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MODEL FOLLOWING CONTROL SYSTEM WITH TIME DELAYS

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Design of model following control system (MFCS) for nonlinear system with time delays and disturbances is discussed. In this paper, the method of MFCS will be extended to nonlinear system with time delays. We set the nonlinear part $f(v(t))$ of the controlled object as $||f(v(t))|| \leq \alpha + \beta||v(t)||^{\gamma}$, and show the bounded of internal states by separating the nonlinear part into $\gamma \geq 0$. Some preliminary numerical simulations are provided to demonstrate the effectiveness of the proposed method.

Keywords: time delays, model following control system (MFCS), internal stable, nonlinear system

Classification: 93C10

1. INTRODUCTION

For nonlinear systems with time delays, especially in recent years, control using model following control system (MFCS) has attracted wide attention. Because of its inherent capability for modelling and controlling highly uncertain, nonlinear and complex systems. MFCS design is the method developed based on the idea proposed by Wu, Wang and Okubo [20, 21, 24]. As is well known, time delays are inherent features of many physical processes, e. g., Chaos system [4, 16], engine system [6] and electrical heater [5] and, biological system [8], environmental system [9], network system [22, 25], etc. Many of these processes are also significantly nonlinear, which motivates the research into the control of nonlinear systems with time delays [2, 6]. Time delay systems are also called systems with aftereffect or dead time, hereditary systems, equations with deviating argument or differential-difference equations. They belong to the class of functional differential equations which are infinite dimensional and dependent on the past history of the dynamics [25]. Design of MFCS for uncertain linear systems have been proposed in [9, 11, 12, 13, 23]. In [24] the model following control problem has been formulated as an eigenstructure assignment problem, tunable CGT design scheme substantially enhances the capability of the controller to optimise model-matching errors. The paper proposes a new algorithm based on model following control to recover the uncompensated slave disturbance on time delayed motion control systems having contact with

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environment [14]. A robust fuzzy MFCS is proposed for the control of robot manipulators. The application field to n-link robot manipulators with torque disturbance and measurement noise is addressed. The control objective is obtained by tailoring a nominal adaptation process of parameters to implement appropriate function approximation and facilitating a self-tuning mechanism on the consequent membership functions to overcome the equivalent uncertainty [18]. These methods are developed to make the ultimate bounded of the tracking error arbitrarily small or guarantee that tracking error decreases asymptotically to zero.

In this paper, we set the nonlinear part $f(v(t))$ of the controlled object as $||f(v(t))|| \leq \alpha+\beta||v(t)||^{\gamma}$, and show the bounded of inner states by separating the nonlinear part into $\gamma \geq 0$. The design of the control system is performed using an easy algebraic algorithm of matrices whose elements are polynomials of the operator. The bounded property of internal states for the control system is given and the utility of this control design is guaranteed. It is confirmed on the basis of example that the output of the control system asymptotically follows the reference model in the case of the existence of the disturbances.

This paper is organized as follows. In the next section, the controlled object and the reference model are described. Design of MFCS for nonlinear system with time delays is then described in Section 3. In Section 4, we prove the bounded property of the internal states. Simulations are reported (An illustrative example) in Section 5. Finally, the conclusions are summarized in Section 6.

2. EXPRESSIONS OF THE PROBLEMS

The controlled object is described below, which is a nonlinear system with time delays:

$$\dot{x}(t) = \sum_{i=0}^{k} A_i x(t-h_i) + \sum_{i=0}^{k} B_i u(t-h_i) + B_f f(v(t)) + d(t), \tag{1}$$

$$y(t) = \sum_{i=0}^{k} C_i x(t-h_i) + d_0(t), \tag{2}$$

$$v(t) = \sum_{i=0}^{k} C_{f_i} x(t-h_i), \tag{3}$$

where $A_i \in R^{n\times n}$, $B_i \in R^{n\times l}$, $B_f \in R^{n\times l}$, $C_i \in R^{l\times n}$, $C_{f_i} \in R^{l\times n}$ are the constant matrices of appropriate dimensions; t is the time $x(t) \in R^n$, $u(t) \in R^l$ and $y(t) \in R^l$ are the system state vector, the control input vector and the available states output vector of the system , respectively; $d(t) \in R^l$ and $d_0(t) \in R^l$ are the bounded linear disturbances; $v(t) \in R^{l_f}$ and $f(v(t)) \in R^{l_f}$ are the grants output and the nonlinear part, respectively; $h_i (0 = h_0 < h_1 < \cdots < h_k)$ are the time delays.

The reference model is given below, which is assumed controllable and observable [21]:

$$\dot{x}_m(t) = A_m x_m(t) + B_m r_m(t), \tag{4}$$

$$y_m(t) = C_m x_m(t), \tag{5}$$

where, $x_m(t) \in R^{n_m}$, $r_m(t) \in R^{l_m}$ and $y_m(t) \in R^l$ are the reference model state vector, the reference model input vector and the reference model output vector of the system, respectively and $A_m \in R^{n_m \times n_m}$, $B_m \in R^{n_m \times I}$, $C_m \in R^{I \times n_m}$ are the constant matrices. The output error is given as [21]

$$e(t) = y(t) - y_m(t). \tag{6}$$

The aim of the design of the control system is to obtain a control law which makes the output error is zero and keeps the internal states is bounded.

