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IMPULSIVE STABILIZATION AND SYNCHRONIZATION OF UNCERTAIN FINANCIAL HYPERCHAOTIC SYSTEMS

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In this paper the issue of impulsive stabilization and synchronization of uncertain financial hyperchaotic systems with parameters perturbation is investigated. Applying the impulsive control theory, some less conservative and easily verified criteria for the stabilization and synchronization of financial hyperchaotic systems are derived. The control gains and impulsive intervals can be variable. Moreover, the boundaries of the stable region are also estimated according to the equidistant impulse interval. Theoretical analysis and numerical simulations are shown to demonstrate the validity and feasibility of the proposed method.

Keywords: financial hyperchaotic system, impulse, stabilization, synchronization

Classification: 34D06, 34D35, 34C15

1. INTRODUCTION

Since Pecora and Carrol ([18]) introduced a method to synchronize two identical chaotic systems with different initial conditions, synchronization in chaotic dynamic systems has received particularly attention in various fields including secure communication, chemical reactions, biological systems, information science, plasma technologies, ([2, 5, 15, 19]) etc. In the past two decades, many effective control techniques have also been developed for synchronization of chaotic systems, such as linear feedback control ([16]), nonlinear feedback control ([17]), time-delay feedback control ([10]), adaptive control ([22]), impulsive control ([29]), and intermittent control ([7]), the cited references therein.

In comparison with continuous control of chaos, the discontinuous control method, which includes impulsive control, occasional bang-bang control, occasional proportional feedback and intermittent control, has attracted more interest recently due to its easy implementation in engineering control. The main idea of impulsive control ([13, 23, 24]) is to change the states of a system by the sudden jumps instantaneously. Impulsive control may provide a relatively highly efficient method for some cases in which the systems can not endure continuous disturbance. As a matter of fact, impulsive phenomena exist in many biological science and mechanics fields in practice. What is more important, impulsive control allows synchronization between chaotic systems only by small impulses being sent to the received systems at the discrete impulsive instances and

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which can reduce the information redundancy in the transmitted signal and increase robustness against the disturbances. Recently, impulsive control and synchronization of chaotic systems have been deeply studied, and many valuable results have been obtained ([6, 11, 12, 25, 26, 27, 31]). Yang and Chua ([25]) presented a theory of impulsive synchronization of two chaotic systems and a promising application of impulsive synchronization of chaotic systems to a secure digital communication scheme. Yang et al. used the theory of comparison system and impulsive differential equations to study the stabilization and synchronization of Lorenz system in ([26]) and ([27]), respectively. Itoh et al. ([11]) presented some experimental results on impulsive synchronization of chaotic circuits, which suggests that various applications of impulsive control and synchronization of chaotic system are feasible. Furthermore, they studied the impulsive control for synchronization of some continuous systems under the assumption that the synchronization errors are sufficiently small ([12]). Impulsive synchronization of uncertain chaotic systems was studied via adaptive strategy, which can relax the restrictions on the impulsive interval ([6, 31]).

As we all know, in nonlinear areas, researchers are striving to utilize the theory of nonlinear dynamics, especially the chaos theory, to study the complexity of economic and financial systems in recent years ([1, 3, 4, 8, 9, 14, 20, 21, 28, 30, 32]). Since Strotz et al. ([21]). has done the pioneering work in this area, various economics chaotic models have been proposed, such as the Kaldorian model ([20]), the IS-LM model ([9]), the hyperchaotic finance systems ([8, 28]). Economic chaotic systems are inevitably influenced by external disturbances stemmed from environmental interference ([30]), external disturbances may lead to the destabilization of economic and financial chaotic systems and cause undesirable results. So, control and synchronization ([1, 3, 4, 14, 32]) of the financial chaotic or hyperchaotic system have more significance. To the best of our knowledge, the stabilization and synchronization of uncertain financial hyperchaotic system with parameters perturbation is seldom discussed via impulsive control, where both control gains and impulsive intervals are variable. Therefore, it is necessary to study the global stabilization of economic and financial chaotic systems under the presence of external disturbance.

Motivated by the above discussions, the aim of this paper is to discuss the asymptotical stabilization and synchronization of uncertain financial hyperchaotic system with parameters perturbation. Based on the comparison theorem, we propose an impulsive law with variable control gains and impulsive intervals to achieve the stabilization and synchronization of uncertain financial hyperchaotic system. Furthermore, simulations are given to illustrate the effectiveness of the proposed method. The main contributions of this paper can be listed as follows: (a) based on the comparison theorem, the asymptotical stability of the uncertain financial hyperchaotic system is obtained, the method can be extended to other financial chaotic (hyperchaotic) systems. (b) Based on the novel financial hyperchaotic system, the synchronization of two uncertain financial chaotic systems is investigated, which can be employed in some other financial chaotic (hyperchaotic) systems.

The organization of this paper is as follows: In section 2, the theoretical model, some definitions and lemmas are presented. In section 3, the asymptotical stabilization of the financial chaotic system is analyzed, moreover, impulsive synchronization of two financial chaotic systems is also investigated. Two examples are provided to illustrate the effectiveness of the obtained scheme in section 4. Conclusion is given in the final section.

