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STABILITY ANALYSIS OF A THREE-DIMENSIONAL ENERGY DEMAND-SUPPLY SYSTEM UNDER DELAYED FEEDBACK CONTROL

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This paper considers a three-dimensional energy demand-supply system which typically demonstrates the relationship between the amount of energy supply and that of energy demand for the two regions in China. A delayed feedback controller is proposed to stabilize the system which was originally unstable even under some other controllers. The stability properties of the equilibrium points are subsequently analyzed and it is found that the Hopf bifurcation appears under some conditions. By using the center manifold theorem and normal form method, we obtain the explicit formulae revealing the properties of the periodic solutions of Hopf bifurcation to show stabilizing effects of the delayed feedback controller. Numerical simulations illustrate effectiveness of our results.

Keywords: a three-dimensional energy demand-supply system, stability, equilibrium point, delayed feedback control, Hopf bifurcation

Classification: 93D05, 93C15, 93C05, 93C95, 93D15

1. INTRODUCTION

In order to reduce the gap in gas supplies between different parts of China, natural gas is being transferred between the western and eastern regions of the country. This paper aims to provide an efficient method to maintain the balance throughout the process of gas energy transfer. First, we consider the mathematical model of energy supply and energy demand. In paper [6] a three-dimensional energy resources system was established, which shows the relationship between the amount of energy supply and that of energy demand for the two regions in China:

$$\begin{cases} \dot{x}(t) = a_1 x(t)\left(1 - \dfrac{x(t)}{M}\right) - a_2(y(t) + z(t)), \\ \dot{y}(t) = -b_1 y(t) - b_2 z(t) + b_3 x(t)[N - (x(t) - z(t))], \\ \dot{z}(t) = c_1 z(t)(c_2 x(t) - c_3). \end{cases} \tag{1.1}$$

In this model t is time, $x(t)$ represents the amount of the energy demand shortage of Region A, $y(t)$ represents the amount of the energy increment supplied from Region

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B to Region A, $z(t)$ represents the amount of the energy import of Region A, M is the maximum value of energy demand shortage of Region A, N is the threshold value of energy demand shortage of Region A ($M > N > 0$), $a_1(> 0)$ is the elasticity coefficient of the amount of energy consumption of Region A, $a_2(> 0)$ is the coefficient of the effect of energy supply of Region B on energy demand of Region A, $b_1(> 0)$ is the coefficient of the effect of energy supply of Region B on the supply rate of Region A, $b_2(> 0)$ is the coefficient of the effect of energy import of Region B on energy supply rate of Region B, $b_3(> 0)$ is the coefficient of the effect of energy demand of Region A on energy supply rate of Region B, $c_1(> 0)$ is a constant equal to energy import rate of Region A, $c_2(> 0)$ is the profit obtained from a unit of energy, $c_3(> 0)$ is the cost of supplying energy.

The previously introduced energy-demand has drawn significant attention because of its wide applications and theoretical significance([9] − [16]). For example, the Hopf bifurcation of energy system has been analyzed and the subcritical periodic bifurcation solution has been obtained under certain given conditions ([9]). By using the energy demand-supply data of the city of Shanghai between 1999 and 2005, the artificial neural network method has been applied to determine the parameters in the energy system. Moreover, the relationship between the energy supply and demand has been established by assessing the dynamics of the energy system ([10]). By analyzing a stochastic energy demand-supply system, the influence of random factors and different parameter selection on stability of the system has been explored([16]). In addition, a new variable can be added to the energy demand-supply system. For example, such variable can represent a renewable energy resource, with which the three-dimensional energy demand-supply system becomes four-dimensional([2, 17]). However, until recently few studies have concentrated on the stability analysis of the energy demand-supply system under stabilizing controllers, especially, the effective ones.

There exists complex attractors which differ from those of Lorenz, Rössler and Chen in the System (1.1) ([6]). For these chaotic energy systems, such controllers as linear feedback control, non-autonomous feedback control and adaptive control can not make the originally unstable equilibrium points and periodic orbits stable ([11]). The phenomenon of parametric perturbation of the system has been accounted for and an effective non-autonomous feedback controller for hyper chaos has been designed to make it stable ([12]). Subsequently, in order to treat this phenomenon, people have proposed numerous methods of designing controllers ([24]), among which the delayed feedback control has attracted much interest ([1, 13]). Compared with the traditional method of OGY ([4]), delayed feedback control needs no phase-space reconstruction and may be more easily designed. This study suggests how to make the chaotic energy system stable by imposing delayed feedback control on the energy system.

This paper is organized as follows: in part 2 we provides the design of a delayed feedback controller, with which the system can be stabilized, and the stability properties of the equilibrium point are analyzed; in part 3 properties of Hopf bifurcation are illustrated by explicit formulae; in part 4 numerical results are simulated to show effectiveness of our conclusion.

2. STABILIZATION OF THE SYSTEM UNDER DELAYED FEEDBACK CONTROL

It is proposed in [6] that system (1.1) has complex attractors different from that of Lorenz, Rössler and Chen. In this section, we design delayed feedback control for the system (1.1). Since the rate of change of energy demand is in proportion to the increasing amount of energy demand, the item: $k(x(t)-x(t-\tau))$ is added in the right side of the first equation in order to make the system stable for the appropriate the values of k and τ. That is, delayed feedback control is imposed on the first equation of the system (1.1):

$$\begin{cases} \dot{x}(t) = a_1 x(t)\Big(1-\dfrac{x(t)}{M}\Big) - a_2(y(t)+z(t)) + k(x(t)-x(t-\tau)), \\ \dot{y}(t) = -b_1 y(t) - b_2 z(t) + b_3 x(t)[N-(x(t)-z(t))], \\ \dot{z}(t) = c_1 z(t)(c_2 x(t) - c_3), \end{cases} \tag{2.1}$$

where $\tau > 0$ is time delay and k is feedback gain. Obviously, the system is just (1.1) when $\tau = 0$.

