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QUANTIZED DISTRIBUTED OUTPUT REGULATION OF MULTI-AGENT SYSTEMS

Xiaoli Wang and Yumin Chen

Motivated by digital communication channel, we consider the distributed output regulation problem for linear multi-agent systems with quantized state measurements. Quantizers take finitely many values and have an adjustable "zoom" parameter. Quantized distributed output regulation concerns designing distributed feedback by employing quantized technique for multi-agent systems such that all agents can track an active leader, and/or distributed disturbance rejection. With the solvability conditions satisfied, both hybrid static and dynamic feedback with quantized strategy are developed.

Keywords: multi-agent systems, distributed output regulation, active leader, quantized control

Classification: 35R35, 49J40, 60G40

1. INTRODUCTION

The study of distributed control laws for groups of agents has emerged as a new and challenging research field. It attracts many researchers from rather diverse disciplines including ecology, physics, biology, social sciences and high technology. On the one hand, researchers have been fascinated by the animal aggregation such as flocking, swarming, and schooling, which is believed to use local and simple coordination rules resulting in remarkable and complex intelligent behavior at the group level ([2]); and on the other hand, engineers on computer and electrical science and technology have been working on the distributed design for large-scale man-made systems (such as smart grids or sensor networks), whose performance emerging based on the design at the individual level ([4]). Therefore, the analysis and control of multi-agent systems, including consensus, formation, and flocking have been widely investigated.

Recently, distributed output regulation problem has been proposed with regarding some coordination problems as its special case as discussed in ([7, 11, 14, 15, 16, 17]). It is mainly about distributed feedback design for multi-agent systems to achieve asymptotically tracking and distributed disturbance rejection. Its background includes multi-agent consensus with environmental inputs and active leader following model. In this problem, the exogenous information (maybe generated by the leader or some disturbance sources)

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is not available to all the agents, which makes many results in conventional output regulation ([9]) failed to be applied to. This may explain why output regulation of linear or nonlinear multi-agent systems has become a hot topic with many publications in recent years.

On the other hand, the subject of quantized feedback stabilization problem for linear systems is a very active and fruitful research area ([1, 10, 12]). It is motivated by numerous applications where the communication between the plant and the controller is limited due to capacity or security constraints. A quantizer is thought as a device that can convert a real-valued signal into a piecewise constant one taking on a finite set of values and having an adjustable "zoom" parameter. For example, a quantizer is used to represent a camera, this corresponds to "zooming-in" or "zooming-out", i. e., varying the focal length, while the number of pixels remains fixed ([10]). In ([1]), the authors investigated the hybrid control for linear system where the quantizer is with rectilinear quantization regions, while in [18], the authors considered impulsive hybrid control with quantized?input and output feedback. In ([10]), the authors considered a more general types of quantizer with quantization regions having arbitrary shapes. More recently, multi-agent consensus under quantization has been studied ([3, 13]). In [13], the authors proposed a quantized gossip algorithm, while the authors discussed the multi-agent consensus with a high-order active leader in [a5]. In [8], the authors introduced coding/decoding strategies in quantized consensus. However, there are very few results on the quantized distributed output regulation control strategy for the general linear multi-agent systems case, to our knowledge.

This paper is concerned with distributed output regulation problem of multi-agent systems subject to quantization. As we know, there is no similar work published. Here we focus on the hybrid feedback strategy. Similar to that done in [10], the control strategy is composed of two stages. The first, "zooming-out" stage: the system is open-loop. It consists in increasing the range of the quantizer until the state of the system can be adequately measured. The second, "zooming-in" stage: it applies distributed feedback and at the same time decreases the quantization error in order to drive the state to the origin. This results in a hybrid control law, where the zoom parameter is a discrete variable whose transitions are triggered by a suitable Lyapunov function.

The paper is organized as follows. In Section 2, problem formulation and preliminary are introduced, while in Section 3, solvability for quantized distributed output regulation of multi-agent systems is obtained. Finally, the concluding remarks are given in Section 4.