2. PRELIMINARIES

In this section, some preliminaries including the novel hyperchaotic system model ([28]), some necessary definitions and lemmas are presented, which are used throughout this paper.

Consider the following impulsive system:

$$\begin{cases} \dot{x}(t) = f(t,x), & t \neq t_k, \\ \Delta x = I_k(x), & t = t_k, \\ x(t_0^+) = x(t_0),\ t_0 \geq 0, & k = 1,2,\dots, \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state variable, $f : I \times \Omega \to R^n$ is right continuous function. $x(t_k^-) = \lim_{t \to t_k^-} x(t)$ and $x(t_k^+) = x(t_k)$. The instant sequence t_k satisfies $0 < t_1 < t_2 < \cdots < t_k < \cdots$, $t_k \to \infty$ as $k \to \infty$. $I_k : R^n \to R^n$ are continuous functions, $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$.

Definition 2.1. Let $V : R^n \times R^+ \to R^+$, then V is said to belong to class υ_0 if

(i) V is continuous in each of the sets $R^n \times [t_{k-1}, t_k)$, and for each $x \in R^n,\ k = 1,2,\dots,$ $\lim_{(y,t)\to(x,t_k^-)} V(y,t) = V(x,t_k^-)$ exists,

(ii) V is locally Lipschitzian in $x \in R^n$, and $V(x,t) = 0$ if and only if $x = 0$.

From this definition, one can see that a function V associated with the impulsive system (1) is similar to a Lyapunov function for the stability analysis of an ordinary differential equation.

Definition 2.2. For $(x,t) \in R^n \times [t_{k-1}, t_k)$, the right and upper Dini's derivative of $V(x,t_0) \in V_0$ is defined as follows:

$$D^+V(x,t) = \lim_{h\to 0} \sup \frac{1}{h}[V(x + hf(x,t), t+h) - V(x,t)].$$

Definition 2.3. Let $V \in \upsilon_0$ and assume that

$$\begin{cases} D^+V(x,t) \leq g(t, V(x,t)), & t \neq t_k, \\ V(x + \Delta x, t) \leq \Psi_k(V(x,t)), & t = t_k, \end{cases} \tag{2}$$

where $g : R^+ \times R^+ \to R$ is continuous and $\Psi_k : R^+ \to R^+$ is nondecreasing. Then the following system:

$$\begin{cases} \dot{\omega} = g(\omega, t), & t \neq t_k, \\ \omega(t_k^+) \leq \Psi_k(\omega(t_k)), & t = t_k, \\ \omega(t_0^+) = \omega_0 \geq 0, \end{cases} \tag{3}$$

is the comparison system of system (1).

Definition 2.4. $S(\rho) = \{x \in^n |\|x\|\}$ where $\|\cdot\|$ denotes the Euclidean norm on R^n.

Lemma 2.5. (Yang [23]) Assume that the following conditions are satisfied

(i) $V : S(\rho) \times R^+ \to R^+,\ \rho > 0,\ V \in \upsilon_0,\ D^+V(x,t) \leq g(V(x,t),t),\ t \neq t_k$. Where $g : R^+ \times R^+ \to R,\ g(0,t) = 0$ and $g(\cdot)$ is continuous function in $R^+ \times [t_{k-1}, t_k)$ and for each $x \in R^n,\ k = 1, 2, \ldots$.

(ii) there exists a $\rho_0 > 0$ such that $X \in S(\rho_0)$ implies that $X + I_k(X) \in S(\rho_0)$ for all k, $V(X + I_k(X), t) \leq \Psi_k[V(x,t)],\ t = t_k,\ X \in S(\rho_0)$.

(iii) $\beta(\|X\|) \leq V(X,t) \leq \alpha(\|X\|)$ on $S(\rho_0) \times R^+$ where $\alpha(\cdot), \beta(\cdot) \in K$ (K is the class of continuous functions $a : R^+ \to R^+$ such that $a(0) = 0$).

Then the stability properties of the trivial solution of comparison system (3) imply the corresponding stability properties of the trivial solution of (1).

Lemma 2.6. (Yang [23]) Let $g(\omega,t) = \dot{\lambda}(t)\omega,\ \lambda \in C^1[R^+, R^+],\ \dot{\lambda}(t) \geq 0,\ \Psi_k(\omega) = d_k\omega,\ d_k > 0$ for all $k \in N$, then the origin of system (1) is asymptotically stable if $\lambda(t_{k+1}) + ln(\gamma d_k) \leq \lambda(t_k)$, for all N, where $\gamma > 1$ is satisfied.

