_{5i}|^{1+\alpha_1} - \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij} + a_{ji}) x_{5i} \operatorname{sig}^{\alpha_1}(x_{5i} - x_{5j}) + \sum_{i=1}^{n} x_{5i} e_{1i} \\ = &-k_4 \sum_{i=1}^{n} |x_{5i}|^{1+\alpha_1} - \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_{5i} \operatorname{sig}^{\alpha_1}(x_{5i} - x_{5j}) \\ &- \tfrac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{n} a_{ij} x_{5j} \operatorname{sig}^{\alpha_1}(x_{5j} - x_{5i}) + \sum_{i=1}^{n} x_{5i} e_{1i} \\ = &-k_4 \sum_{i=1}^{n} |x_{5i}|^{1+\alpha_1} - \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \left\{ (x_{5i} - x_{5j}) \operatorname{sig}^{\alpha_1}(x_{5i} - x_{5j}) \right\} + \sum_{i=1}^{n} x_{5i} e_{1i} \\ = &-k_4 \sum_{i=1}^{n} |x_{5i}|^{1+\alpha_1} - \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} |x_{5i} - x_{5j}|^{1+\alpha_1} + \sum_{i=1}^{n} x_{5i} e_{1i} \\ \leq &\sum_{i=1}^{n} |x_{5i}||e_{1i}|.\end{aligned} \tag{19}$$

Since e_{1i} converges to zero in finite time, thus e_{1i} is bounded, we denote $|e_{1i}| \leq \gamma$, where $\gamma > 0$ is a constant.

If $|x_{5i}| > 1$, one obtains

$$\dot{V}(x_{4i}, x_{5i}) \leq \sum_{i=1}^{n} |x_{5i}|^2 |e_{1i}| \leq 2\gamma V(x_{4i}, x_{5i}). \tag{20}$$

If $|x_{5i}| \leq 1$, equation (19) can be written as

$$\dot{V}(x_{4i}, x_{5i}) \leq l_1 |e_{1i}| \leq l_1 \gamma, \quad l_1 \geq n. \tag{21}$$

Therefore, for any x_{ji} $(j = 4, 5, i \in \Gamma)$, we have

$$\dot{V}(x_{4i}, x_{5i}) \leq 2\gamma V(x_{4i}, x_{5i}) + l_1 \gamma. \tag{22}$$

From (22), one has

$$V(x_{4i}, x_{5i}) \leq \left(V(x_{4i}(0), x_{5i}(0)) + \frac{l_1}{2} \right) e^{2\gamma t} - \frac{l_1}{2}. \tag{23}$$

Therefore, x_{ji} $(j = 4, 5, i \in \Gamma)$ are bounded when $t \leq t_0$. Thus, control law (16) makes subsystem (11b) finite time stable. This completes the proof. □

3.2. Design of control law u_{1i}

In this subsection, we consider third-order subsystem (11a) and design finite-time tracking control laws u_{1i} to guarantee that system (11a) is finite time stable in the presence of external disturbances.

3.2.1. Design of finite-time disturbance observer for $\omega_r d_{1i}$

Because there exists disturbance $d_{1i}(i \in \Gamma)$ in third-order subsystem (11a), similar to section 3.1, let us first design a finite-time disturbance observer to estimate $\omega_r d_{1i}$, which can be written as follows [19, 28]

$$\dot{\bar{z}}_{0i} = \bar{\nu}_{0i} + \bar{u}_{1i} + \omega_r u_{1i} + g_{31}, \tag{24a}$$

$$\bar{\nu}_{0i} = -\bar{\lambda}_{0i} L_{2i}^{\frac{1}{2}} |\bar{z}_{0i} - x_{3i}|^{\frac{1}{2}} \operatorname{sign}(\bar{z}_{0i} - x_{3i}) + \bar{z}_{1i}, \tag{24b}$$

$$\dot{\bar{z}}_{1i} = -\bar{\lambda}_1 L_{2i} \operatorname{sign}(\bar{z}_{1i} - \bar{\nu}_{0i}), \tag{24c}$$

where $\bar{u}_{1i}$, g_{31} are defined as (12) and (13c), $\bar{z}_{0i}$ and $\bar{z}_{1i}$ are the estimates of x_{3i} and $\omega_r d_{1i}$, respectively, L_{2i} is an upper boundary of $\frac{d(\omega_r d_{1i})}{dt}$.