Next we will make the system asymptotically stabilized to the equilibrium point O. In order to do this, the energy demand of Region A needs be in balance with the energy supply of Region B, moreover, the energy import of Region A needs trend to zero as time increases infinitely. This section consists of the following parts: firstly stability properties of the system (2.1) at the equilibrium point O are analyzed, then by using the center manifold theorem and normal form method the explicit formulae are given to show the properties of Hopf bifurcation, and numerical simulations illustrate effectiveness of our results.

2.1. Stability of the system under delayed feedback control

The characteristic equation of the system (2.1) at the point O is

$$(c_1c_3+\lambda)[(a_1+k-ke^{-\lambda\tau}-\lambda)(b_1+\lambda)-a_2b_3N] = 0. \tag{2.2}$$

Obviously, $\lambda = -c_1c_3$ is a negative real root of the equation (2.2). Thus we only consider the roots of:

$$(a_1+k-ke^{-\lambda\tau}-\lambda)(b_1+\lambda)-a_2b_3N = 0, \tag{2.3}$$

which can be reorganized as:

$$\lambda^2 + (b_1-a_1-k)\lambda - a_1b_1 - b_1k + a_2b_3N + (k\lambda+b_1k)e^{-\lambda\tau} = 0. \tag{2.4}$$

Substituting

$$\begin{array}{ll} A = b_1-a_1-k, & B = -a_1b_1-b_1k+a_2b_3N, \\ C = k, & D = b_1k, \end{array}$$

into the equation (2.4), we have

$$\lambda^2 + A\lambda + B + (C\lambda+D)e^{-\lambda\tau} = 0. \tag{2.5}$$

We have the following Lemma ([5]).

Lemma 2.1. For the equation:

$$P(\lambda, e^{-\lambda\tau}) = \lambda^2 + A\lambda + B + (C\lambda + D)e^{-\lambda\tau},$$

only if there exists a root of zero or a pair of roots of pure imaginary number, its total multiplications changes in the open right half plane as time delay τ changes in a range.

Suppose $\pm iw (w > 0)$ be a pair of pure imaginary roots of the equation (2.5), then we have

$$\begin{cases} D\cos(w\tau) + Cw\sin(w\tau) = w^2 - B, \\ -D\sin(w\tau) + Cw\cos(w\tau) = -Aw, \end{cases} \tag{2.6}$$

which indicate that

$$w^4 + (A^2 - C^2 - 2B)w^2 + B^2 - D^2 = 0. \tag{2.7}$$

For the equation (2.7), we obtain Lemma 2.2 below.

Lemma 2.2. (1) If one of the three conditions

$$B^2 - D^2 < 0, \quad \begin{cases} B^2 - D^2 = 0, \\ A^2 - C^2 - 2B < 0, \end{cases} \quad \begin{cases} \Delta = (A^2 - C^2 - 2B)^2 - 4(B^2 - D^2) = 0, \\ A^2 - C^2 - 2B < 0, \end{cases}$$

holds, there exists only one root for the equation (2.7). That is, the equation (2.5) only has a pair of pure imaginary roots.

(2) If all of the inequalities $\begin{cases} \Delta = (A^2 - C^2 - 2B)^2 - 4(B^2 - D^2) > 0, \\ B^2 - D^2 > 0, \\ A^2 - C^2 - 2B < 0, \end{cases}$

hold, equation (2.7) has two roots which means that there exists two pairs of pure imaginary roots for the equation (2.5).

If there exists a solution for the equation (2.7), time delay τ can be obtained according to ω:

$$\tau_n = \frac{\arccos\left(\dfrac{Dw^2 - BD - ACw^2}{D^2 + C^2w^2}\right) + 2n\pi}{w}, \tag{2.8}$$

where $0 < \arccos\left(\dfrac{Dw^2 - BD - ACw^2}{D^2 + C^2w^2}\right) < 2\pi$ and n is a nonnegative integer.

Lemma 2.3.

$$\operatorname{sign}\left\{\frac{\mathrm{d}(\mathrm{Re}\lambda)}{\mathrm{d}\tau}|_{\lambda=iw}\right\} = \operatorname{sign}\left\{\frac{A^2 + 2(w^2 - B)}{A^2w^2 + (w^2 - B)^2} - \frac{C^2}{C^2w^2 + D^2}\right\}.$$

P r o o f. Make derivatives of the equation (2.5) on both sides about the variable τ shows that

$$[2\lambda + A + [C - \tau(C\lambda + D)]e^{-\lambda\tau}]\frac{\mathrm{d}\lambda}{\mathrm{d}\tau} = \lambda(C\lambda + D)e^{-\lambda\tau}$$

which together with (2.5) gives that

$$e^{-\lambda\tau} = -\frac{\lambda^2 + A\lambda + B}{C\lambda + D}.$$

Both of two equations above indicate that

$$\Big(\frac{\mathrm{d}\lambda}{\mathrm{d}\tau}\Big)^{-1} = -\frac{2\lambda + A}{\lambda(\lambda^2 + A\lambda + B)} + \frac{C}{\lambda(C\lambda + D)} - \frac{\tau}{\lambda}.$$