Lemma 2.7. Given any real matrices A, B of appropriate dimensions and a positive scalar $c > 0$ the following inequality is satisfied:

$$A^TB + B^TA \leq cA^TA + c^{-1}B^TB.$$

Proof. For any real $a \neq 0$, it is clear that we have

$$(aA - a^{-1}B)^T(aA - a^{-1}B) = a^2A^TA - A^TB - B^TA + a^{-2}B^TB \geq 0.$$

Let $a^2 = c$, the inequality is true. This completes the proof. □

The novel financial hyperchaotic system ([28]) can be described by the following differential equations:

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 + x_4 \\ \dot{x}_2 = 1 - bx_2 - x_1^2 \\ \dot{x}_3 = -x_1 - cx_3 \\ \dot{x}_4 = -dx_1x_2 - kx_4 \end{cases} \tag{4}$$

in which a, b, c, d and k are constant parameters. The system (4) has three equilibrium points $p_0\left(0, \frac{1}{b}, 0, 0\right)$ and $P_{1,2}(\pm\theta, \frac{k + ack}{c(k-d)}, \frac{\mp\theta}{c}, \frac{\mathrm{d}\theta(1+ac)}{cd - ck})$ when the parameters a, b, c, d and k satisfy $\frac{kb + cd + abck - ck}{c(d-k)} > 0$ and $\theta = \sqrt{\frac{kb + abck}{c(d-k)} + 1}$. When $a = 0.9, b = 0.2, c = 1.5, d = 0.2$ and $k = 0.17$, the novel financial system (4) has hyperchaotic behavior, as shown in Figure 1. More dynamic behaviors have been analyzed about the financial system (4) in ([28]).

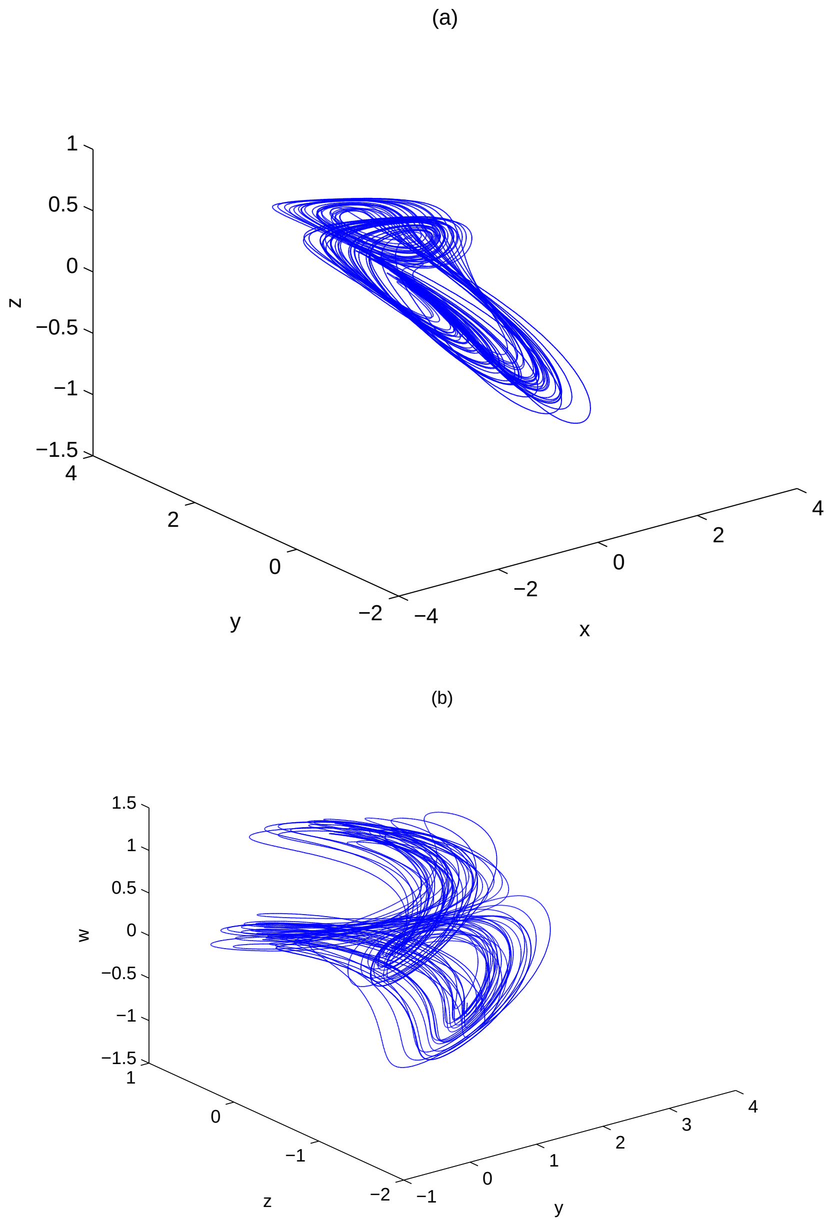

Fig. 1. The financial hyperchaotic attractor.

3. MAIN RESULTS

In this section, simple but effective impulsive control is designed to control hyperchaos to the three unstable equilibrium points p_0 and $p_{1,2}$. Since these equilibrium points are not original, to analyze the asymptotical stabilization of system, we will transform these equilibrium points to zero.