2. PROBLEM FORMULATION AND PRELIMINARY

In this paper, concern the following systems:

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i + Dw \\ \dot{w} = \Gamma w, \ y_0 = Fw \qquad\qquad i = 1, \ldots, N, \\ e_i = Cx_i - y_0 \end{cases} \tag{1}$$

where the first equation is the dynamics of N agents, with $x_i \in \mathbb{R}^n, u_i \in \mathbb{R}^m$ as the states and controls of the agent i, $i = 1, \ldots, N$; the second equation is the exogenous dynamic system (or simply, exosystem) with the internal state $w \in \mathbb{R}^l$ and the measured output $y_0 \in \mathbb{R}^q$; the third equation defines the regulated output $e_i \in \mathbb{R}^q$, $q \leq n$ for agent i, $i = 1, \ldots, N$, which may be unavailable by measurement. Without loss of generality, we assume $C \in \mathbb{R}^{q \times n}$ is of full row rank (i. e., $rank(C) = q$) in the sequel.

First of all, we introduce some basic concepts and notations in graph theory (referring to [5] for details). A digraph is denoted as $\mathcal{G} = (\mathcal{O}, \mathcal{E})$, where $\mathcal{O} = \{1, 2, \cdots, N\}$ is the set of nodes and $\mathcal{E}$ is the set of edges. $(i, j) \in \mathcal{E}$ denotes an edge leaving from node i and entering into node j if node i can get information from node j. In this case node j is said to be a neighbor of node i. The special case of digraph is undirected graph if $(i, j) \in \mathcal{E}$ once $(j, i) \in \mathcal{E}$. A path in digraph $\mathcal{G}$ is an alternating sequence $i_1\,(i_1, i_2)\,i_2\,(i_2, i_3) \cdots (i_{k-1}, i_k)\,i_k$ of nodes i_j and edges $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, 2, \cdots, k-1$. If there exists a path from node i to node j, then node j is said to be reachable from node i. A node which is reachable from every other node of $\mathcal{G}$ is called a globally reachable node of $\mathcal{G}$.

Here we consider a system consisting of N agents and a leader (denoted as node 0). The corresponding digraph is denoted as $\bar{\mathcal{G}}$. Regarding the N agents as the nodes, the relationships between N agents can be conveniently described by an undirected graph $\mathcal{G}_0$ which is a subgraph of $\bar{\mathcal{G}}$. $\mathcal{N}_i$ $(i = 1, \ldots, N)$ is called the neighbor set of agent i. The weighted adjacency matrix of $\mathcal{G}_0$ is denoted as $A^0 = (a_{ij})_{N \times N} \in \mathbb{R}^{N \times N}$, where $a_{ii} = 0$ and $a_{ij} \geq 0$ ($a_{ij} = 1$ if there is an edge from agent i to agent j). Its degree matrix $D^0 = diag\{\bar{a}_1^0, \ldots, \bar{a}_N^0\} \in \mathbb{R}^{N \times N}$ is a diagonal matrix, where diagonal elements $\bar{a}_i^0 = \sum_{j=1}^N a_{ij}$ for $i = 1, \ldots, N$. Then the Laplacian of the weighted graph is defined as $L = D^0 - A^0$. Moreover, let us consider the digraph $\bar{\mathcal{G}}$ contains N agents and the leader with directed edges from some agents to the leader by the connection weights $a_{i0} > 0$ if agent i can get information from the leader, otherwise $a_{i0} = 0$ (note that $\bar{\mathcal{G}}$ is directed though $\mathcal{G}_0$ is undirected). Set an $N \times N$ diagonal matrix $A_0 = diag\{a_{10}, \ldots, a_{N0}\}$. Define a matrix $H = L + A_0$ to describe the connectivity of the whole graph $\bar{\mathcal{G}}$. Obviously, we have $H\mathbf{1} = A_0\mathbf{1}$.

The following lemma is about the matrix $H = L + A_0$ ([6]).

Lemma 2.1. H is positive definite if and only if node 0 is globally reachable in $\bar{\mathcal{G}}$.

The following assumption is needed for analysis.

Assumption 2.2. Node 0 (that is, the leader) is globally reachable in $\bar{\mathcal{G}}$.

Based on Lemma 2.1, we denote all the positive eigenvalues of H as $\bar{\lambda}_i$, $i = 1, \ldots, N$.

In this paper, we mainly discuss the distributed output regulation problem with quantized signal. By a quantizer we mean a piecewise constant function $q : R^n \to \mathcal{Q}$, where $\mathcal{Q}$ is a finite subset of R^n. Moreover, we assume that there exist positive real numbers M, Δ such that the following two conditions hold:

1. If
$$\|z\| \leq M, \tag{2}$$
then
$$\|z - q(z)\| \leq \Delta. \tag{3}$$

2. If
$$\|z\| > M, \tag{4}$$
then
$$\|q(z)\| > M - \Delta. \tag{5}$$

M, Δ are referred as the range of q and the quantization error, respectively. Assume that $q(0) = 0$ to preserve the equilibrium at the origin.