Therefore,

$$\begin{aligned}
\operatorname{sign}\Big\{\frac{\mathrm{d}(\mathrm{Re}\lambda)}{\mathrm{d}\tau}|_{\lambda=iw}\Big\} &= \operatorname{sign}\Big\{\mathrm{Re}\Big[\Big(\frac{\mathrm{d}\lambda}{\mathrm{d}\tau}\Big)^{-1}|_{\lambda=iw}\Big]\Big\}\\
&= \operatorname{sign}\Big\{\mathrm{Re}\Big[-\frac{2\lambda + A}{\lambda(\lambda^2 + A\lambda + B)} + \frac{C}{\lambda(C\lambda + D)} - \frac{\tau}{\lambda}\Big]|_{\lambda=iw}\Big\}\\
&= \operatorname{sign}\Big\{\frac{A^2 + 2(w^2 - B)}{A^2w^2 + (w^2 - B)^2} - \frac{C^2}{C^2w^2 + D^2}\Big\}.
\end{aligned}$$

□

Combing Theorem 1 of the paper [18] with Lemma 2.1, Lemma 2.2 and Lemma 2.3, we have the following theorem.

Theorem 2.4. (1) For any $\tau > 0$, there is no root or only one root for the equation (2.7), the equilibrium point O is unstable.

(2) If there exists two roots for the equation (2.7), that is, w_1 and w_2 (without loss of generality assuming $w_1 > w_2$), and suppose $\tau_1^{(n)}$ and $\tau_2^{(n)}$ represent time delay respectively corresponding to w_1 and w_2 where n is a nonnegative integer, there is divided into two situations below.

Situation I: When $\tau_1^{(0)} < \tau_2^{(0)}$ the equilibrium point O is unstable for any $\tau > 0$.

Situation II: When $\tau_1^{(0)} > \tau_2^{(0)}$ there exists a nonnegative integer l satisfying $\tau_2^{(l)} < \tau_1^{(l)} < \tau_1^{(l+1)}$. If $\tau \in \bigcup_{k=0}^{m}(\tau_2^{(k)}, \tau_1^{(k)})$, the equilibrium point O is asymptotically stable, and if $\tau \in \bigcup_{k=0}^{m}(\tau_1^{(k-1)}, \tau_2^{(k)})\bigcup(\tau_1^{(m)}, +\infty)$, $(\tau_1^{(-1)} = 0)$, the equilibrium point O is unstable where m is the minimum of all of values of for l. When $\tau = \tau_{1,2}^{(j)}$, $(j = 0, 1, 2, \ldots, m)$, Hopf bifurcation appears.

Proof. (1) According to Lemma 2.1, if there is no root of the equation (2.7), the number of roots with positive real parts is equal to the number of that when $\tau = 0$. Since when $\tau = 0$ the chaotic system (1.1) is unstable the characteristic equation (2.2) always has roots with positive real parts. Thus when $\tau > 0$ the equation (2.2) always has roots with positive real parts. Therefore, the equilibrium point O is unstable for any $\tau > 0$.

The paper [18] shows that when the equation (2.7) has one root w the inequality $\frac{\mathrm{d}(\mathrm{Re}\lambda)}{\mathrm{d}\tau}|_{\lambda=iw} > 0$ holds. Together with Lemma 2.1, the characteristic equation (2.2) always has roots with positive real parts and then the equilibrium point O is unstable for any $\tau > 0$.

(2) When the equation (2.7) has two roots w_1 and w_2, without loss of generality assume that $w_1 > w_2$, furthermore, $\tau_1^{(n)}$ and $\tau_2^{(n)}$ represent time delay respectively corresponding to w_1 and w_2 where n is a nonnegative integer, we have that $\frac{\mathrm{d}(\mathrm{Re}\lambda)}{\mathrm{d}\tau}|_{\lambda=iw_1,\tau=\tau_1^{(n)}} > 0$ and $\frac{\mathrm{d}(\mathrm{Re}\lambda)}{\mathrm{d}\tau}|_{\lambda=iw_2,\tau=\tau_2^{(n)}} < 0$ according to the paper [18]. In addition, the period of $\tau_1^{(n)}$ is less than that of $\tau_2^{(n)}$.

Situation I: In the case that $\tau_1^{(0)} < \tau_2^{(0)}$ we now consider stability properties of the equilibrium points O. The number of roots with positive real parts for the characteristic equation will not decrease as τ increases ([18]). Since the characteristic equation (2.2) always has roots with positive real parts when $\tau = 0$, the characteristic equation always has roots with positive real parts when $\tau > 0$. Therefore, the equilibrium point O is unstable for any $\tau > 0$.