For the equilibrium point p_0, we have a the following transformation:

$$\begin{cases} X_1 = x_1 \\ X_2 = x_2 - \dfrac{1}{b} \\ X_3 = x_3 \\ X_4 = x_4. \end{cases} \tag{5}$$

Then financial hyperchaotic system (4) is rewritten as follows:

$$\begin{cases} \dot{X}_1 = X_3 + \left(X_2 + \dfrac{1}{b} - a\right) X_1 + X_4 \\ \dot{X}_2 = -bX_2 - X_1^2 \\ \dot{X}_3 = -X_1 - cX_3 \\ \dot{X}_4 = -\dfrac{d}{b} x_1 - dX_1X_2 - kX_4. \end{cases} \tag{6}$$

By decomposing the linear and nonlinear parts of the uncertain financial hyperchaotic system in (6), we can rewrite it as

$$\dot{X} = (A + \Delta A(t))X + f(X) \tag{7}$$

where $X = (X_1, X_2, X_3, X_4)^T$, $\Delta A(t) \in R^{n\times n}$ parameters perturbation matrix bounded by $\Delta A^T(t)\Delta A(t) \leq \gamma^2 I_n$, $A \in R^{n\times n}$ is a system matrix, and $f(X)$ is a continuous nonlinear function,

$$A = \begin{pmatrix} -\dfrac{1}{b} - a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ -\dfrac{d}{b} & 0 & 0 & -k \end{pmatrix} \quad \text{and} \quad f(x) = \begin{pmatrix} X_1X_2 \\ -X_1^2 \\ 0 \\ -dX_1X_2 \end{pmatrix}. \tag{8}$$

The impulsively controlled financial hyperchaotic system is then given by

$$\begin{cases} \dot{X} = (A + \Delta A(t))X + f(X), & t \neq t_k, \\ \Delta X = I_k(X) = B_kX, & t = t_k, k \in N, \end{cases} \tag{9}$$

where $B_k \in R^{n\times n}$ is the impulses gain matrix, the impulses be separated by interval $t_{k+1} - t_k = \delta_k$. Since chaotic (hyperchaotic) signals are bounded, so there exists a positive number M such that $|X_4| \leq M$ for all t.

Remark 3.1. The implementation of the nonlinear controller in practical systems is difficult due to the results of physical limitations. However, impulsive controller has a relatively simple structure and is easy to stabilize the financial system. In an impulsive control scheme, only the impulses are sent to the controlled system at the discrete impulsive instance, which can reduce the information redundancy in the transmitted signal, and increase the robustness against the disturbances and decrease the control cost.

To establish the sufficient conditions for stability of impulsive differential systems, we use the following theorem to guarantee that the uncertain financial hyperchaotic system is asymptotically stabilized. Then, we have the following results.

Theorem 3.2. Let λ_1 be the largest eigenvalue of $A^T + A$, λ_k be the largest eigenvalue of $(I + B_k)^T(I + B_k)$, $\lambda = \max(\lambda_k)$. If the following inequality holds

$$\ln(\xi\lambda) + (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + dM)\delta_k \leq 0, \ \xi > 1, \tag{10}$$

then the origin of the impulsive controlled system (9) is asymptotically stable.

P r o o f. Select the following Lyapunov type function defined as

$$V(t, x) = X^T X.$$

From the equality (9), for $t \neq t_k$, we have

$$\begin{aligned} D^+V(t, X) &= \dot{X}^T X + X^T \dot{X} \\ &= [(A + \Delta A(t))X + f(X)]^T X + X^T[(A + \Delta A(t))X + f(X)] \\ &= X^T(A^T + A + \Delta A^T(t) + \Delta A(t))X + f(X)^T X + X^T f(X). \end{aligned}$$

By using lemma 2.7, we have

$$\begin{aligned} (\Delta A^T(t))X + X^T(\Delta A(t)) &\leq \varepsilon_1 X^T \Delta A^T(t)\Delta A(t)X + \varepsilon_1^{-1}X^T X \\ &\leq \varepsilon_1\gamma^2 X^T X + \varepsilon_1^{-1}X^T X. \end{aligned}$$

Then, we have

$$\begin{aligned} D^+V(t, X) &\leq X^T(A^T + A + \varepsilon_1\gamma^2 I + \varepsilon_1^{-1}I)X + 2dX_1X_2X_4 \\ &\leq X^T(A^T + A + \varepsilon_1\gamma^2 I + \varepsilon_1^{-1}I)X + dM(X_1^2 + X_2^2) \\ &\leq (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1})X^T X + dMX^T X \\ &= (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + dM)V(X, t). \end{aligned}$$

Hence, condition 1 of Lemma 2.5 is satisfied with $g(\omega, t) = (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + dM)\omega$.
Since $\|X + I_k(X)\| = \|X + B_kX\| \leq \|I + B_k\|\|X\|$ and $\|I + B_k\|$ is finite, $\rho_0 > 0$ and $X \in S(\rho_0)$ such that $X + I_k(X) \in S(\rho_0)$.