As mentioned in [10], the form of the quantized measurements is usually given as the form
$$\mu q\Big(\frac{z}{\mu}\Big),$$
where $\mu > 0$.

Large μ leads to large quantization range and quantization error, little μ leads to little quantization range and quantization error. The range of it is $M\mu$ and the quantization error is $\Delta\mu$.

In the literature about quantized feedback stabilization of linear systems, quantized feedback controllers are constructed based on quantized measurements of state. However, in this paper we concern with the scenario of an independent active leader, who does not need the quantized feedback control updates. Thus, a more sophisticated quantized feedback strategy needs to be developed to continuously update every agent's partial control input. The relative position measurement considering quantized strategy becomes:
$$\hat{z}_i = \sum_{j \in \mathcal{N}_i} a_{ij}\Big(C\mu q\Big(\frac{x_i}{\mu}\Big) - C\mu q\Big(\frac{x_j}{\mu}\Big)\Big) + a_{i0}(Cx_i - Fw), \quad i = 1, \ldots, N. \tag{6}$$

Then the distributed output regulator is taken as follows:
$$u_i = K_z \hat{z}_i + K_x \mu q\Big(\frac{x_i}{\mu}\Big), \quad i = 1, \ldots, N, \tag{7}$$
in static state feedback;
$$\begin{cases} u_i = K_z \hat{z}_i + K_x \mu q\Big(\frac{x_i}{\mu}\Big) + K_v v_i \\ \dot{v}_i = E_z \hat{z}_i + E_x \mu q\Big(\frac{x_i}{\mu}\Big) + E_v v_i \end{cases} \quad v_i \in \mathbb{R}^s, \quad i = 1, \ldots, N \tag{8}$$
in dynamic state feedback. It is noted that both $v_i(t)$ and $u_i(t)$ use the broadcasted measurements $C\mu q\Big(\frac{x_i}{\mu}\Big) - C\mu q\Big(\frac{x_j}{\mu}\Big)$ from neighboring followers and the continuous-time measurement $Cx_i - Fw$ from the leader.

Denote
$$\digamma_i(t) = \mu\Big(q\Big(\frac{x_i}{\mu}\Big) - \frac{x_i}{\mu}\Big), \ \digamma = \begin{pmatrix}\digamma_1^T & \cdots & \digamma_N^T\end{pmatrix}^T.$$
Note that
$$\mu q\Big(\frac{x_i}{\mu}\Big) - \mu q\Big(\frac{x_j}{\mu}\Big) = x_i(t) - x_j(t) + \digamma_i(t) - \digamma_j(t),$$

and then the closed-loop system can be rewritten as

$$\begin{cases} \dot{\xi} = A_c\xi + B_c w + D_c \digamma \\ \dot{w} = \Gamma w \\ e = C_c \xi - F_c w \end{cases} \tag{9}$$

with

$$\xi = x = (x_1^T \cdots x_N^T)^T,$$
$$A_c = I_N \otimes (A + BK_x) + H \otimes (BK_z C), B_c = \mathbf{1} \otimes D - (H\mathbf{1}) \otimes (BK_z F),$$
$$D_c = L \otimes (BK_z C) + I_N \otimes (BK_x),\ C_c = I_N \otimes C,\ F_c = \mathbf{1} \otimes F,\ \mathbf{1} = (1, \ldots, 1)^T$$

in static state feedback case;

$$\xi = \begin{pmatrix} x^T & v^T \end{pmatrix}, \qquad v = (v_1^T \cdots v_N^T)^T,$$

$$A_c = \begin{pmatrix} I_N \otimes (A + BK_x) + H \otimes (BK_z C) & I_N \otimes (BK_v) \\ I_N \otimes E_x + H \otimes (E_z C) & I_N \otimes E_v \end{pmatrix},$$

$$B_c = \begin{pmatrix} \mathbf{1} \otimes D - (H\mathbf{1}) \otimes (BK_z F) \\ -(H\mathbf{1}) \otimes (E_z F) \end{pmatrix},$$

$$D_c = \begin{pmatrix} L \otimes (BK_z C) + I_N \otimes (BK_x) & 0 \\ 0 & L \otimes (E_z C) + I_N \otimes E_x \end{pmatrix},$$

$$C_c = (I_N \otimes C \ \ 0),\ F_c = \mathbf{1} \otimes F,$$

in dynamic state feedback case.