Situation II: In the case that $\tau_1^{(0)} > \tau_2^{(0)}$ we consider stability for the point O. Since the period of $\tau_1^{(n)}$ is less than that of $\tau_2^{(n)}$, there exists a nonnegative integer l satisfying $\tau_2^{(l)} < \tau_1^{(l)} < \tau_1^{(l+1)}$. Assume that m is the the minimum for all of values of l. According to analysis of Theorem 1 for the paper [18] from page 218 to page 219 and Lemma 2.1, all roots of the characteristic equation (2.2) have negative real parts when $\tau \in \bigcup_{k=0}^{m}(\tau_2^{(k)}, \tau_1^{(k)})$ while the equation (2.2) has roots with positive real parts when $\tau \in \bigcup_{k=0}^{m}(\tau_1^{(k-1)}, \tau_2^{(k)}) \bigcup(\tau_1^{(m)}, +\infty)$, $(\tau_1^{(-1)} = 0)$. Therefore, if $\tau \in \bigcup_{k=0}^{m}(\tau_2^{(k)}, \tau_1^{(k)})$, the equilibrium point O is asymptotically stable, and if $\tau \in \bigcup_{k=0}^{m}(\tau_1^{(k-1)}, \tau_2^{(k)}) \bigcup(\tau_1^{(m)}, +\infty)$, $(\tau_1^{(-1)} = 0)$, the equilibrium point O is unstable. When $\tau = \tau_{1,2}^{(j)}$, $(j = 0, 1, 2, \ldots, m)$, the characteristic equation (2.2) has only one pair of pure imaginary roots while other roots have negative real parts, furthermore, the inequalities $\frac{\mathrm{d}(\mathrm{Re}\lambda)}{\mathrm{d}\tau}|_{\lambda=iw_1,\tau=\tau_1^{(j)}} \neq 0$ and $\frac{\mathrm{d}(\mathrm{Re}\lambda)}{\mathrm{d}\tau}|_{\lambda=iw_2,\tau=\tau_2^{(j)}} \neq 0$ hold. Therefore, Hopf bifurcation appears for the system (2.1) when $\tau = \tau_{1,2}^{(j)}$, $(j = 0, 1, 2, \ldots, m)$. □

2.2. Properties of Hopf bifurcation

Theorem 2.4 tells us that Hopf bifurcation may appear in the case of the equilibrium point $O(0, 0, 0)$ for $\tau = \tau_i^{(j)}$, $(i = 1, 2;\ j = 0, 1, 2, \ldots, m)$. In this section we consider the properties of Hopf bifurcation by using the center manifold theorem and normal form method ([19, 20]).

For the equilibrium point O, define functions as follows

$$u_1(t) = x(t\tau),\ u_2(t) = y(t\tau),\ u_3(t) = z(t\tau).$$

We transform the systems (2.1) equivalently into the system below

$$\dot{u}(t) = L_\mu(u_t) + F(\mu, u_t),\ \mu \in \mathbb{R}, \tag{2.9}$$

where $\dot{u}(t)=(\dot{u}_1(t),\ \dot{u}_2(t),\ \dot{u}_3(t))^T,\ u_t=u(t+\theta)=(u_1(t+\theta),\ u_2(t+\theta),\ u_3(t+\theta))^T$, $\theta\in[-1,0]$, $\mathbb{C}=C([-1,0],\mathbb{R}^3)$, $L_\mu:\mathbb{C}\to\mathbb{R}^3$, and $F:\mathbb{R}\times\mathbb{C}\to\mathbb{R}^3$ are given by

$$L_\mu(\phi)=\tau\begin{bmatrix} a_1+k & -a_2 & -a_2 \\ b_3N & -b_1 & -b_2 \\ 0 & 0 & -c_1c_3 \end{bmatrix}\begin{bmatrix}\phi_1(0)\\ \phi_2(0)\\ \phi_3(0)\end{bmatrix}+\tau\begin{bmatrix} -k & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\phi_1(-1)\\ \phi_2(-1)\\ \phi_3(-1)\end{bmatrix},$$

$$F(\mu,\phi)=\tau\begin{bmatrix} -\frac{a_1}{M}\phi_1^2(0) \\ -b_3\phi_1^2(0)+b_3\phi_1(0)\phi_3(0) \\ c_1c_2\phi_1(0)\phi_3(0)\end{bmatrix},\quad \phi(\theta)=(\phi_1(\theta),\ \phi_2(\theta),\ \phi_3(\theta))^T.$$

According to Riesz Representation Theorem, there exists a bounded variation function matrix $\eta(\theta,\mu),\theta\in[-1,0]$ which satisfies that

$$L_\mu(\phi)=\int_{-1}^{0}\mathrm{d}\eta(\theta,\mu)\phi(\theta),\forall\phi\in\mathbb{C}. \tag{2.10}$$

Next in order to be decomposed in the phase space, the systems (2.9) are subsequently transformed into the ordinary differential equations.

For $\phi(\theta)\in\mathbb{C}$, define

$$A(\mu)\phi(\theta)=\begin{cases}\frac{\mathrm{d}\phi(\theta)}{\mathrm{d}\theta}, & \theta\in[-1,0),\\ \int_{-1}^{0}\mathrm{d}\eta(\xi,\mu)\phi(\xi), & \theta=0,\end{cases}$$

$$R(\mu)\phi(\theta)=\begin{cases}0, & \theta\in[-1,0),\\ F(\mu,\phi), & \theta=0.\end{cases}$$

Then the system (2.9) can be transformed into the abstract differential equation:

$$\dot{u}_t=A(\mu)u_t+R(\mu)u_t, \tag{2.11}$$

where $u_t=u(t+\theta),\theta\in[-1,0]$.

For $\psi(s)\in C([-1,0],(\mathbb{R}^3)^*)$, define

$$A^*\psi(s)=\begin{cases}-\frac{\mathrm{d}\psi(s)}{\mathrm{d}s}, & s\in(0,1],\\ \int_{-1}^{0}\mathrm{d}\eta^T(t,0)\psi(-t), & s=0,\end{cases}$$

where $\eta^T(t,0)$ is defined in (2.10).