On the other hand, when $t = t_k$, we have

$$\begin{aligned}(D^+V(X + B_kX, t_k^+) &= X^T(t_k)(I + B_k)^T(I + B_k)X(t_k)\\ &\leq \lambda_k X^T(t_k)X(t_k)\\ &\leq \lambda V(X, t_k).\end{aligned}$$

Hence condition 2 of Lemma 2.5 is satisfied with $\Psi_k(\omega) = \lambda\omega$. We can see that condition 3 of Lemma 2.5 is also satisfied. It follows from Lemma 2.5 that the asymptotic stability of system (9) is implied by the following comparison system:

$$\begin{cases}\dot{\omega} = (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + dM)\omega, & t \neq t_k,\\ \omega(t_k^+) = \lambda\omega(t_k), & t = t_k,\\ \omega(t_0^+) = \omega(0) \geq 0.\end{cases} \tag{11}$$

From (10), we have $\ln(\xi b) + \int_k^{k+1}(\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + dM)\,\mathrm{d}t \leq 0,\ \xi > 1$, for all $k \in N$. It follows from Lemma 2.6 that the trivial solution of (9) is asymptotically stable. □

Remark 3.3. heorem 3.2 gives the sufficient conditions for the asymptotic stability of controlling the systems to the origin. The results are new and comprehensive for the impulsive control of the financial hyperchaotic system. Furthermore, we can obtain an estimate of the upper bound of impulsive interval δ_k, the largest impulsive interval $\delta_{\max} = -\dfrac{\ln(\xi\lambda)}{\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + dM}$ $(0 < \lambda, \lambda_i < 1)$, depends on the value ξ, the parameters perturbation $\Delta A(t)$, the largest eigenvalue of $(I + B_k)^T(I + B_k)$, and the largest eigenvalue of $A^T + A$.

For the equilibrium point p_1 , we have a the following transformation:

$$\begin{cases}X_1 = x_1 - \theta\\ X_2 = x_2 - \dfrac{k + ack}{ck - cd}\\ X_3 = x_3 + \dfrac{\theta}{c}\\ X_4 = x_4 - \dfrac{\mathrm{d}\theta(1 + ac)}{cd - ck}.\end{cases} \tag{12}$$

Then financial hyperchaotic system (4) is rewritten as follows:

$$\begin{cases}\dot{X}_1 = \dfrac{k + ack}{ck - cd}X_1 + \theta X_2 + X_4 + X_1X_2\\ \dot{X}_2 = -2\theta X_1 - bX_2 - X_1^2\\ \dot{X}_3 = -X_1 - cX_3\\ \dot{X}_4 = \dfrac{\mathrm{d}\theta(1 + ac)}{cd - ck}x_1 - \mathrm{d}\theta X_2 - kX_4 - dX_1X_2\end{cases} \tag{13}$$

where $A = \begin{pmatrix} \dfrac{k+ack}{ck-cd} & \theta & 1 & 1 \\ -2\theta & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ \dfrac{\mathrm{d}\theta(1+ac)}{cd-ck} & -\mathrm{d}\theta & 0 & -k \end{pmatrix}$ and $f(x) = \begin{pmatrix} X_1X_2 \\ -X_1^2 \\ 0 \\ -dX_1X_2 \end{pmatrix}$.

For the equilibrium points $p_{1,2}$, the rest of proof is the same as that of theorem 3.2, omitted here. And we have also similar as the results of theorem1.

In order to achieve financial synchronous development of developed and developing countries or different areas, we need to solve more problems of synchronization of financial hyperchaotic systems.

Next, we are now in a position to discuss the impulsive synchronization of two financial hyperchaotic systems. In order to deal with impulsive synchronization, we need to design control input for a response system so that the response system achieves synchronization with the drive system, provided that the two systems start from different initial conditions. The drive system is given by (4), the response impulsively controlled financial hyperchaotic system is defined as:

$$\begin{cases} \dot{y} = (A + \Delta A(t))y + f(y), & t \neq t_k, \\ \Delta y = I_k(e) = B_k e, & t = t_k, k \in N, \end{cases} \tag{14}$$

where $e^T = (e_1, e_2, e_3, e_4) = (y_1 - x_1, y_2 - x_2, y_3 - x_3, y_4 - x_4)$ is the synchronization error.

If we define

$$\Psi(x,y) = f(y) - f(x) = \begin{pmatrix} y_1y_2 - x_1x_2 \\ -y_1^2 + x_1^2 \\ 0 \\ -dy_1y_2 + dx_1x_2 \end{pmatrix}. \tag{15}$$

Then the error system of the impulsive synchronization is given by

$$\begin{cases} \dot{e} = (A + \Delta A(t))e + \Psi(x,y), & t \neq t_k, \\ \Delta e = I_k(e) = B_k e, & t = t_k, k \in N. \end{cases} \tag{16}$$

We use the following theorem to guarantee that the origin of (16) is asymptotically stable.

Theorem 3.4. Let λ_1 be the largest eigenvalue of $A^T + A$, let λ_k be the largest eigenvalue of $(I + B_k)^T(I + B_k)$. If the following inequality holds

$$\ln(\xi\lambda) + (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + 3N + 2Nd)\delta_k \leq 0, \ \xi > 1, \tag{17}$$

then the origin of the synchronization error system (16) is asymptotically stable.