Then we will give the definition of quantized distributed output regulation problem as follows.

Definition 2.3. The quantized distributed output regulation problem is achieved for system (1) under hybrid feedback control (7) or (8), respectively, if,

1) when $w = 0$, $\digamma = 0$, system (9) is asymptotically stable (that is, A_c is stable);

2) for any initial condition $(x_1(0), \ldots, x_N(0))$ and $(w_1(0), \ldots, w_l(0))$ (and even $(v_1(0), \ldots, v_N(0))$ in the dynamic feedback case)

$$\lim_{t \to +\infty} e_i(t) = 0, \quad i = 1, \ldots, N.$$

3. SOLVABILITY FOR QUANTIZED DISTRIBUTED OUTPUT REGULATION

In this section we will give our main result. The following assumption is given for the following analysis.

Assumption 3.1. (1) There is a regulation matrix of regulation equation

$$\begin{cases} X_c\Gamma = A_cX_c + B_c \\ C_cX_c = F_c, \end{cases} \tag{10}$$

with A_c, B_c defined in (9), and $C_c = I_N \otimes C,\ F_c = \mathbf{1} \otimes F$ in the static feedback case, or $C_c = (I_N \otimes C\ \ 0),\ F_c = \mathbf{1} \otimes F$ in the dynamic feedback case;

(2) A_c is stable.

Set $\bar{\xi} = \xi - X_cw$, and then we have the first equation of (9) rewritten as

$$\dot{\bar{\xi}} = A_c\bar{\xi} + D_c\digamma. \tag{11}$$

When $\digamma = 0$, (11) becomes

$$\dot{\bar{\xi}} = A_c\bar{\xi}. \tag{12}$$

Since A_c is stable, there exists positive definite matrices $P,\ Q$ such that

$$A_cP + PA_c^T = -Q.$$

Then the Lyapunov function for systems (12) is $V(\bar{\xi}) = \bar{\xi}^TP\bar{\xi}$ for system (12) with $\dot{V}|_{(12)} \le -\bar{\xi}^TQ\bar{\xi}$ negative definite.

The following lemma is very important showing that the behavior of the system (11) for a fixed μ.

Lemma 3.2. Under Assumption 3.1, fix an arbitrary $\epsilon > 0$ and assume that M is large enough compared to Δ so that we have

$$\sqrt{\lambda_{\min}(P)}M > \sqrt{\lambda_{\max}(P)}\Theta\Delta(1+\epsilon), \tag{13}$$

where

$$\Theta = \frac{2\|\hbar\|}{\lambda_{\min}(Q)},$$

with

$$\hbar = P[L \otimes (BK_zC) + I_N \otimes (BK_x)] \tag{14}$$

in static feedback case;

$$\hbar = P\begin{pmatrix} L \otimes (BK_zC) + I_N \otimes (BK_x) & 0 \\ 0 & L \otimes (E_zC) + I_N \otimes E_x \end{pmatrix}, \tag{15}$$

in dynamic feedback case.
Then the ellipsoids

$$\mathcal{R}_1 = \{\bar{\xi} : \bar{\xi}^TP\bar{\xi} \le \lambda_{\min}(P)M^2\mu^2\}, \tag{16}$$

and

$$\mathcal{R}_2 = \{\bar{\xi} : \bar{\xi}^TP\bar{\xi} \le \lambda_{\max}(P)\Theta^2\Delta^2(1+\epsilon)^2\mu^2\} \tag{17}$$

are invariant regions for the system (11). Moreover, all solutions of (11) that start in $\mathcal{R}_1$ enter the smaller $\mathcal{R}_2$ in finite time.