3. DESIGN OF MODEL FOLLOWING CONTROL SYSTEM FOR NONLINEAR SYSTEM WITH TIME DELAYS

Let $p = d/dt$, $\sigma = [\sigma_0, \sigma_1, \sigma_2, \cdots, \sigma_k]^T$, $\sigma_i = e^{-ph_i}, (i = 0, 1, \cdots, k)$ and $\sigma_i x(t) = x(t - h_i), (i = 0, 1, \cdots, k)$. Using σ, (1) $\sim$ (3) are rewritten as [27]

$$\dot{x}(t) = A(\sigma)x(t) + B(\sigma)u(t) + B_f f(v(t)) + d(t), \tag{7}$$
$$y(t) = C(\sigma)x(t) + d_0(t), \tag{8}$$
$$v(t) = C_f(\sigma)x(t), \tag{9}$$

where $A(\sigma) = \sum_{i=0}^{k} A_i\sigma_i$, $B(\sigma) = \sum_{i=0}^{k} B_i\sigma_i$, $C(\sigma) = \sum_{i=0}^{k} C_i\sigma_i$, and $C_f(\sigma) = \sum_{i=0}^{k} C_{f_i}\sigma_i$. From (7) $\sim$ (9), we have

$$\begin{aligned} y(t) = C(\sigma)[pI - A(\sigma)]^{-1}B(\sigma)u(t) + C(\sigma)[pI - A(\sigma)]^{-1} \\ \cdot B_f f(v(t)) + C(\sigma)[pI - A(\sigma)]^{-1}d(t) + d_0(t), \end{aligned} \tag{10}$$
$$y_m(t) = C_m[pI - A_m]^{-1}B_m r_m(t), \tag{11}$$
$$\begin{aligned} v(t) = C_f(\sigma)[pI - A(\sigma)]^{-1}B(\sigma)u(t) + C_f(\sigma)[pI - A(\sigma)]^{-1} \\ \cdot B_f f(v(t)) + C_f(\sigma)[pI - A(\sigma)]^{-1}d(t), \end{aligned} \tag{12}$$

where

$$C(\sigma)[pI - A(\sigma)]^{-1}B(\sigma) = N(\sigma, p)/D(\sigma, p),\ C(\sigma)[pI - A(\sigma)]^{-1}B_f = N_f(\sigma, p)/D(\sigma, p),$$
$$C_m[pI - A_m]^{-1}B(\sigma) = N_m(p)/D_m(p),\ D(\sigma, p) = |pI - A(\sigma)|,\ D_m(p) = |pI - A_m|.$$

It follows that

$$D(\sigma, p)y(t) = N(\sigma, p)u(t) + N_f(\sigma, p)f(v(t)) + w(t), \tag{13}$$
$$D_m(p)y_m(t) = N_m(p)r_m(t). \tag{14}$$

The disturbance $w(t)$ is thus obtained [21]:

$$w(t) = C(\sigma)adj[pI - A(\sigma)]d(t) + D(\sigma, p)d_0(t). \tag{15}$$

Then, it follows that

$$N(\sigma, p) = C(\sigma)adj[pI - A(\sigma)]B(\sigma), \tag{16}$$
$$N_f(\sigma, p) = C(\sigma)adj[pI - A(\sigma)]B_f, \tag{17}$$
$$N_m(p) = C_m adj[pI - A_m]B_m. \tag{18}$$

Furthermore, we have

$$N(\sigma,p) = diag(p^{\eta_i})N_r(\sigma) + \tilde{N}(\sigma,p), \tag{19}$$
$$N_r(\sigma) = \bar{N}_r(\sigma) + \hat{N}_r, \tag{20}$$
$$N_f(\sigma,p) = diag(p^{\eta_{f_i}})N_{f_r}(\sigma) + \tilde{N}_f(\sigma,p), \tag{21}$$
$$N_m(p) = diag(p^{\eta_{m_i}})N_{m_r} + \tilde{N}_m(p), \tag{22}$$

where $\partial_{r_i}\tilde{N}(\sigma,p) < \eta_i$, $\partial_{r_i}\tilde{N}_f(\sigma,p) < \eta_{f_i}$, $\partial_{r_i}\tilde{N}_m(p) < \eta_{m_i}$, $\partial_{r_i}(\cdot)$ is the minimum degree of $(\cdot)$. $\bar{N}_r(\sigma)$ is a fixed matrix of $l \times l$, and $|\hat{N}_r| \neq 0$. Without loss of generality, we assume that [10],

$$D_d(p)d(t) = 0, D_d(p)d_0(t) = 0. \tag{23}$$

Here, $D_d(p)$ is a scalar characteristic polynomial monic of the disturbance. From (15), we have [20]