P r o o f. It is easy to know that the state variables of system (14) are bounded. We assume that $N = \max|x_1|, |x_2|, |y_1|$. Observe that the error system (16) is almost the same as the system in (9) except $\Psi(x,y)$. Similarly, let us construct the following Lyapunov function

$$V(e,t) = e^T e.$$

For $t \neq t_k$, we have

$$\begin{aligned}
D^+V(t,e) &= \dot{e}^T e + e^T \dot{e} \\
&= [(A + \Delta A(t))e + \Psi(x,y)]^T e + e^T [(A + \Delta A(t))e + \Psi(x,y)] \\
&= e^T (A^T + A + \Delta A^T(t) + \Delta A(t))e + \Psi(x,y)^T e + e^T \Psi(x,y) \\
&\leq (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1})e^T e + 2e_1(y_1y_2 - x_1x_2) \\
&\quad + 2e_2(x_1^2 - y_1^2) - 2de_4(y_1y_2 - x_1x_2) \\
&\leq (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1})e^T e + 2N(e_1^2 + |e_1||e_2|) + 2dN(|e_2||e_4| + |e_1||e_4|) \\
&\leq (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1})e^T e + N(3e_1^2 + e_2^2) + dN(e_1^2 + e_2^2 + 2e_4^2|) \\
&\leq (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + 3N + 2Nd)V(t,e).
\end{aligned}$$

Hence, condition 1 of Lemma 2.5 is satisfied with $g(\omega,t) = (\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + 3N + 2Nd)\omega$. The rest of proof is the same as the corresponding proof of theorem 3.2, omitted here. □

Remark 3.5. If we assume $\{t_k\}$ is equidistant, and $t_k - t_{k-1} = \delta$, the estimate of the upper bound of impulsive interval $\delta_{\max} = -\dfrac{\ln(\xi\lambda)}{\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + 3N + 2Nd}$, $0 < \lambda, \lambda_i < 1$.

Remark 3.6. From conditions (10) and (17), we note that the more uncertain the system parameters, the shorter the impulsive intervals should be designed. If the bounds of parametric uncertainties are unavailable, Theorems 3.2 and 3.4 still provide theoretical bases to ensure the stabilization and synchronization if the length of impulsive interval is chosen to be small enough. This implies that all the impulsive schemes have certain degree of robustness and can be applicable.

4. NUMERICAL SIMULATIONS

In this section, to verify theoretical results obtained in the previous section, the corresponding numerical simulations will be performed. In the following numerical simulations, the fourth-order Runge-Kutta method of Matlab is applied to solve the systems with time step 0.001. Choose $a = 0.9, b = 0.2, c = 1.5, d = 0.2$ and $k = 0.17$ to ensure the existence of hyperchaos in system (4). For simplicity, we assume t_k is equidistant, and $t_k - t_{k-1} = \delta$. Let $\varepsilon_1 = 1$. In ([28]) typical phase portraits of this system are shown. It is also shown that in hyperchaotic region of parameters, this system is a forced dissipative system with bounded states ($M \leq 4$) as $t \to \infty$.

Example 4.1. Consider the impulsively controlled uncertain financial hyperchaotic system (6):

$$\begin{pmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\\ \dot{X}_4\end{pmatrix} = \begin{pmatrix}4.1+0.4\sin t & 0 & 1 & 1\\ 0 & -0.2-0.3\cos t & 0 & 0\\ -1 & 0 & -1.5-0.5\cos t & 0\\ -1 & 0 & 0 & -0.17-0.1\sin t\end{pmatrix}$$

$$\times\begin{pmatrix}X_1\\ X_2\\ X_3\\ X_4\end{pmatrix} + \begin{pmatrix}X_1X_2\\ -X_1^2\\ 0\\ -dX_1X_2\end{pmatrix} \tag{18}$$

From Eq.(18), we can get $A = \begin{pmatrix}4.1 & 0 & 1 & 1\\ 0 & -0.2 & 0 & 0\\ -1 & 0 & -1.5 & 0\\ -1 & 0 & 0 & -0.17\end{pmatrix}$, $f(x) = \begin{pmatrix}X_1X_2\\ -X_1^2\\ 0\\ -dX_1X_2\end{pmatrix}$ and $\Delta A(t) = \begin{pmatrix}0.4\sin t & 0 & 0 & 0\\ 0 & -0.3\cos t & 0 & 0\\ 0 & 0 & -0.5\cos t & 0\\ -0.5\sin t & 0 & 0 & -0.1\sin t\end{pmatrix}$, it is obvious that $\Delta A^T(t)A(t) \leq 0.3I$. Choose $B_k = B = \begin{pmatrix}k & 0 & 0 & 0\\ 0 & k & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$, then $(I+B)^T(I+B) = \begin{pmatrix}(k+1)^2 & 0 & 0 & 0\\ 0 & (k+1)^2 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$, with the largest eigenvalue $\lambda = (k+1)^2$. Then estimations of the boundaries of stable regions are given by

$$0 < \delta < -\frac{\ln\xi + \ln\lambda}{\lambda_1 + \varepsilon_1\gamma^2 + \varepsilon_1^{-1} + dM}, \quad -2 < k < 0, \tag{19}$$

where $M = 4$. By computing, we obtain easily $\lambda_1 = 8.2$.

It is easy to find that the stable regions for different ξ are different, as shown in Figure 2. The entire region below the curve corresponding to $\xi = 1$ is the predicted stable region. When $\xi \to \infty$, the stable region shrinks to a line $k = -1$. For any selected in the stable region, system (6) will be stabilized at the origin. For simulation, we take the initial conditions $(x_1(0), x_2(0), x_3(0), x_4(0)) = (-0.2, 0.1, -0.5, 0.3)$ and $k = -1.5$. The simulation result is shown in Figure 3. There result shows that the stability has been achieved via impulse.