According to Assumptions 2.3 and 2.5, the existence of L_{2i} is reasonable. Then, we have the following results.

Lemma 3.3. (Shtessel, Shkolnikov and Levant [19, 28]) For third-order subsystem (11a) with disturbance observer (24), there exist the observer gains $\bar{\lambda}_{0i}$, $\bar{\lambda}_{1i}$ and L_{2i} such that the estimated states $\bar{z}_{0i}$, $\bar{z}_{1i}$ converge to x_{3i} and $\omega_r d_{1i}$, respectively.

3.2.2. Design of finite-time control law u_{1i}

Based on disturbance observer (24), we can get the following theorem.

Theorem 3.4. For subsystem (11a), If Assumptions 2.2 − 2.5 hold, and the controller is given as follows

$$\begin{aligned} u_{1i} = & -\tfrac{k_3}{\omega_r}\left(x_{3i}^{\frac{p}{2-p}} + k_2^{\frac{p}{2-p}}\left(x_{2i}^p + k_1^p \sum_{j\in N_i} a_{ij}(x_{1i}-x_{1j}) + k_1^p x_{1i}\right)\right)^{\frac{3}{p}-2} + \dot{\omega}_r x_{1i} \\ & -\left(\tfrac{\ddot{\omega}_r}{\omega_r^2} - \omega_r - \tfrac{2\dot{\omega}_r^2}{\omega_r^3}\right)x_{2i} - \tfrac{2\dot{\omega}_r}{\omega_r^2}x_{3i} + \dot{v}_r - \tfrac{\bar{z}_{1i}}{\omega_r}. \end{aligned} \tag{25}$$

Then, the closed-loop system (11a) with control law (25) is finite time stable in the presence of external disturbances.

Proof. According to Lemma 3.3, it is known that there exists a time $t_1 > 0$ such that $\bar{z}_{1i} = \omega_r d_{1i}$ for any $t > t_1$. Similar to Theorem 3.2, in the following, we only need to prove states x_{ji} $(j = 1, 2, 3, i \in \Gamma)$ are bounded during $t \leq t_1$.

Substituting composite control law (25) into (11a), one obtains

$$\begin{aligned} \dot{x}_{1i} = & \ x_{2i} - \tfrac{1}{\omega_r}x_{2i}x_{5i} + v_r \sin x_{4i}, \\ \dot{x}_{2i} = & \ x_{3i} + \omega_r x_{1i}x_{5i} - \omega_r v_r(\cos x_{4i} - 1), \\ \dot{x}_{3i} = & -k_3\left(x_{3i}^{\frac{p}{2-p}} + k_2^{\frac{p}{2-p}}\left(x_{2i}^p + k_1^p \sum_{j\in N_i} a_{ij}(x_{1i}-x_{1j}) + k_1^p x_{1i}\right)\right)^{\frac{3}{p}-2} + x_{2i}x_{5i}\omega_r \\ & -\omega_r^2 v_r \sin x_{4i} + \dot{\omega}_r x_{1i}x_{5i} - \dot{\omega}_r v_r(\cos x_{4i} - 1) - (\bar{z}_{1i} - \omega_r d_{1i}). \end{aligned} \tag{26}$$

Consider the finite time bounded function

$$B(X_{1i}) = \frac{1}{2}x_{1i}^2 + \frac{1}{2}x_{2i}^2 + \frac{1}{2}x_{3i}^2. \tag{27}$$