P r o o f. Consider the derivative of $V(\bar{\xi})$ along the system (11):

$$\begin{aligned} \dot{V}|_{(11)} &= -\bar{\xi}^T Q \bar{\xi} + 2\bar{\xi}^T \hbar F \\ &\leq -\lambda_{\min}(Q)\bar{\xi}^T \bar{\xi} + 2\bar{\xi}^T \hbar F \\ &\leq -\lambda_{\min}(Q)\|\bar{\xi}\|^2 + 2\|\bar{\xi}\|\|\hbar\|\triangle\mu. \end{aligned}$$

Then using (2), (3), we have the following formula:

$$\Theta\Delta(1+\epsilon)\mu \leq \|\bar{\xi}\| \leq M\mu \Rightarrow \dot{V}(\bar{\xi}) \leq -\|\bar{\xi}\|\lambda_{\min}(Q)\Theta\Delta\epsilon\mu. \tag{18}$$

Define

$$\mathcal{B}_1 = \{\bar{\xi} : \|\bar{\xi}\| \leq M\mu\},$$

and

$$\mathcal{B}_2 = \{\bar{\xi} : \|\bar{\xi}\| \leq \Theta\Delta(1+\epsilon)\mu\}.$$

Then using (13), we can have

$$\mathcal{B}_2 \subset \mathcal{R}_2 \subset \mathcal{R}_1 \subset \mathcal{B}_1.$$

From (18), it is easy to see that $\mathcal{R}_1, \mathcal{R}_2$ are both invariant. Moreover, from (18), we have: if $\bar{\xi}(t_0) \in \mathcal{R}_1$, then $\bar{\xi}(t_0+T) \in \mathcal{R}_2$, where

$$T = \frac{\lambda_{\min}(P)M^2 - \lambda_{\max}(P)\Theta^2\Delta^2(1+\epsilon)^2}{\lambda_{\min}(Q)\Theta^2\Delta^2(1+\epsilon)\epsilon}. \tag{19}$$

□

Theorem 3.3. Under Assumption 3.1, assume M is large enough compared to Δ such that

$$\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}M > 2\Delta \max\left\{1, \frac{\|\hbar\|}{\lambda_{\min}(Q)}\right\}, \tag{20}$$

with $\hbar$ defined in (14) in static feedback case and (15) in dynamic feedback case. Then there exists hybrid static feedback controller (7) and dynamic feedback controller (8) that can solve the quantized distributed output regulation of multi-agent systems.

P r o o f. *The zooming-out stage.* Set $u = 0$. Let $\mu(0) = 1$. Then increase μ in a piecewise constant fashion, fast enough to dominate the rate of growth of $\|e^{(I_N \otimes A)t}\|$. For example, one can fix a positive number τ and let $\mu(t) = 1$ for $t \in [0, \tau)$, $\mu(t) = \tau e^{2\|I_N \otimes A\|\tau}$ for $t \in [\tau, 2\tau)$, $\mu(t) = 2\tau e^{2\|I_N \otimes A\|2\tau}$ for $t \in [2\tau, 3\tau)$, and so on. Then there exists $t \geq 0$, such that

$$\left\|\frac{\bar{\xi}(t)}{\mu(t)}\right\| \leq \sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}M - 2\Delta.$$

Using the conditions (2), (3), we have

$$\left\| q\left(\frac{\bar{\xi}(t)}{\mu(t)}\right) \right\| \leq \sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}} M - \Delta.$$

Assume that the above inequality is satisfied at $t = t_0$. From (2), (3), (4), (5), we have

$$\left\| \frac{\bar{\xi}(t_0)}{\mu(t_0)} \right\| \leq \sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}} M,$$

thus $\bar{\xi}(t_0) \in \mathcal{R}_1$ with $\mu = \mu(t_0)$. Note that this event can be detected using only the available quantized measurements.

The zooming-in stage. Choose $\epsilon > 0$ such that (13) is satisfied ((20) makes it possible). Since $\bar{\xi}(t_0) \in \mathcal{R}_1$ with $\mu = \mu(t_0)$, then we employ the distributed feedback law (7) in static case or (8) in dynamic case. Let $\mu(t) = \mu(t_0)$, $t \in [t_0, t_0 + T)$, where T is given in (19). Then we have $\bar{\xi}(t_0 + T) \in \mathcal{R}_2$ with $\mu = \mu(t_0)$. When $t \in [t_0 + T, t_0 + 2T)$, let

$$\mu(t) = \Omega\mu(t_0),$$

where

$$\Omega = \frac{\sqrt{\lambda_{\max}(P)}\Theta\Delta(1+\epsilon)}{\sqrt{\lambda_{\min}(P)}M}.$$