Remark 4.2. Furthermore, we can observe two facts about the stabilizations via impulse compared to the stabilizations of feedback control method ([28]): on the one hand, they converge to zero faster; on the other hand, their fluctuation is smaller. If we adopt the feedback control strategy, the other conditions are chosen as the mentioned above,

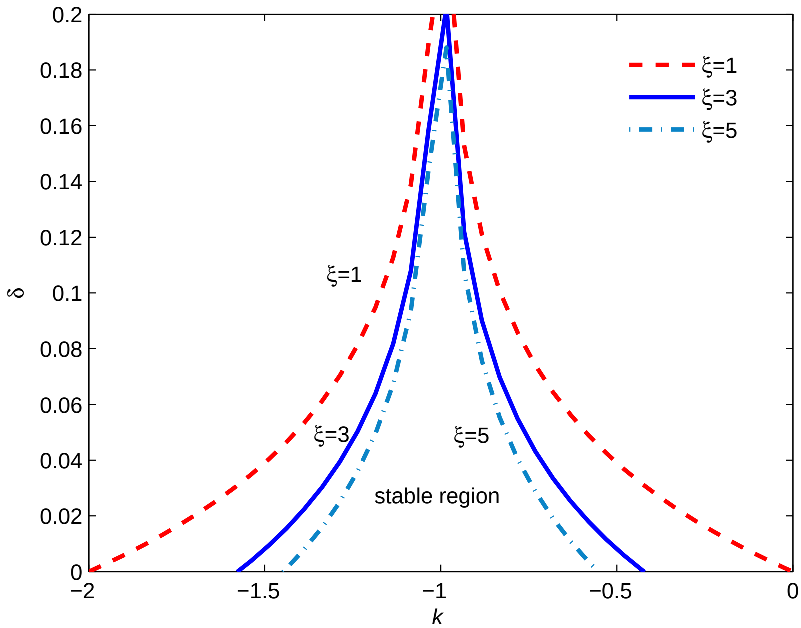

Fig. 2. Estimate of the boundaries of stable region with different ξ used in (19).

the state trajectories are shown in Figure 4. Clearly, from Figures 3 and 4, it is easy to find that the stabilization with impulse have smaller fluctuation than those of feedback control method. This proved that the impulsive control method is better than the feedback control method. To further explain the advantages of impulsive control method, it is obvious that the stable time under the impulse control is shorter than that of the feedback method. Those above results show the correctness and effectiveness of our methods via impulse.

Example 4.3. Similarly, in this example the same parameters as those in example 4.1 are used. We choose the impulse matrix B_k as

$$B_k = \begin{pmatrix} -0.5 & -0.5 & 0 & 0 \\ 0 & -0.5 & 0 & 0 \\ 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & -0.5 \end{pmatrix},$$

then $(I+B)^T(I+B) = \begin{pmatrix} 0.25 & -0.25 & 0 & 0 \\ -0.25 & 0.5 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0.25 \end{pmatrix}.$

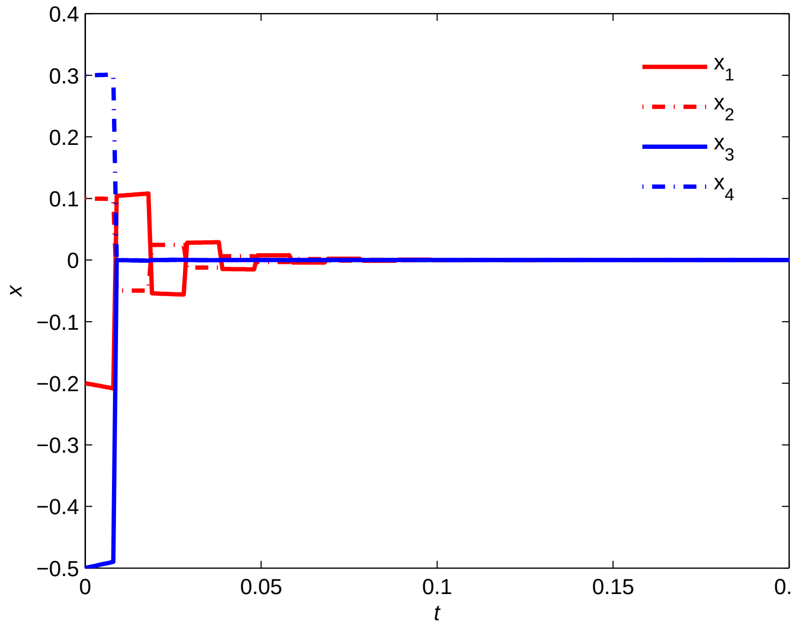

Fig. 3. The trajectories of financial hyperchaotic system (6) under impulsive control with $\delta = 0.01$ and $k = -1.5$.