Denote $\bar{e}_{1i} = \bar{z}_{1i} - \omega_r d_{1i}$. Taking the derivative of $B(X_{1i})$ along system (26) yields

$$\begin{aligned} \dot{B}(X_{1i}) = & \ x_{1i}\dot{x}_{1i} + x_{2i}\dot{x}_{2i} + x_{3i}\dot{x}_{3i} \\ = & \ x_{1i}\big(x_{2i} - \tfrac{1}{\omega_r}x_{2i}x_{5i} + v_r \sin x_{4i}\big) + x_{2i}\big(x_{3i} + \omega_r x_{1i}x_{5i} - \omega_r v_r(\cos x_{4i} - 1)\big) \\ & +x_{3i}\big(\bar{u}_{1i} + \omega_r x_{2i}x_{5i} - \omega_r^2 v_r \sin x_{4i} + \dot{\omega}_r x_{1i}x_{5i} - \dot{\omega}_r v_r(\cos x_{4i} - 1)\big) \\ & -x_{3i}(\bar{z}_{1i} - \omega_r d_{1i}) \\ \leq & \ x_{1i}x_{2i} + (\omega_r - \tfrac{1}{\omega_r})x_{1i}x_{2i}x_{5i} + |v_r x_{1i}| + x_{2i}x_{3i} + 2|\omega_r v_r x_{2i}| \\ & +\omega_r x_{2i}x_{3i}x_{5i} + |\omega_r^2 v_r x_{3i}| + \dot{\omega}_r x_{1i}x_{3i}x_{5i} + 2|\dot{\omega}_r v_r x_{3i}| - x_{3i}\bar{e}_{1i} \\ & -k_3 x_{3i}\big(x_{3i}^{\frac{p}{2-p}} + k_2^{\frac{p}{2-p}}\big(x_{2i}^p + k_1^p \sum_{j\in N_i} a_{ij}(x_{1i}-x_{1j}) + k_1^p x_{1i}\big)\big)^{\frac{3}{p}-2}. \end{aligned}$$

First of all, based on Theorem 3.2 and Lemma 3.3, the states x_{ji} $(j = 4, 5, i \in \Gamma)$ and $\bar{e}_{1i}$ reach zero in finite time. Thus, x_{5i} and $\bar{e}_{1i}$ are bounded, we denote $|x_{5i}(t)| \leq x_5^{\max}$ and $|\bar{e}_{1i}| < \gamma_1$, where $x_5^{\max} > 0$ and $\gamma_1 > 0$ are two constants. Let $\eta_1 = \|X_{1i}(t)\| =$

$\sqrt{x_{1i}^2 + x_{2i}^2 + x_{3i}^2} \geq \eta > 1$, then we have $|x_{ji}(t)| \leq \eta_1 \leq \eta_1^2$ and $|x_{ji}(t)||x_{li}(t)| \leq \frac{\eta_1^2}{2}, (j, l = 1, 2, 3, i \in \Gamma)$. With this in mind, we obtain

$$\begin{aligned}
\dot{B}(X_{1i}) \leq\ & \tfrac{\eta_1^2}{2} + \tfrac{\eta_1^2}{2}|\omega_r - \tfrac{1}{\omega_r}|x_5^{\max} + \eta_1^2|v_r| + \tfrac{\eta_1^2}{2} + 2\eta_1^2|\omega_r v_r| + \tfrac{\eta_1^2}{2}|\omega_r|x_5^{\max} + \eta_1^2|\omega_r^2 v_r| \\
& + \tfrac{\eta_1^2}{2}|\dot{\omega}_r|x_5^{\max} + 2\eta_1^2|\dot{\omega}_r v_r| + \eta_1^2 \bar{e}_{1i} + k_3\eta_1 \left| \eta_1^{\frac{p}{2-p}} + k_2^{\frac{p}{2-p}}\eta_1^p + k_1^p k_2^{\frac{p}{2-p}} k' \eta_1 \right|^{\frac{3}{p}-2} \\
\leq\ & \eta_1^2 + \tfrac{\eta_1^2}{2}|\omega_r - \tfrac{1}{\omega_r}|x_5^{\max} + \eta_1^2|v_r| + 2\eta_1^2|\omega_r v_r| + \tfrac{\eta_1^2}{2}|\omega_r|x_5^{\max} + \eta_1^2|\omega_r^2 v_r| \\
& + \tfrac{\eta_1^2}{2}|\dot{\omega}_r|x_5^{\max} + 2\eta_1^2|\dot{\omega}_r v_r| + \eta_1^2\gamma_1 + k_3\eta_1 \left| \eta_1^{\frac{p}{2-p}} + k_2^{\frac{p}{2-p}}\eta_1^{\frac{p}{2-p}} + k_1^p k_2^{\frac{p}{2-p}} k' \eta_1^{\frac{p}{2-p}} \right|^{\frac{3}{p}-2} \\
\leq\ & \eta_1^2 + \tfrac{\eta_1^2}{2}|\omega_r - \tfrac{1}{\omega_r}|x_5^{\max} + \eta_1^2|v_r| + 2\eta_1^2|\omega_r v_r| + \tfrac{\eta_1^2}{2}|\omega_r|x_5^{\max} + \eta_1^2|\omega_r^2 v_r| \\
& + \tfrac{\eta_1^2}{2}|\dot{\omega}_r|x_5^{\max} + 2\eta_1^2|\dot{\omega}_r v_r| + \tfrac{\eta_1^2}{2}2\gamma_1 + k_3\eta_1^2 \left| 1 + k_2^{\frac{p}{2-p}} + k_1^p k_2^{\frac{p}{2-p}} k' \right|^{\frac{3}{p}-2} \\
=\ & \tfrac{\eta_1^2}{2}\Big(2 + |\omega_r - \tfrac{1}{\omega_r}|x_5^{\max} + 2|v_r| + 4|\omega_r v_r| + |\omega_r|x_5^{\max} + 2|\omega_r^2 v_r| + |\dot{\omega}_r|x_5^{\max} \\
& + 4|\dot{\omega}_r v_r| + 2\gamma_1 + 2k_3\big(1 + k_2^{\frac{p}{2-p}} + k_1^p k_2^{\frac{p}{2-p}} k'\big)^{\frac{3}{p}-2} \Big),
\end{aligned} \tag{28}$$