From (13), $\Omega < 1$, and then $\mu(t_0 + T) < \mu(t_0)$. $\mathcal{R}_2$ with the old value $\mu(t_0)$ is the same as the $\mathcal{R}_1$ with the new value $\mu(t_0 + T)$. This means that we can continue the analysis for $t \geq t_0 + T$ as above. That is to say $\bar{\xi}(t_0 + 2T) \in \mathcal{R}_2$ with $\mu = \mu(t_0 + T)$. For $t \in [t_0 + 2T, t_0 + 3T)$, let $\mu(t) = \Omega\mu(t_0 + T) \ldots$ The stability of the equilibrium 0 in the sense of Lyapunov follows directly from the adjustment of μ. Moreover

$$\mu(t) \to 0, \quad t \to \infty \Rightarrow e(t) \to 0, \quad t \to \infty.$$

The proof is completed. □

Remark 3.4. The proof of Theorem 3.3 is mainly inspired by [10]. Our hybrid feedback policy is composed of two stages. The first, zooming-out stage consists in increasing the range of the quantizer until the state $\bar{\xi}$ of the system (11) can be adequately measured; at this stage, the feedback gains of the systems equal zero, The second, zooming-in stage involves applying feedback gains and decreasing the quantization error in such a way as to drive the state $\bar{\xi}$ to the origin. The obtained feedback policy is a hybrid quantized one.

Theorem 3.5. Under Assumption 2.2, the following are equivalent:

(i) The quantized distributed output regulation of system (1) with $D_1 = \cdots = D_N = D$ can be solved;

(ii) The conventional quantized output regulation of the N systems

$$\begin{cases} \dot{\tilde{x}}_i = A\tilde{x}_i + B\tilde{u}_i + Dw, \quad i = 1, \dots, N \\ \tilde{e}_i = C\tilde{x}_i - Fw, \ \tilde{z}_i = \bar{\lambda}_i\Big(C\mu q\Big(\frac{x_i}{\mu}\Big) - Fw\Big) \\ \dot{w} = \Gamma w \end{cases} \tag{21}$$

with $\bar{\lambda}_i, \ i = 1, \dots, N$ as the eigenvalues of H, is solvable.

Proof. Let T be a transformation such that $U = THT^{-1}$ is a diagonal matrix with the eigenvalues of H along the diagonal. Clearly, $T \otimes I_n$ transforms $H \otimes I_n$ into $U \otimes I_n$. Setting

$$\tilde{x} = (T \otimes I_n)x, \ \tilde{v} = (T \otimes I_s)v,$$

we restate the matrices A_c, B_c in terms of $\tilde{x}, \tilde{v}$ as follows:

$$\tilde{A}_c = \begin{pmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{pmatrix}, \quad \tilde{B}_c = \begin{pmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{pmatrix},$$

where

$$\begin{aligned} \tilde{A}_{11} &= I_N \otimes (A + BK_x) + U \otimes (BK_zC), & \tilde{A}_{12} &= I_N \otimes (BK_v), \\ \tilde{A}_{21} &= I_N \otimes E_x + U \otimes (E_zC), & \tilde{A}_{22} &= I_N \otimes E_v, \\ \tilde{B}_1 &= -(UT\mathbf{1}) \otimes (BK_zF), & \tilde{B}_2 &= -(UT\mathbf{1}) \otimes (E_zF). \end{aligned} \tag{22}$$

For $U \otimes I_n$, the diagonal blocks are each $\bar{\lambda}_i I_n$, where $\bar{\lambda}_i$ is the ith eigenvalue of H, so the N diagonal subsystems can be written as (21).

Denote

$$\tilde{\xi}_i = \begin{pmatrix} \tilde{x}_i^T & \tilde{v}_i^T \end{pmatrix}^T, \ i = 1, \dots, N.$$

Then the closed-loop system can be rewritten as

$$\begin{cases} \dot{\tilde{\xi}}_i = A_i\tilde{\xi}_i + \bar{B}_i + D_i F, \ i = 1, \dots, N \\ \dot{w} = \Gamma w \\ \tilde{e}_i = C\tilde{x}_i - y_0 = C\tilde{x}_i - Fw, \end{cases} \tag{23}$$

with

$$A_i = A + BK_x + \bar{\lambda}_i BK_zC, \ \bar{B}_i = -\bar{\lambda}_i BK_zF + D, \ D_i = BK_x + \bar{\lambda}_i BK_zC \tag{24}$$

in the case of static state feedback, or with

$$A_i = \begin{pmatrix} A + BK_x + \bar{\lambda}_i BK_zC & BK_v \\ E_x + \bar{\lambda}_i E_zC & E_v \end{pmatrix}, \ \bar{B}_i = \begin{pmatrix} -\bar{\lambda}_i BK_zF + D \\ -\bar{\lambda}_i E_zF \end{pmatrix}, \tag{25}$$