$$D_d(p)w(t) = 0. \tag{24}$$

Choose a stable polynomial $T(p)$ which satisfies the following conditions: (I) The degree of $T(p)$ is $\rho \geq n_d + 2n - n_m - 1 - \eta_i$; (II) The coefficient of the maximum degree term of $T(p)$ is the same as the one of $D(p)$. Consider the following equation [21]:

$$T(p)D_m(p) = D_d(p)D(\sigma,p)R(\sigma,p) + S(\sigma,p), \tag{25}$$

where $\partial T(p) = \rho$, $\partial D_m(p) = n_m$, $\partial D_d(p) = n_d$, $\partial D(\sigma,p) = n$, $\partial R(\sigma,p) = \rho + n_m - n_d - n$, $\partial S(\sigma,p) \leq n_d + n - 1$. $T(p)$, $D_m(p)$, $D_d(p)$, $D(\sigma,p)$, $R(\sigma,p)$ and $S(\sigma,p)$ be monic polynomials. Then the following form is obtained:

$$\begin{aligned} T(p)D_m(p)e(t) &= D_d(p)D(\sigma,p)R(\sigma,p)y(t) + S(\sigma,p)y(t) - T(p)N_m(p)r_m(t) \\ &= \Big(D_d(p)R(\sigma,p)N(\sigma,p) - Q(p)N_r(\sigma)\Big)u(t) + D_d(p)R(\sigma,p)N_f(\sigma,p) \\ &\quad \cdot f(v(t)) + Q(p)N_r(\sigma)u(t) + S(\sigma,p)y(t) - T(p)N_m(p)r_m(t). \end{aligned} \tag{26}$$

Here, $Q(p) = diag(p^{\rho+n_m-n+\eta_i}) + \tilde{Q}(p)$ is a polynomial and stable matrix, and $\partial_{r_i}\tilde{Q}(p) < \rho + n_m - n + \eta_i$, The next control law $u(t)$ can be obtained by making the right-hand side of (26) equals to zero. Thus,

$$\begin{aligned} u(t) = &-\hat{N}_r^{-1}\bar{N}_r(\sigma)u(t) - \hat{N}_r^{-1}Q^{-1}(p)(D_d(p)D(\sigma,p)N(\sigma,p) \\ &-Q(p)N_r(\sigma))u(t) - \hat{N}_r^{-1}Q^{-1}(p)D_d(p)D(\sigma,p)N_f(\sigma,p)f(v(t)) \\ &-\hat{N}_r^{-1}Q^{-1}(p)S(\sigma,p)y(t) + u_m(t), \end{aligned} \tag{27}$$

$$u_m(t) = \hat{N}_r^{-1}Q^{-1}(p)T(p)N_m(p)r_m(t), \tag{28}$$

where $u_m(t)$ is the external signal. To avoid using derivatives of signals in control input $u(t)$, the degree of the polynomial must be satisfied: $n_m - \eta_{m_i} \geq n - \eta_i$ and $\eta_i \geq \eta_{f_i}$. Therefore, $u(t)$ of (27) is obtained from $e(t) \to 0 (t \to \infty)$. The MFCS can be realized if the system internal states are bounded.

4. BOUNDED PROPERTY OF INTERNAL STATES

The state space expression of $u(t)$ is given by:

$$\begin{aligned} u(t) = &-E_0(\sigma)u(t) - H_1(\sigma)\xi_1(t) - E_2(\sigma)y(t) - H_2(\sigma)\xi_2(t) \\ &-E_3(\sigma)f(v(t)) - H_3(\sigma)\xi_3(t) + u_m(t), \end{aligned} \tag{29}$$

$$u_m(t) = E_4 r_m(t) + H_4\xi_4(t). \tag{30}$$

In addition, the following conditions must be satisfied [21]:

$$\dot{\xi}_1(t) = F_1\xi_1(t) + G_1u(t), \tag{31}$$
$$\dot{\xi}_2(t) = F_2\xi_2(t) + G_2y(t), \tag{32}$$
$$\dot{\xi}_3(t) = F_3\xi_3(t) + G_3f(v(t)), \tag{33}$$
$$\dot{\xi}_4(t) = F_4\xi_4(t) + G_4r_m(t). \tag{34}$$

The connections between the polynomial matrices and the system matrices are given by:

$$E_0(\sigma) = \hat{N}_r^{-1}\bar{N}_r(\sigma), \tag{35}$$
$$H_1(\sigma)[pI - F_1]^{-1}G_1 = \hat{N}_r^{-1}Q^{-1}(p)(D_d(p)R(\sigma,p)N(\sigma,p) - Q(p)N_r(\sigma)), \tag{36}$$
$$E_2(\sigma) + H_2(\sigma)[pI - F_2]^{-1}G_2 = \hat{N}_r^{-1}Q^{-1}(p)S(\sigma,p), \tag{37}$$
$$E_3(\sigma) + H_3(\sigma)[pI - F_3]^{-1}G_3 = \hat{N}_r^{-1}Q^{-1}(p)D_d(p)R(\sigma,p)N_f(\sigma,p), \tag{38}$$
$$E_4 + H_4[pI - F_4]^{-1}G_4 = \hat{N}_r^{-1}Q^{-1}(p)T(p)N_m(p), \tag{39}$$

where $|pI - F_i| = |Q(p)|(i = 1,2,3)$. Let

$$z^T(t) = \begin{bmatrix} x^T(t) & \xi_1^T(t) & \xi_2^T(t) & \xi_3^T(t) & u^T(t) \end{bmatrix}. \tag{40}$$