where $k' = 1 + 2\sum_{j \in N_i} a_{ij}$. $b' = \max\{b_1, b_2, \ldots, b_n\}$. Consider Assumptions 2.3 – 2.4, then $\dot{B}(X_{1i})$ can be rewritten as

$$\dot{B}(X_{1i}) \leq KB(X_{1i}), \tag{29}$$

where

$$\begin{aligned}
K =\ & 2 + |\omega_r^{\max} + \tfrac{1}{\omega_r^{\min}}|x_5^{\max} + 2v_r^{\max} + 4\omega_r^{max} v_r^{\max} + \omega_r^{\max} x_5^{\max} + 2(\omega_r^{\max})^2 v_r^{\max} \\
& + \omega_1^{max} x_5^{\max} + 4\omega_1^{\max} v_r^{\max} + 2\gamma_1 + 2k_3\big(1 + k_2^{\frac{p}{2-p}} + k_1^p k_2^{\frac{p}{2-p}} k'\big)^{\frac{3}{p}-2}.
\end{aligned} \tag{30}$$

On the other hand, if $\eta_1 \leq 1$, there exists a constant $L > 0$ such that $\dot{B}(X_{1i}) \leq L$. Thus, for any x_{ji} $(j = 1, 2, 3, i \in \Gamma)$, we have

$$\dot{B}(X_{1i}) \leq KB(X_{1i}) + L. \tag{31}$$

From (31), we can obtain

$$B(X_{1i}) \leq \left(B(X_{1i}(0)) + \frac{L}{K} \right) e^{Kt} - \frac{L}{K}. \tag{32}$$

Therefore, x_{ji} $(j = 1, 2, 3, i \in \Gamma)$ are bounded when $t \leq t_1$. Thus, control law (25) can make subsystem (11a) finite time stable. This completes the proof. □

By virtue of Theorems 3.2 and 3.4, we have the following main result.

Theorem 3.5. Consider systems (4) – (5) satisfying Assumptions 2.2 – 2.5, then control laws (16) and (25) make system (4) globally track the reference trajectory (5) in finite time, where the control parameters are chosen as in Remark 2.6.

P r o o f. From the proof procedure of Theorems 3.2 and 3.4, control laws (16) and (25) make the states $x_{ji} = 0$ ($j = 1, 2, 3, 4, 5, i \in \Gamma$) of system (11) converge to zero in finite time, i. e., system (4) globally tracks the reference trajectory (5) in finite time. Thus, the proof is completed. □

4. SIMULATION RESULTS

In this section, a numerical example is provided to illustrate our theoretical results derived in the previous section. The information exchange among mobile robots is shown in Figure 1, where $1, 2, 3, 4, 5$ are five mobile robots. We choose $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise.