For $\phi\in\mathbb{C}$ and $\psi\in C([-1,0],(\mathbb{R}^3)^*)$, define the bilinear function

$$\langle\psi,\phi\rangle=\bar{\psi}(0)\phi(0)-\int_{-1}^{0}\int_{\xi=0}^{\theta}\bar{\psi}(\xi-\theta)\,\mathrm{d}\eta(\theta,0)\phi(\xi)\,\mathrm{d}\xi. \tag{2.12}$$

From the discussions in the section above, we know that $\pm i\omega\tau_i^{(j)}$, $(i=1,2;\ j=0,1,2,\dots,m)$ are eigenvalues of $A(0)$, and they are also the eigenvalues of $A^*(0)$ because it is adjoint operator of $A(0)$. Suppose $q(\theta)=q(0)e^{i\omega\tau_i^{(j)}\theta}$ is an eigenvector of $A(0)$ corresponding to $i\omega\tau_i^{(j)}$ and $q^*(s)=Dq^*(0)e^{i\omega\tau_i^{(j)}s}$ is the eigenvector of $A^*(0)$ corresponding to $-i\omega\tau_i^{(j)}$ where $q(0) = [1, \alpha, \beta]^T$, and $q^*(0) = [1, p, g]$. Moreover, $\langle q^*(s), q(\theta)\rangle = 1$, and $\langle q^*(s), \overline{q}(\theta)\rangle = 0$. Then we obtain that

$$\begin{aligned} &\alpha = \frac{b_3 N}{b_1 + i\omega}, && \beta = 0,\\ &p = \frac{a_2}{i\omega - b_1}, && g = \frac{a_2 + b_2 p}{i\omega - c_1 c_3},\\ &\bar{D} = [1 + \alpha\overline{p} - \tau_i^{(j)} k e^{-i\omega\tau_i^{(j)}}]^{-1}. \end{aligned}$$

When $\mu = 0$, for any solution of the system (2.11) define

$$z(t) = \langle q^*, u_t\rangle,\ W(t,\theta) = u_t(\theta) - 2\mathrm{Re}\{z(t)q(\theta)\}, \tag{2.13}$$

on the center manifold C_0. We have

$$W(t,\theta) = W(z(t), \bar{z}(t), \theta),$$

where

$$W(z(t), \bar{z}(t), \theta) = W_{20}(\theta)\frac{z^2}{2} + W_{11}(\theta)z\bar{z} + W_{02}(\theta)\frac{\bar{z}^2}{2} + \cdots, \tag{2.14}$$

z and $\bar{z}$ are local coordinates for center manifold C_0 in the directions of q^* and $\overline{q}^*$.

Then the flow of the system (2.9) on the center manifold can be determined by the following equations:

$$\dot{z}(t) = i\omega\tau_i^{(j)} z(t) + \bar{q}^*(0)F(0, W(t,0) + 2\mathrm{Re}[z(t)q(0)]). \tag{2.15}$$

Denote $G(z,\bar{z}) = \bar{q}^*(0)F(0, W(t,0) + 2\mathrm{Re}[z(t)q(0)])$, since

$$W(t,0) + 2\mathrm{Re}[z(t)q(0)] = u_t$$

we get that

$$F(0, W(t,0) + 2\mathrm{Re}[z(t)q(0)]) = F(0, u_t),$$

$$G(z(t), \bar{z}(t)) = \tau_i^{(j)}\bar{D}(1, \bar{p}, \bar{g})\begin{bmatrix} -\frac{a_1}{M}u_{1t}^2(0) \\ -b_3 u_{1t}^2(0) + b_3 u_{1t}(0)u_{3t}(0) \\ c_1 c_2 u_{1t}(0)u_{3t}(0) \end{bmatrix} \tag{2.16}$$

which can be represented respectively as power series of $z(t)$ and $\bar{z}(t)$:

$$G(z(t), \bar{z}(t)) = g_{20}\frac{z^2}{2} + g_{11}z(t)\bar{z}(t) + g_{02}\frac{\bar{z}^2}{2} + \cdots, \tag{2.17}$$

$$F(0, u_t) = f_{20}\frac{z^2}{2} + f_{11}z(t)\bar{z}(t) + f_{02}\frac{\bar{z}^2}{2} + \cdots. \tag{2.18}$$

From (2.13) and (2.14), we have that

$$\begin{aligned} u_t(\theta) = {}&(1, \alpha, \beta)^T e^{i\omega\tau_i^{(j)}\theta} z + (1, \bar{\alpha}, \bar{\beta})^T e^{-i\omega\tau_i^{(j)}\theta}\bar{z} + W_{20}(\theta)\tfrac{z^2}{2} + W_{11}(\theta)z(t)\bar{z}(t)\\ &+ W_{02}(\theta)\tfrac{\bar{z}^2}{2} + \cdots, \end{aligned} \tag{2.19}$$

where $u_t(\theta) = [u_{1t}(\theta), u_{2t}(\theta), u_{3t}(\theta)]^T$.

Substituting (2.19) into (2.16) and comparing their coefficients with that of (2.17) and (2.18) respectively give that:

$$g_{20} = \bar{q^*}(0) f_{20} = -2\bar{D}\tau_i^{(j)}\Big(\frac{a_1}{M} + \bar{p}b_3\Big),$$

$$g_{11} = \bar{q^*}(0) f_{11} = -2\bar{D}\tau_i^{(j)}\Big(\frac{a_1}{M} + \bar{p}b_3\Big),$$

$$g_{02} = \bar{q^*}(0) f_{02} = -2\bar{D}\tau_i^{(j)}\Big(\frac{a_1}{M} + \bar{p}b_3\Big),$$

$$g_{21} = \bar{q^*}(0) f_{21} = 2\bar{D}\tau_i^{(j)}\Big[W_{11}^{(3)}(0) + \frac{1}{2}W_{20}^{(3)}(0) - \Big(\frac{a_1}{M} + \bar{p}b_3\Big)(W_{20}^{(1)}(0) + 2W_{11}^{(1)}(0))\Big],$$

where g_{21} is determined by W_{20} and W_{11}.