The initial conditions are given by $(x_1(0), x_2(0), x_3(0), x_4(0)) = (-0.2, 0.1, -0.5, 0.3)$, $(y_1(0), y_2(0), y_3(0), y_4(0)) = (0.3, -0.2, 0.5, -0.3)$ and $\delta = 0.01$. By computing $\lambda = 0.6546$, the other constants are the same as example 4.1. The simulation result is shown in Figure 5. Figure 5 displays the synchronization error of the drive system and the response system. Since the stability boundary estimates are the same as those in example 4.1, we do not repeat them here. We can find that impulsive synchronization was achieved rapidly.

Remark 4.4. Based on similar reasons as in example 4.1, we can get the same stability boundary estimates of impulsive intervals. When the other parameters are the same as those used in example 4.1, and the initial conditions of two financial hyperchaotic systems are not the same, it is easy to see that impulsive synchronization is achieved rapidly.

Remark 4.5. We have investigated the issue on the stabilization and synchronization of financial hyperchaotic system via an impulsive method. In comparison with the schemes reported in the literature ([3, 4, 28, 32]), our method does not require complex mathematical analysis. Moreover, we can see that the stable conditions in this paper are

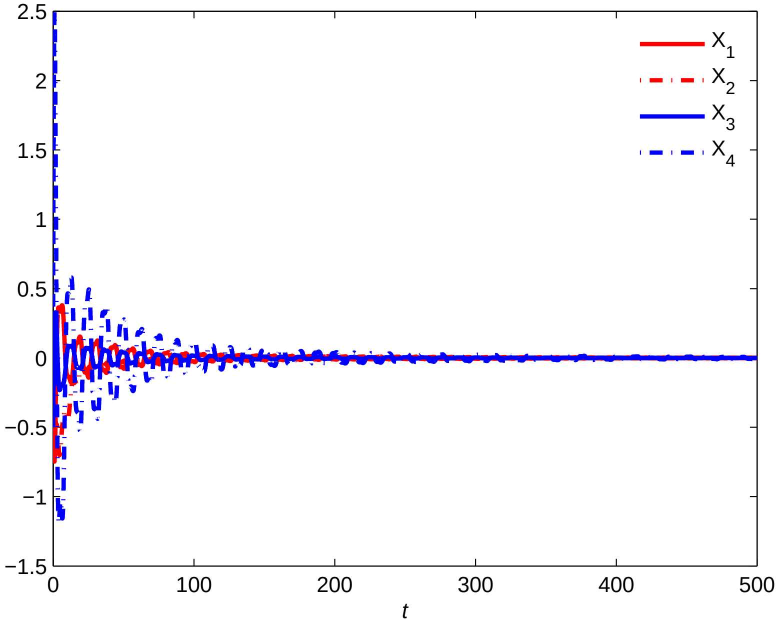

Fig. 4. The trajectories of financial hyperchaotic system (6) under feedback control.

simpler and less conservative. Simulation results in this paper show that the proposed method is effective and less conservative.

5. CONCLUSION

In this paper, we have studied the stabilization and synchronization of uncertain financial hyprechaotic systems with parameters perturbation. The control gains and impulsive intervals are both variable. The conditions for global asymptotic stability have been derived, and the upper bound of impulsive interval is also given. Some examples have been shown, to verify the theoretical results and the effectiveness of the proposed synchronization scheme. It is believed that the conditions are practical and the designed method is effective. The main advantage of our results exists in the convenience of determining the control gain from the criteria. The results obtained are helpful for stabilization development of financial systems and financial markets. However, in the real world, we will not only meet synchronization of two identical hyperchaotic financial systems, but also encounter more often the one of two different hyperchaotic financial systems. So in the future we will extend the proposed method to two different uncertain hyperchaotic financial systems.

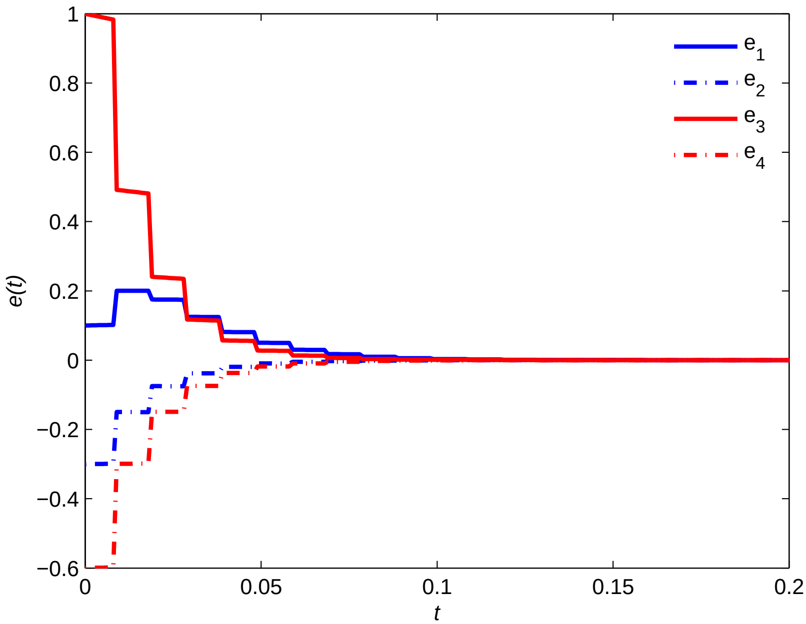

Fig. 5. Time response of the synchronization of error system.

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