Removing the $u(t)$ from (7) $\sim$ (9), (29) $\sim$ (33), we have

$$E\dot{z}(t) = A_s(\sigma)z(t) + B_s(\sigma)f(v(t)) + d_s(t), \tag{41}$$
$$y(t) = C_{s0}(\sigma)z(t) + d_{s0}(t), \tag{42}$$
$$v(t) = C_s(\sigma)z(t), \tag{43}$$

where

$$C_{s0}(\sigma) = [C(\sigma),0,0,0,0],\ C_s(\sigma) = [C_f(\sigma),0,0,0,0],\ B_s(\sigma) = [B_f,0,0,G_3,-E_3(\sigma)],$$
$$d_{s0}(\sigma) = d_0(t),0,0,0,0]^T,\ d_s(t) = [d(t),0,G_2d_0(t),0,u_m(t) - E_2(\sigma)d_0(t)]^T$$

and

$$E = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{44}$$

$$A_s(\sigma) = \begin{bmatrix} A(\sigma) & 0 & 0 & 0 & B(\sigma) \\ 0 & F_1 & 0 & 0 & G_1 \\ G_2C(\sigma) & 0 & F_2 & 0 & 0 \\ 0 & 0 & 0 & F_3 & 0 \\ -E_2C(\sigma) & -H_1(\sigma) & -H_2(\sigma) & -H_3(\sigma) & -I - E_0(\sigma) \end{bmatrix}. \tag{45}$$

The bounded property of internal states is turned out to be that $z(t)$ is bounded. The nonlinear function $f(v(t))$ is available and satisfies the following constraint:

$$||f(v(t))|| \le \alpha + \beta||v(t)||^{\gamma}, \tag{46}$$

where $\alpha \ge 0$, $\beta \ge 0$, $\gamma \ge 0$, $||\cdot||$ is the Euclidean norm. The system (41) $\sim$ (43) internal states are bounded if we can prove $z(t)$ is bounded. The characteristic polynomial of $A_s(\sigma)$ is calculated next.

From (44) and (45), $|pE - A_s(\sigma)|$ can be shown as follows (see Appendix A)

$$|pE - A_s(\sigma)| = |\hat{N}_r|^{-1}T(p)^l D_m(p)^l |Q(p)|^2 V_s(\sigma, p). \tag{47}$$

Here $V_s(\sigma, p)$ is the zeros polynomial of $C(\sigma)[pI - A(\sigma)]^{-1}B(\sigma)$ (left coprime decomposition),

$$C(\sigma)[pI - A(\sigma)]^{-1}B(\sigma) = W^{-1}(\sigma, p)U(\sigma, p) = N(\sigma, p)/D(\sigma, p),$$
$$|V(\sigma, p)| = V_s(\sigma, p) = |W(\sigma, p)||N(\sigma, p)/D(\sigma, p)| = D(\sigma, p)|N(\sigma, p)|/D^l(\sigma, p)$$

and $deg|N(\sigma, p)| = \sum_{i=1}^{l} \eta_i$. It follows that

$$V_s(\sigma, p) = |N(\sigma, p)|D^{1-l}(\sigma, p). \tag{48}$$

One can conclude that $A_s(\sigma)$ is a stable system matrix due to the fact that $T(p)$, $D_m(p)$, $|Q(p)|$ and $V_s(\sigma, p)$ are stable polynomials. From (41) and (43), we have

$$\begin{aligned} v(t) = {} & C_s(\sigma)[pE - A_s(\sigma)]^{-1}B_s(\sigma)f(v(t)) \\ & + C_s(\sigma)[pE - A_s(\sigma)]^{-1}d_s(t) = H(p)f(v(t)) + d_v(t). \end{aligned} \tag{49}$$

Here $H(p)$ is the transfer function from $f(v(t))$ to $v(t)$, and $d_v(t) = C_s(\sigma)[pE - A_s(\sigma)]^{-1}d_s(t)$.

Lemma 1. $d_v(t) = C_s(\sigma)[pE - A_s(\sigma)]^{-1}d_s(t)$ is bounded.