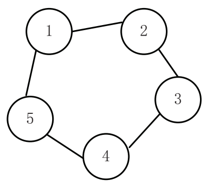

Fig. 1. Undirected and connected graph G.

In simulation we take the initial states as
$x_i(0) = (0.1507, -0.18, 0.2396, 0.1108, -0.4914)^T$, $\theta_i(0) = (1.86, 2.1, 2.3, 1.9, 2.27)^T$, $y_i(0) = (-0.0856, 0.0106, 0.4823, -0.015, -0.3479)^T$, $\omega_i(0) = (1.8, 1.95, 2.4, 1.7, 2.15)^T$, $v_i(0) = (1.2375, 0.7, 1.85, 1.05, 0.075)^T$. The initial values of disturbance observers are chosen as $z_{0i}(0) = (1.8, 1.95, 2.4, 1.7, 2.15)^T$, $z_{1i}(0) = (0, 0, 0, 1, 0)^T$, $\bar{z}_{0i}(0) = (1.86, 2.1, 2.3, 1.9, 2.27)^T$, and $\bar{z}_{1i}(0) = (0, 1, 0, 1, 0)^T$.

The reference velocities for system (5) are selected as in [20]: $v_r = 1.5 - \frac{1.5t}{t+10} m/s$, $\omega_r = 1 + \frac{2t}{t+10} rad/s$. The external disturbances are designed as $d_{11} = \sin(t), d_{12} = \cos(2t), d_{13} = 0.5\sin(t), d_{14} = \cos(t), d_{15} = 0.3\sin(t)$, $d_{21} = 0.2\sin(t), d_{22} = \cos(t), d_{23} = \sin(2t), d_{24} = 0.3\cos(3t), d_{25} = 0.8\sin(2t)$. Choose the gains as $\lambda_{01} = \lambda_{02} = \lambda_{03} = \lambda_{04} = \lambda_{05} = 6$, $\lambda_{11} = \lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{15} = 6$, $\bar{\lambda}_{01} = \bar{\lambda}_{02} = \bar{\lambda}_{03} = \bar{\lambda}_{04} = \bar{\lambda}_{05} = 4$, $\bar{\lambda}_{11} = \bar{\lambda}_{12} = \bar{\lambda}_{13} = \bar{\lambda}_{14} = \bar{\lambda}_{15} = 4$. Let $p = \frac{13}{11}$, $\alpha_1 = \frac{2}{5}$, $\alpha_2 = \frac{1}{4}$ and $k_1 = 0.3, k_2 = 5, k_3 = 9, k_4 = 4, k_5 = 4$. The simulation results are shown in Figure 2 – Figure 9.

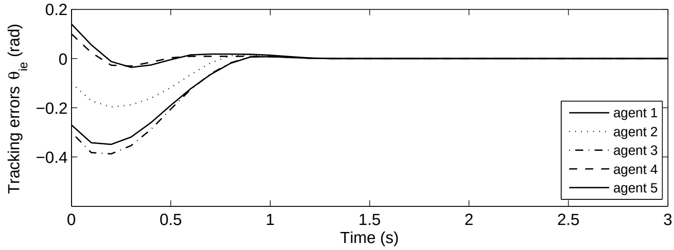

Fig. 2. Response curves of tracking errors θ_{ie} $(i = 1, 2, \ldots, 5)$.

Figures 2 – 3 show the tracking errors of $x_{ie}, y_{ie}, \theta_{ie}$ $(i = 1, 2, \ldots, 5)$ respect to time for each mobile robot. In Figure 4, the time response of v_i and ω_i of system (4) with FTDO are plotted, showing that the expected tracking velocities have been achieved for system (4) with the external disturbances under FTDO. Figures 5−8 show the disturbance estimated by finite-time disturbance observers (FTDO)(15) and (24), we can see that the observer exhibits excellent tracking performance. Figure 9 shows the control outputs of u_{1i} and u_{2i}, respectively.