From the paper [20] we obtain that

$$W_{20}(\theta) = \frac{ig_{20}}{w\tau_i^{(j)}} q(0) e^{i\omega\tau_i^{(j)}\theta} + \frac{i\bar{g}_{02}}{3\omega\tau_i^{(j)}} \bar{q}(0) e^{-i\omega\tau_i^{(j)}\theta} + E_1 e^{2i\omega\tau_i^{(j)}\theta},$$

$$W_{11}(\theta) = -\frac{ig_{11}}{\omega\tau_i^{(j)}} q(0) e^{i\omega\tau_i^{(j)}\theta} + \frac{i\bar{g}_{11}}{\omega\tau_i^{(j)}} \bar{q}(0) e^{-i\omega\tau_i^{(j)}\theta} + E_2,$$

where

$$E_1 = [2i\omega\tau_i^{(j)} I - \textstyle\int_{-1}^0 e^{2i\omega\tau_i^{(j)}\theta}\, \mathrm{d}\eta(\theta, 0)]^{-1} f_{20},$$
$$E_2 = -[\textstyle\int_{-1}^0 \mathrm{d}\eta(\theta, 0)]^{-1} f_{11}.$$

From $g_{20} = \bar{q^*}(0) f_{20}$ and $g_{11} = \bar{q^*}(0) f_{11}$, we have that

$$f_{20} = f_{11} = 2\tau_i^{(j)} \begin{bmatrix} -\frac{a_1}{M} \\ -b_3 \\ 0 \end{bmatrix}.$$

In addition, according to the definition of $A(\mu)$, when $\mu = 0$ these two equalities below hold:

$$\int_{-1}^0 e^{2i\omega\tau_i^{(j)}\theta}\, \mathrm{d}\eta(\theta, 0) = \tau_i^{(j)} \begin{bmatrix} a_1 + k - ke^{-2i\omega\tau_i^{(j)}} & -a_2 & -a_2 \\ b_3 N & -b_1 & -b_2 \\ 0 & 0 & -c_1 c_3 \end{bmatrix}$$

and

$$\int_{-1}^0 \mathrm{d}\eta(\theta, 0) = \tau_i^{(j)} \begin{bmatrix} a_1 & -a_2 & -a_2 \\ b_3 N & -b_1 & -b_2 \\ 0 & 0 & -c_1 c_3 \end{bmatrix}.$$

Substituting respectively f_{20}, f_{11}, $\int_{-1}^0 e^{2i\omega\tau_0\theta}\, \mathrm{d}\eta(\theta, 0)$ and $\int_{-1}^0 \mathrm{d}\eta(\theta, 0)$ into E_1 and E_2 gives that

$$E_1^{(1)} = \frac{2a_1(b_1+2\omega i) - 2a_2b_3M}{M(2\omega i + b_1)(a_1 + k - ke^{-2\omega i\tau_i^{(j)}} - 2\omega i) - a_2b_3MN},$$

$$E_1^{(2)} = \frac{2a_1b_3N - 2b_3M(a_1 + k - ke^{-2\omega i\tau_i^{(j)}} - 2\omega i)}{M(2\omega i + b_1)(a_1 + k - ke^{-2\omega i\tau_i^{(j)}} - 2\omega i) - a_2b_3MN},$$

$$E_1^{(3)} = 0,$$

$$E_2^{(1)} = \frac{2a_2b_3M - 2a_1b_1}{M(a_2b_3N - a_1b_1)},$$

$$E_2^{(2)} = \frac{2a_1b_3M - 2a_1b_3N}{M(a_2b_3N - a_1b_1)},$$

$$E_2^{(3)} = 0,$$

with which $E_1 = (E_1^{(1)},\ E_1^{(2)},\ E_1^{(3)})^T,\ E_2 = (E_2^{(1)},\ E_2^{(2)},\ E_2^{(3)})^T$.

Substitute E_1 and E_2 into $W_{20}(\theta)$ and $W_{11}(\theta)$ respectively, and substitute $W_{20}(\theta)$ and $W_{11}(\theta)$ into g_{21}, then all of g_{20}, g_{11}, g_{02} and g_{21} can be obtained by the system (2.1). Denote $c_1(0) = \frac{i}{2\omega\tau_0}(g_{20}g_{11} - 2|g_{11}|^2 - \frac{|g_{02}|^2}{3}) + \frac{g_{21}}{2}$, then we get the parameters that determine the properties of Hopf bifurcation as follows:

$$\begin{aligned}
\mu_{i2}^{(j)} &= -\frac{\mathrm{Re}\{\mathrm{c}_1(0)\}}{\mathrm{Re}\{\lambda'(\tau_{\mathrm{i}}^{(\mathrm{j})})\}}, \\
\beta_{i2}^{(j)} &= 2\mathrm{Re}\{\mathrm{c}_1(0)\}, \\
\tau_{i2}^{(j)} &= -\frac{\mathrm{Im}\{\mathrm{c}_1(0)\} + \mu_2\mathrm{Im}\{\lambda'(\tau_{\mathrm{i}}^{(\mathrm{j})})\}}{\omega\tau_i^{(j)}}.
\end{aligned} \tag{2.20}$$

According to positiveness or negativeness of parameters above and the paper [20], we get the properties of Hopf bifurcation by the following theorem.