$$D_i = \begin{pmatrix} \bar{\lambda}_i BK_z + BK_x & 0 \\ 0 & \bar{\lambda}_i E_zC + E_x \end{pmatrix}$$

in the case of dynamic state feedback. Because the elements of the transformed system matrix $\tilde{A}_c$ are block diagonal, A_c is stable if and only if $A_i,\ i=1,\dots,N$ are stable. From above discussion the quantized distributed regulation of system (1) is equivalent to the conventional quantized output regulation of system (21) using Definition 2.3.

$\square$

Based on Theorem 3.5, in order to solve the quantized distributed regulation of system (1), we can consider the solvability of the conventional quantized output regulation of system (21). Then Assumption 3.1 can be replaced by the following simple form.

Assumption 3.6. There is a unique matrix $\bar{X}$ satisfying

$$\begin{cases} \bar{X}\Gamma = \bar{A}\bar{X} + \bar{D} \\ \bar{C}\bar{X} = F, \end{cases} \tag{26}$$

where in the case of static feedback

$$\bar{A} = A + BK_x,\ \ \bar{D} = D,\ \ \bar{C} = C \tag{27}$$

and $A_i = A + BK_x + \bar{\lambda}_i BK_z C,\ i = 1,\dots,N$ are stable; in the case of dynamic feedback

$$\bar{A} = \begin{pmatrix} A + BK_x & BK_v \\ E_x & E_v \end{pmatrix},\ \bar{D} = \begin{pmatrix} D \\ 0 \end{pmatrix},\ \bar{C} = (C \quad 0_{q\times s}) \tag{28}$$

and $A_i = \begin{pmatrix} A + BK_x + \bar{\lambda}_i BK_z C & BK_v \\ E_x + \bar{\lambda}_i E_z C & E_v \end{pmatrix}$, $i = 1,\dots,N$ are stable.

Considering systems (21), it is a conventional output regulation, and the equivalence can be obtained in a conventional way by noting that A_c is stable if and only if $A_i,\ i = 1,\dots,N$ are stable. In fact, the regulation equation of system (21) is

$$\begin{cases} X_i\Gamma = A_i X_i + \bar{B}_i \\ \bar{C}X_i = F \end{cases} \qquad \bar{C} = (C, 0_{q\times s}) \tag{29}$$

with $A_i,\ \bar{B}_i$ defined in (23), which is equivalent to (26).

The following lemma is very important characterizing the behavior of the system (21) for a fixed μ.

Lemma 3.7. Under Assumption 3.6, fix an arbitrary $\epsilon > 0$ and assume that M is large enough compared to Δ so that we have

$$\sqrt{\min_{\{i=1,\dots,N\}}(\lambda_{\min}(P_i))}M > \sqrt{\max_{\{i=1,\dots,N\}}(\lambda_{\max}(P_i))}\Theta\Delta(1+\epsilon). \tag{30}$$

Here

$$\Theta = \frac{2\max_{\{i=1,\dots,N\}}\|\hbar_i\|}{\min_{\{i=1,\dots,N\}}(\lambda_{\min}(Q_i))},$$

with

$$\hbar_i = P_i(\bar{\lambda}_i BK_z C + BK_x) \tag{31}$$

in static feedback case;

$$\hbar_i = P_i \begin{pmatrix} \bar{\lambda}_i BK_z C + BK_x & 0 \\ 0 & \bar{\lambda}_i E_z C + E_x \end{pmatrix}, \tag{32}$$

in dynamic feedback case. Then the ellipsoids $\mathcal{R}_1 = \mathcal{R}_{11} \cap \cdots \cap \mathcal{R}_{1N}$ with

$$\mathcal{R}_{1i} = \{\hat{\xi}_i : \hat{\xi}_i^T P \hat{\xi}_i \le \lambda_{\min}(P_i) M^2 \mu^2\}, \tag{33}$$

and $\mathcal{R}_2 = \mathcal{R}_{21} \cap \cdots \cap \mathcal{R}_{2N}$ with

$$\mathcal{R}_{2i} = \{\hat{\xi}_i : \hat{\xi}_i^T P_i \hat{\xi}_i \le \lambda_{\max}(P_i) \Theta^2 \Delta^2 (1+\epsilon)^2 \mu^2\} \tag{34}$$

are invariant regions for the system (21). Moreover, all solutions of (21) that start in the $\mathcal{R}_1$ enter the smaller $\mathcal{R}_2$ in finite time.