Proof. See Appendix B. □

It follows from (44) $\sim$ (45), that

$$\begin{aligned} H(p) &= C_s(\sigma)[pE - A_s(\sigma)]^{-1}B_s(\sigma) \\ &= C_f(\sigma)\{I + [pI - A(\sigma)]^{-1}B(\sigma)[I + E_0(\sigma) + H_1(\sigma)[pI - F_1]^{-1} \\ &\quad \cdot G_1]^{-1}[E_2(\sigma) + H_2(\sigma)[pI - F_2]^{-1}G_2]C(\sigma)\}^{-1}\{[pI - A(\sigma)]^{-1}B_f \\ &\quad -[pI - A(\sigma)]^{-1}B(\sigma)[I + E_0(\sigma) + H_1(\sigma)[pI - F_1]^{-1}G_1]^{-1} \\ &\quad \cdot[E_3(\sigma) + H_3(\sigma)[pI - F_3]^{-1}G_3]\}. \end{aligned} \tag{50}$$

Substituting (35) $\sim$ (38) into (50), we have

$$\begin{aligned} H(p) &= C_f(\sigma)\{I+[pI-A(\sigma)]^{-1}B(\sigma)[I+\hat{N}_r^{-1}\bar{N}_r(\sigma)-\hat{N}_r^{-1}N_r(\sigma) \\ &\quad +\hat{N}_r^{-1}Q^{-1}(p)D_d(p)R(\sigma,p)N(\sigma,p)]^{-1}\hat{N}_r^{-1}Q^{-1}(p)S(\sigma,p)C(\sigma)\}^{-1} \\ &\quad \cdot\{[pI-A(\sigma)]^{-1}B_f-[pI-A(\sigma)]^{-1}B(\sigma)[\hat{N}_r^{-1}Q^{-1}(p)D_d(p) \\ &\quad \cdot R(\sigma,p)N(\sigma,p)]^{-1}\hat{N}_r^{-1}Q^{-1}(p)D_d(p)R(\sigma,p)N_f(\sigma,p)\}. \end{aligned} \tag{51}$$

Then, from $N(\sigma,p)=D(\sigma,p)C(\sigma)[pI-A(\sigma)]^{-1}B(\sigma)$ and $N_f(\sigma,p)=D(\sigma,p)C(\sigma)[pI-A(\sigma)]^{-1}B_f(\sigma)$, we can get

$$H(p)=C_f(\sigma)[I+X(\sigma,p)\rho(\sigma,p)]^{-1}[I-X(\sigma,p)][pI-A(\sigma)]^{-1}B_f, \tag{52}$$

where $\rho(\sigma,p)$ is a scalar polynomial, and

$$\rho(\sigma,p) = S(\sigma,p)/D(\sigma,p)R(\sigma,p)D_d(p), \tag{53}$$

$$X(\sigma,p) = [pI-A(\sigma)]^{-1}B(\sigma)\{C(\sigma)[pI-A(\sigma)]^{-1}B(\sigma)\}^{-1}C(\sigma). \tag{54}$$

Therefore

$$X^2(\sigma,p)=X(\sigma,p), \tag{55}$$

$$[I+X(\sigma,p)\rho(\sigma,p)]^{-1}[I-X(\sigma,p)]=I-X(\sigma,p). \tag{56}$$

From (52) $\sim$ (56), we have

$$\begin{aligned} H(p) &= C_f(\sigma)[I+X(\sigma,p)[pI-A(\sigma)]^{-1}B_f \\ &= C_f(\sigma)[pI-A(\sigma)]^{-1}B_f-C_f(\sigma)[pI-A(\sigma)]^{-1}B(\sigma) \\ &\quad \cdot\{C(\sigma)[pI-A(\sigma)]^{-1}B(\sigma)\}^{-1}C(\sigma)[pI-A(\sigma)]^{-1}B_f \\ &= \begin{bmatrix} C_f(\sigma) & 0 \end{bmatrix}\begin{bmatrix} pI-A(\sigma) & -B(\sigma) \\ -C(\sigma) & 0 \end{bmatrix}\begin{bmatrix} B_f \\ 0 \end{bmatrix}. \end{aligned} \tag{57}$$

Since $(C_v(\sigma),A_v(\sigma),B_v)$ is the minimal realization of $H(p)$, where $A_v(\sigma)$ is a stable system matrix, the state-space realization of the system (41) $\sim$ (43) is given by

$$\dot{z}_v(t) = A_v(\sigma)z_v(t)+B_vf(v(t)), \tag{58}$$

$$v(t) = C_v(\sigma)z_v(t)+d_v(t). \tag{59}$$

In addition the transfer function of the system (58) and (59) can be calculated from

$$H(p)=\frac{N_v(\sigma,p)}{D_v(\sigma,p)}=C_v(\sigma)[pE-A_v(\sigma)]^{-1}B_v(\sigma). \tag{60}$$

The following lemma gives an important inequality. To save space, we omit its proof here.