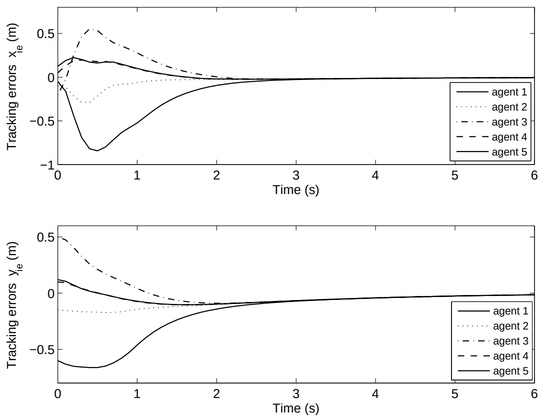

Fig. 3. Response curves of tracking errors x_{ie} and y_{ie} $(i = 1, 2, \ldots, 5)$.

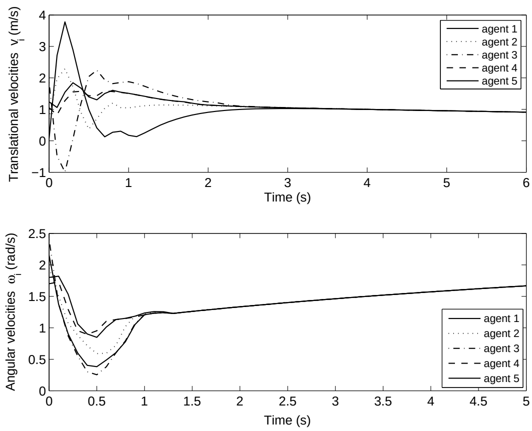

Fig. 4. Response curves of velocities v_i and ω_i $(i = 1, 2, \ldots, 5)$.

5. CONCLUSIONS

This paper has studied finite-time tracking control problem for a group of nonholonomic mobile robots in dynamic model with external disturbances, where a kind of finite-time disturbance observer has been introduced to estimate the external disturbances for each mobile robot. First of all, the unified tracking error has been transformed a fifth-order system consisting of two subsystems, i. e., a third-order subsystem and a second-order

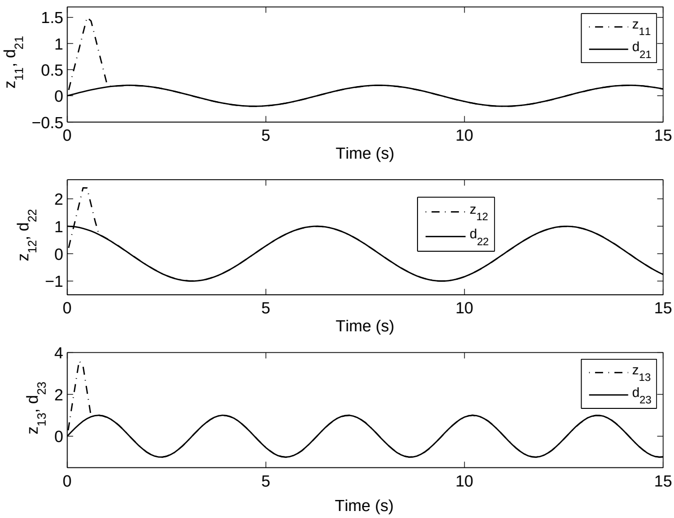

Fig. 5. Disturbance estimated by finite time disturbance observers (FTDO)(15).

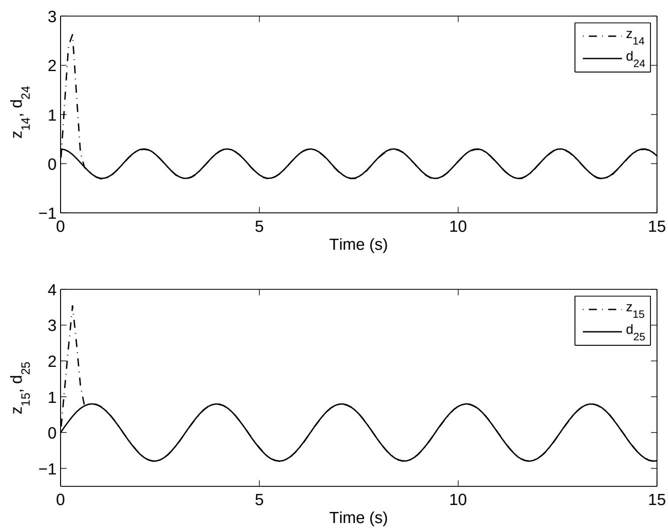

Fig. 6. Disturbance estimated by finite time disturbance observers (FTDO)(15).