Based on Lemma 3 and Assumption 3, following the proof of Theorem 2, we can easily obtain the following theorem.

Theorem 3.8. Under Assumption 2.2 and Assumption 3.6, assume M is large enough compared to Δ such that

$$\sqrt{\frac{\min_{\{i=1,\ldots,N\}}(\lambda_{\min}(P_i))}{\max_{\{i=1,\ldots,N\}}(\lambda_{\max}(P_i))}} M > 2\Delta \max\left\{1, \frac{\|\hbar\|}{\min_{\{i=1,\ldots,N\}}(\lambda_{\min}(Q_i))}\right\}, \tag{35}$$

with $\hbar$ defined in (31) in static feedback case and (32) in dynamic feedback case. Then there exists hybrid static feedback controller (7) and dynamic feedback controller (8) that can solve the distributed output regulation of multi-agent systems.

Before the end of this section, we provide an example to illustrate our theoretical results.

Example 1. Consider the followers with dynamics

$$\begin{cases} \ddot{x}_i = u_i \in R, \quad i = 1, \ldots N \\ y_i = x_i \end{cases} \tag{36}$$

and the leader with dynamics

$$\begin{cases} \ddot{x}_0 = w_0 \in R \\ \dot{w}_0 = 0 \\ y_0 = x_0 \end{cases} \tag{37}$$

The control aim is $\lim_{t\to\infty} x_i(t) - x_0(t) = 0$. In this example,

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad C = (1 \quad 0), \; D = 0, \; F = (1 \quad 0 \quad 0).$$

For simulation, we consider a multi-agent system with four agents. Its topology is described by a graph with adjacency weights as follows: $a_{12} = a_{21} = 1$, $a_{23} = a_{32} = 1$, $a_{43} = a_{34} = 1$ and other weights are set zero. Moreover, $a_{40} = 1$ and $a_{i0} = 0$ $(i < 4)$. Then

$$H = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

whose eigenvalues are $0.1206, 1.0000, 2.3473, 3.5321$. We can take $K_z = \begin{pmatrix} -10.0092 & -10.0092 & -4.1459 \end{pmatrix}$, $K_x = -(1.0000 \quad 1.7321)$. In the simulation, four agents start from random initial conditions and evolve under the control law (7) with a 10-bit quantizer. Set $M = 5$, $\mu(t_0) = 1$ and the quantization interval Δ is $1/1023$.

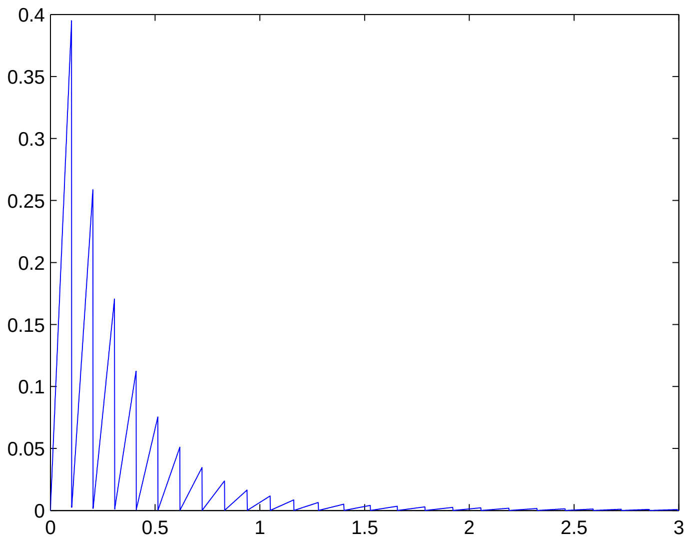

Fig. 1. The evolution of the state measurement error $\|e(t)\|$.

Figure 1 shows the evolution of the state measurement error $\|e(t)\|$.

4. CONCLUSIONS

The distributed output regulation problem for linear multi-agent systems with quantized state measurements is considered in this paper. Quantizers take finitely many values and have an adjustable "zoom" parameter. We focus on the distributed static and dynamic feedback composed of two stages "zooming-out" and "zooming-in" stage. This results in a hybrid control strategy, where the zoom parameter is a discrete variable whose transitions are triggered by the values of the constructed Lyapunov function.

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