Lemma 2. If $0 \leq \gamma < 1$, for any vectors $x \in R^n$ and $y \in R^n$, then

$$||x+y||^\gamma \leq (||x||+||y||)^\gamma \leq ||x||^\gamma + ||y||^\gamma. \tag{61}$$

Let

$$v(t) - d_v(t) = \bar{v}(t), z_v(t) = \Phi(t)\phi(t), \dot{\Phi}(t) = A_v(\sigma)\Phi(t). \tag{62}$$

It follows from (58) that

$$\Phi(t)\dot{\phi}(t) = B_v f(\bar{v}(t) + d_v(t)), \tag{63}$$

where $\Phi^{-1}(t) = \Phi(-t)$, namely $\Phi(t)\Phi(-t) = I$, $||\Phi(t)|| \leq C_1 e^{-\alpha_1 t}$. One can easily verify that

$$\phi(t) = \int_0^t \Phi(-\tau)B_v f(\bar{v}(\tau) + d_v(\tau))\,\mathrm{d}\tau + C. \tag{64}$$

From (62) and (64), we can get that

$$z_v(t) = \Phi(t)\int_0^t \Phi(-\tau)B_v f(\bar{v}(\tau) + d_v(t))\,\mathrm{d}\tau + \Phi(t)z_v(0). \tag{65}$$

Furthermore, we have by (59), (62) and (65),

$$\bar{v}(t) = C_v(\sigma)\Phi(t) + C_v(\sigma)\Phi(t)\int_0^t \Phi(-\tau)B_v f(\bar{v}(\tau) + d_v(\tau))\,\mathrm{d}\tau. \tag{66}$$

Let $C_v(\sigma) = \sum_{j=0}^k C_{vj}\sigma_j$. It follows from (66) that

$$\begin{aligned}\bar{v}(t) = &\sum_{j=0}^k C_{vj}\{\Phi(t-h_j)z_v(0)\}\\ &+\sum_{j=0}^k C_{vj}\int_0^{t-h_j} \Phi(t-h_j-\tau)B_v f(\bar{v}(\tau)+d_v(\tau))\,\mathrm{d}\tau.\end{aligned} \tag{67}$$

By using Lemma 2 and (46), we derive

$$||f(\bar{v}(t)+d_v(t))|| \leq \alpha + \beta||\bar{v}(t)+d_v(t)||^\gamma \leq \alpha + \beta||\bar{v}(t)||^\gamma + \beta||d_v(t)||^\gamma. \tag{68}$$

It follows that

$$\begin{aligned}||\bar{v}(t)|| \leq &\sum_{j=0}^k ||C_{vj}||||\Phi(t-h_j)||||z_v(0)||\\ &+\sum_{j=0}^k ||C_{vj}||\int_0^{t-h_j} ||\Phi(t-h_j-\tau)||||B_v||||f(\bar{v}(\tau)+d_v(\tau))||\,\mathrm{d}\tau.\end{aligned} \tag{69}$$

Let $||\Phi(t)|| \le \rho_1 e^{-\lambda_{\min} t}$. Then $||\Phi(t-h_j-\tau)|| \le \rho_1 e^{-\lambda_{\min}(t-h_j-\tau)}$. Hence, we have

$$||\bar{v}(t)|| \le \sum_{j=0}^{k} ||C_{vj}||\rho_1 e^{\lambda_{\min} h_j} ||z_v(0)|| e^{-\lambda_{\min} t} + \sum_{j=0}^{k} ||C_{vj}||\rho_1 e^{\lambda_{\min} h_j} \cdot \int_0^t e^{-\lambda_{\min}(t-\tau)} ||B_v||(\alpha + \beta||\bar{v}(\tau)||^{\gamma} + \beta||d_v(\tau)||^{\gamma})\,\mathrm{d}\tau. \tag{70}$$

Let $\rho_2 = \sum_{j=0}^{k} ||C_{vj}||\rho_1 e^{\lambda_{\min} h_j}$, $K_1 = \rho_2||z_v(0)||$,$\alpha_2 = \rho_2||B_v||\alpha$, and $\beta_2 = \rho_2||B_v||\beta$. We have

$$||\bar{v}(t)||e^{\lambda_{\min} t} \le K_1 + \alpha_2 \int_0^t e^{\lambda_{\min}\tau}\,\mathrm{d}\tau + \beta_2 \int_0^t e^{\lambda_{\min}\tau} ||\bar{v}(\tau)||^{\gamma}\,\mathrm{d}\tau + \beta_2 \int_0^t e^{\lambda_{\min}\tau} ||d_v(\tau)||^{\gamma}\,\mathrm{d}\tau. \tag{71}$$

Since $\alpha_2 + \beta_2||d_v(\tau)|| \le K_2$, we can easily verify that

$$||\bar{v}(t)||e^{\lambda_{\min} t} \le K_1 + K_2 \int_0^t e^{\lambda_{\min}\tau}\,\mathrm{d}\tau + \beta_2 \int_0^t e^{\lambda_{\min}\tau} ||\bar{v}(\tau)||^{\gamma}\,\mathrm{d}\tau. \tag{72}$$

Throughout this paper, we will use the following Lemma 3.