Theorem 2.5. If $\mu_{i2}^{(j)} > 0$ ($\mu_{i2}^{(j)} < 0$), then the periodic solutions of Hopf bifurcation are supercritical (subcritical). If $\beta_{i2}^{(j)} < 0$ ($\beta_{i2}^{(j)} > 0$), then the periodic solutions of Hopf bifurcation are asymptotically stable (unstable). If $\tau_{i2}^{(j)} > 0$ ($\tau_{i2}^{(j)} < 0$), then the period of bifurcating periodic solutions increases (decreases).

2.3. Simulations

Parameters are selected as following

$$\begin{array}{lllll}
a_1 = 1, & a_2 = 1.5, & b_1 = 0.7, & b_2 = 0.9, & b_3 = 0.7, \\
c_1 = 2, & c_2 = 1.5, & c_3 = 1, & M = 1.5, & N = 1.
\end{array} \tag{2.21}$$

Then the system (2.1) is chaotic as is shown in Figure 1. Initial values are chosen as that $x(0) = 0.5,\ y(0) = 0.2,\ z(0) = 0.2$ in all of figures from Figure 1 to Figure 11.

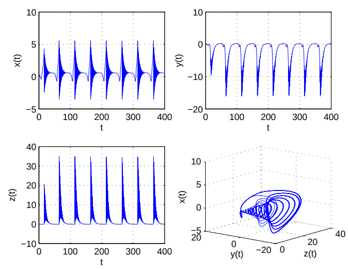

Fig. 1. The systems are chaotic when $\tau = 0$.

In Lemma 2.2, $\Delta > 0$ tells us that $k < -0.121865$ or $k > 0.241865$. Next we simulate under the condition that $k = -0.15$, with which following values can be attained:

$$\omega_1 = 0.7483,\ \tau_1^{(n)} = 4.1981 + 8.3963n,\ n = 0, 1, 2, \ldots$$

$$\omega_2 = 0.5916,\ \tau_2^{(n)} = 2.3721 + 10.6205n,\ n = 0, 1, 2, \ldots$$

Theorem 2.4 shows that the equilibrium point O is asymptotically stable when $\tau \in (2.3721, 4.1981)$, but unstable when $\tau \in (0, 2.3721) \bigcup (4.1981, +\infty)$, moreover, the system (2.1) produces Hopf bifurcation when $\tau = 4.1981$ or $\tau = 2.3721$.

When $\tau = \tau_2^{(0)}$, the parameters which determine the properties of Hopf bifurcation are get as follows:

$$\mu_{22}^{(0)} \approx -30.9262 < 0,\ \beta_{22}^{(0)} \approx -3.606 < 0,\ \tau_{22}^{(0)} \approx 6.7385 > 0,$$

which together with Theorem 2.5 show that the periodic solutions of Hopf bifurcation are subcritical, asymptotically stable and increasing periodically.

When $\tau = \tau_1^{(0)}$, the parameters which determine the properties of Hopf bifurcation are obtained below:

$$\mu_{12}^{(0)} \approx 158.053 > 0,\ \beta_{12}^{(0)} \approx -29.8404 < 0,\ \tau_{12}^{(0)} \approx 1.4254 > 0$$

which together with Theorem 2.5 indicate that the periodic solutions of Hopf bifurcation are supercritical, asymptotically stable and increasing periodically.

Next we simulate trends of energy demand shortage $x(t)$, energy supply increment $y(t)$ and energy import $z(t)$ as time increases and the phase diagram of system (2.1) all of which illustrate effectiveness of Theorem 2.4 and Theorem 2.5.

Set $\tau = 1 \in (0, 2.3721)$, we attain Figure 2 which shows that the system (2.1) still keeps chaotic as t increases and the equilibrium point O is unstable.

Set $\tau = \tau_2^{(0)} = 2.3721$ and $\tau = 2.34 < \tau_2^{(0)}$, then we have Figure 3 and Figure 4 respectively both of which show that the energy import $z(t)$ tends to zero, besides, both the amount of energy demand shortage of Region A $x(t)$ and that of the energy supply increment of Region B $y(t)$ shock periodically near the null point. Obviously in both of the figures Hopf bifurcation appears.

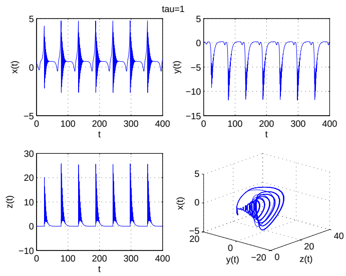

Fig. 2. The equilibrium point O is unstable when $\tau = 1 \in (0, 2.3721)$.

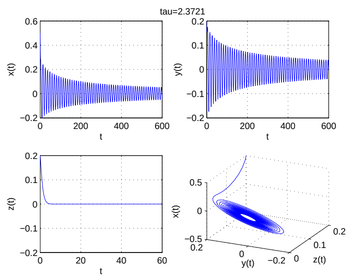

Fig. 3. Hopf bifurcation appears when $\tau = 2.3721$.

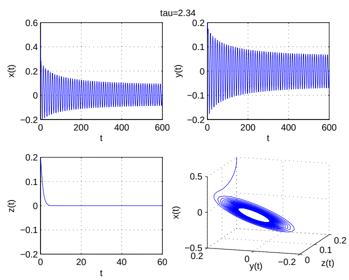

Fig. 4. The changes of Hopf bifurcation when $\tau = 2.34 < 2.3721$.

Set $\tau = 3 \in (2.3721, 4.1981)$, we attain Figure 5 which shows that the values of $x(t)$, $y(t)$ and $z(t)$ all tend to zero if time t is large enough. Therefore in this case the equilibrium point O is asymptotically stable and the chaotic system can be stabilized to the equilibrium point O.