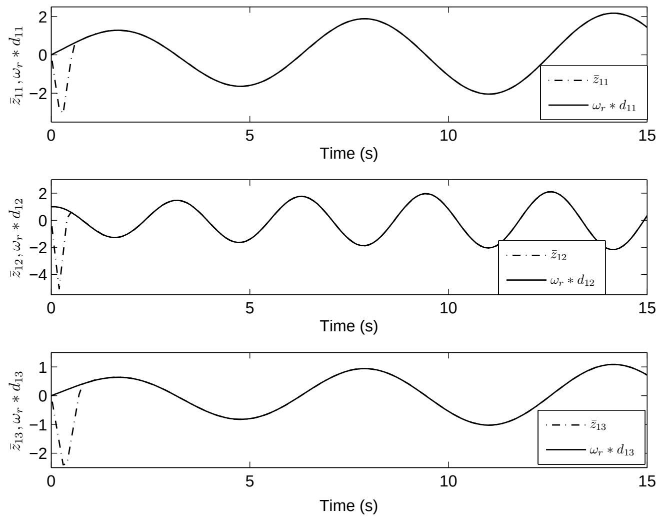

Fig. 7. Disturbance estimated by finite time disturbance observers (FTDO)(24).

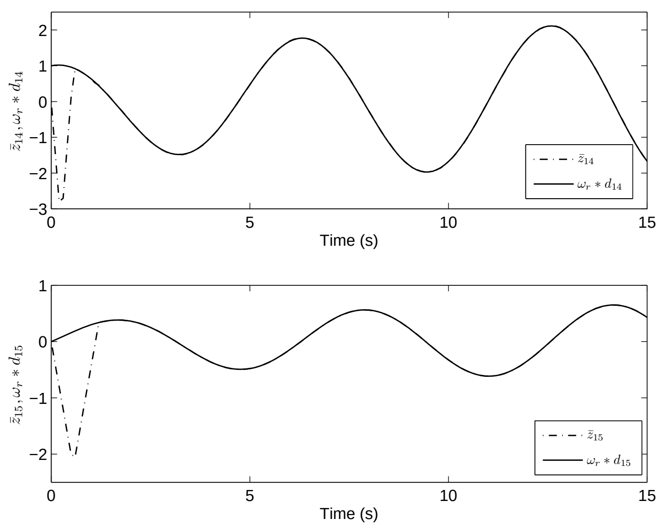

Fig. 8. Disturbance estimated by finite time disturbance observers (FTDO)(24).

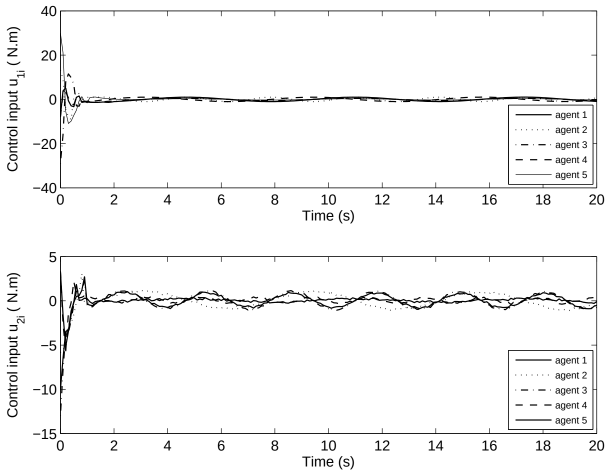

Fig. 9. Response curves of control outputs.

subsystem for each robot. Then, these two subsystems have been discussed respectively, continuous finite-time disturbance observers and finite-time tracking control laws have been designed for each mobile robot. Rigorous proof has shown that these finite-time controllers can make the states of a group of robots converge to a desired value in finite time, i. e., all the robots can track the desired trajectory in a finite time. Simulation results been presented to support the theoretical results. It is worth noting that the communication topology graph here is required to be connected and undirected. Future research will try to solve the case of directed network topology, which is more complicate and general.

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