Lemma 3. For $K_2 \ge 0, \beta_2 \ge 0, \gamma > 0$, the appropriate positive $K_3 > 0, \beta_3 > 0$, and any $x \ge 0$, we have the following inequality

$$K_2 + \beta_2 x^r \le (K_3 + \beta_3 x)^r. \tag{73}$$

Proof. See Appendix C. □

It follows from Lemma 3 that

$$||\bar{v}(t)||)e^{\lambda_{\min} t} \le K_1 + \int_0^t (K_2 + \beta_2||\bar{v}(\tau)||^{\gamma}) e^{\lambda_{\min}\tau}\,\mathrm{d}\tau \le K_1 + \int_0^t (K_3 + \beta_3||\bar{v}(t)||)^{\gamma} e^{\lambda_{\min}\tau}\,\mathrm{d}\tau = U(t). \tag{74}$$

Hence, we have $\dot{U}(t) = (K_3 + \beta_3||\bar{v}(t)||)^{\gamma} e^{\lambda_{\min} t} \le (K_3 + \beta_3 e^{-\lambda_{\min} t} U(t))^{\gamma} e^{\lambda_{\min} t}$ and $||\bar{v}(t)|| \le U(t) e^{-\lambda_{\min} t}$. Let us define

$$V(t) = K_3 + \beta_3 e^{-\lambda_{\min} t} U(t). \tag{75}$$

We can conclude that $e^{\lambda_{\min} t} V(t) = K_3 e^{\lambda_{\min} t} + \beta_3 U(t)$. Applying the above equation, we obtain

$$\lambda_{\min} e^{\lambda_{\min} t} V(t) + e^{\lambda_{\min} t}\dot{V}(t) = K_3 \lambda_{\min} e^{\lambda_{\min} t} + \beta_3 \dot{U}(t) \le K_3 \lambda_{\min} e^{\lambda_{\min} t} + \beta_3 V^{\gamma}(t) e^{\lambda_{\min} t}.$$

Namely, $\lambda_{\min}V(t) + \dot{V}(t) \leq K_3\lambda_{\min} + \beta_3 V^{\gamma}(t)$. Then, we have

$$\dot{V}(t) \leq -\lambda_{\min}V(t) + \beta_3 V^{\gamma}(t) + K_3\lambda_{\min} \leq -k_1 V(t) + k_2, \tag{76}$$

where k_1 and k_2 is positive.

Let $V(t) = e^{-k_1 t}\phi(t)$. The time derivatives of $V(t)$ along the trajectories is $\dot{V}(t) = -k_1 e^{-k_1 t}\phi(t) + e^{-k_1 t}\dot{\phi}(t) \leq -k_1 e^{-k_1 t}\phi(t) + k_2$, where $e^{-k_1 t}\dot{\phi}(t) \leq k_2$, namely,

$$\dot{\phi}(t) \leq k_2 e^{-k_1 t}. \tag{77}$$

Integrating both sides of (77) on the interval [0, t), we have $\int_0^t \dot{\phi}(t)\,\mathrm{d}t \leq \int_0^t k_2 e^{-k_1 t}\,\mathrm{d}t$, and consequently,

$$\phi(t) \leq \frac{k_2}{k_1}(e^{-k_1 t} - 1) + \phi(0). \tag{78}$$

Substituting (78) into $V(t) = e^{-k_1 t}\phi(t)$ yields

$$V(t) \leq e^{-k_1 t}\frac{k_2}{k_1}(e^{k_1 t} - 1) + e^{-k_1 t}\phi(0) \leq \frac{k_2}{k_1}(1 - e^{-k_1 t}) + \phi(0). \tag{79}$$

Let $t = 0$. Then $V(0) \leq \phi(0)$. Hence, we have

$$\begin{aligned} V(t) &\leq V(0)e^{-k_1 t} + \frac{k_2}{k_1}(1 - e^{-k_1 t}) \\ &\leq \{K_3 + \beta_3 ||U(0)||\}e^{-k_1 t} + \frac{k_2}{k_1}(1 - e^{-k_1 t}). \end{aligned} \tag{80}$$

Substituting (75) into (80), we have

$$\begin{aligned} K_3 + \beta_3 e^{-\lambda_{\min} t}U(t) &\leq \{K_3 + \beta_3 ||U(0)||\}e^{-k_1 t} + \frac{k_2}{k_1}(1 - e^{-k_1 t}) \\ &\leq \{K_3 + \beta_3 ||\bar{v}(0)||\}e^{-k_1 t} + \frac{k_2}{k_1}(1 - e^{-k_1 t}). \end{aligned} \tag{81}$$

Furthermore, we have

$$\begin{aligned} ||\bar{v}(t)|| &\leq \frac{1}{\beta_3}(\frac{k_2}{k_1} - K_3)(1 - e^{-k_1 t}) + ||\bar{v}(0)||e^{-k_1 t} \\ &\leq \frac{1}{\beta_3}(\frac{k_2}{k_1} - K_3) + ||\bar{v}(0)|| < \infty. \end{aligned} \tag{82}$$

This implies that, $\bar{v}(t)$ is bounded. From (59) and (62), we can conclude that $v(t)$, $z_v(t)$ and $z(t)$ are also bounded. In general, the above main results are summarized in the next Theorem 1.