Set $\tau = \tau_1^{(0)} = 4.1981$ and $\tau = 4.22 > \tau_1^{(0)}$, then we have Figure 6 and Figure 7 respectively both of which show that $z(t)$ tends to zero if time is large enough, besides, the amounts of $x(t)$ and $y(t)$ turbulence periodically near the null point. Evidently in these cases the system produces Hopf bifurcation.

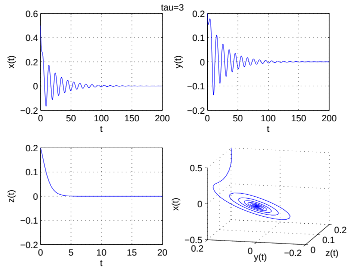

Fig. 5. The system is stabilized to the equilibrium point O when $\tau = 3 \in (2.3721, 4.1981)$.

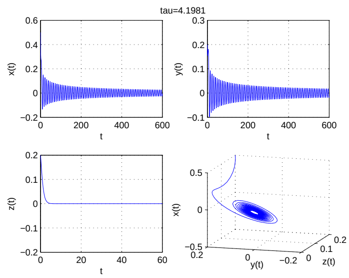

Fig. 6. Hopf bifurcation appears when $\tau = 4.1981$.

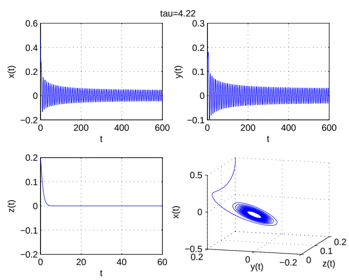

Fig. 7. The changes of Hopf bifurcation when $\tau = 4.22 > 4.1981$.

Set $\tau = 7 \in (4.1981, +\infty)$, we attain Figure 8 which shows that the system (2.1) is chaotic and the equilibrium point O is unstable.

All of Figure 2, Figure 3, Figure 5, Figure 6 and Figure 8 satisfy the case of Theorem 2.4 that there exists two roots. Comparing Figure 5 with Figure 6 tells us that the periodic solutions of Hopf bifurcation are subcritical, asymptotically stable and increasing periodically. Furthermore, comparing Figure 8 with Figure 9 shows that the periodic solutions of Hopf bifurcation are supercritical, asymptotically stable and increasing periodically.

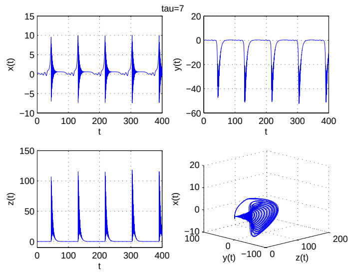

Fig. 8. The equilibrium point O is unstable when $\tau = 7 \in (4.1981, +\infty)$.

Next we choose $k = -0.1$ in which case the equation (2.7) has no root and the equilibrium point O is unstable according to Theorem 2.4. We choose $\tau = 10$ to attain Figure 9 which shows that the system is chaotic.

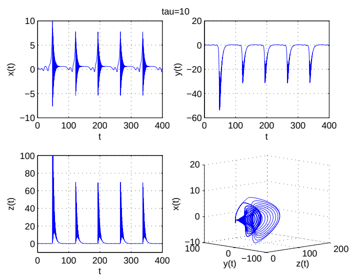

Fig. 9. The equilibrium point O is unstable when $k = -0.1$ and $\tau = 10$.

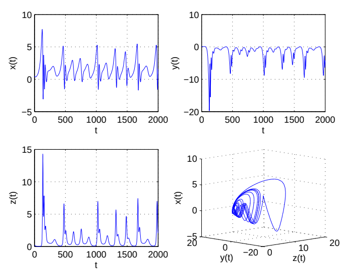

Fig. 10. The systems (1.1) is chaotic for the parameters (2.22).

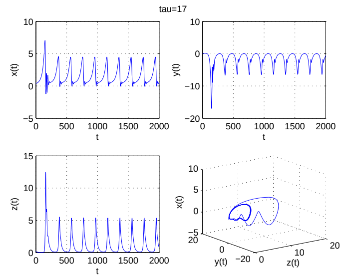

Fig. 11. Set $k = -0.05$ and $\tau = 17$, the equilibrium point O is unstable.

For the case that the equation (2.7) has only one root. We take the following parameters

$$\begin{array}{lllll} a_1 = 0.1, & a_2 = 0.15, & b_1 = 0.11, & b_2 = 0.1, & b_3 = 0.07, \\ c_1 = 0.2, & c_2 = 0.2, & c_3 = 0.3, & M = 1.8, & N = 1, \end{array} \tag{2.22}$$

with which the system (1.1) is chaotic as is shown in Figure 10. Choosing $k = -0.05$ which satisfies the preconditions of Lemma 2.2 gives that

$$w = 0.0972 \quad \text{and} \quad \tau^{(n)} = 15.4165 + 64.6165n$$

where n is a nonnegative integer. Theorem 2.4 tells us that the equilibrium point O is unstable. Choose $\tau = 17$, then we attain Figure 11 which illustrates our results.

3. CONCLUSION

In this paper, a delayed feedback controller was designed for a typical three-dimensional energy demand-supply system to make the chaotic system periodically changing and asymptotically stabilizing. The stable effect of the controller indicates that the supply and demand of energy between the two regions may be in balance under the time-delayed feedback control even without the import of energy. Although the controller is effective illustrated by our results, there may exist other simpler and easier control methods which is our future work.

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