Theorem 1. Let $z(t) \in R^n$, $v(t) \in R^q$, $f(v(t)) \in R^{l_f}$, $d_0(t) \in R^l$, $d(t) \in R^n$ be unknown bounded disturbances, $A(\sigma) \in R^{n\times n}$ is stable matrices. The nonlinear MFCS with time delays is well designed, if the following conditions are held:

$$\begin{aligned} E\dot{z}(t) &= A(\sigma)z(t) + B(\sigma)f(v(t)) + d(t), \\ v(t) &= C(\sigma)z(t) + d_0(t). \end{aligned}$$

(1) $||f(v(t))|| \le \alpha + \beta||v(t)||^\gamma$, $(\alpha \ge 0, \beta \ge 0, 0 \le \gamma < 1)$

(2) $|\hat{N}_r| \neq 0$.

5. AN ILLUSTRATIVE EXAMPLE

In this section, a numerical example is provided to demonstrate the efectiveness of the proposed method. Consider the following nonlinear system with time delays.

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} x(t-h_1) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-h_2) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} f(v(t)) + \begin{bmatrix} 0 \\ d(t) \end{bmatrix}, \\ v(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 5 & 0 \end{bmatrix} x(t-h_4), \\ y(t) &= \begin{bmatrix} 5 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 5 & 0 \end{bmatrix} x(t-h_3) + d_0(t), \\ f(v(t)) &= \frac{v(t)}{1+v(t)^2}. \end{aligned}$$

From σ, the control system can be expressed as

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ -1 & -3+\sigma_1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1+\sigma_2 \end{bmatrix} u(t) \\ &\quad + \begin{bmatrix} 2 \\ 1 \end{bmatrix} f(v(t)) + \begin{bmatrix} 0 \\ d(t) \end{bmatrix}, \\ v(t) &= \begin{bmatrix} 5\sigma_4 & 1 \end{bmatrix} x(t), \\ y(t) &= \begin{bmatrix} 5+5\sigma_3 & 1 \end{bmatrix} x(t) + d_0(t). \end{aligned}$$

In this case, we choose the initial values for
$x_0(t) = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}^T (t \le 0)$, $\xi_i^0(t) = \begin{bmatrix} 0.0 & 0.0 \end{bmatrix}^T (t < 0)(i = 1,2,3)$,
$\xi_4^0(t) = \begin{bmatrix} 0.0 & 0.0 \end{bmatrix}^T (t = 0)$, $u_0(t) = 0.0, (t \le 0)$.

Let $d(t) = 0.9$ $(5 \le t \le 15)$ and $d_0(t) = 0.6$ $(30 \le t \le 43)$ be the bounded disturbances of the system. The reference model is given as follows [10]:

$$\begin{aligned} \dot{x}_m(t) &= \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x_m(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_m(t), \\ y_m(t) &= \begin{bmatrix} 2 & 1 \end{bmatrix} x_m(t). \end{aligned}$$

Here the initial values is $x_m(0) = \begin{bmatrix} 0.0 & 0.0 \end{bmatrix}^T$, and the reference model input is $r_m(t) = 4sin0.5t + 8$. In this example, we choose $T(p) = p + 5$, $Q = (p + 6)^2$ and $D_d(p) = p$. The matrix F_i and G_i are given as follows:

$$F_i = \begin{bmatrix} 0 & 1 \\ -36 & -12 \end{bmatrix}, \qquad G_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Based on (27) ∼ (34) and Theorem 1, the control input $u(t)$ is given as

$$\begin{aligned} u(t) &= -\sigma_2 u(t) - \begin{bmatrix} -36 - 36\sigma_2 & -7 - 7\sigma_2 + 5\sigma_3 + 5\sigma_2\sigma_3 \end{bmatrix} \xi_1 \\ &\quad -(7 + \sigma_1)y(t) - \begin{bmatrix} -222 - 36\sigma_1 & -54 - 12\sigma_1 \end{bmatrix} \xi_2 \\ &\quad -(11 + 10\sigma_3)f(v(t)) \\ &\quad - \begin{bmatrix} -396 - 360\sigma_3 99 - 10\sigma_1 - 85\sigma_3 - 10\sigma_1\sigma_3 \end{bmatrix} \xi_3 + u_m(t), \\ u_m(t) &= r_m(t) + \begin{bmatrix} -26 & -5 \end{bmatrix} \xi_4. \end{aligned}$$

By using $\xi_i(t) = [\xi_{i1}(t), \xi_{i2}(t)]^T (i = 1, 2, 3, 4)$, we derive [1]

$$\begin{aligned} u(t) &= -u(t - h_2) + 36\xi_{11} + 36\xi_{11}(t - h_2) + 7\xi_{12}(t) \\ &\quad +7\xi_{12}(t - h_2) - 5\xi_{12}(t - h_3) - 5\xi_{12}(t - h_2 - h_3) \\ &\quad -7y(t) - y(t - h_1) + 222\xi_{21}(t) + 36\xi_{21}(t - h_1) \\ &\quad +54\xi_{22}(t) + 12\xi_{22}(t - h_1) - 11f(v(t)) - 10f(v(t - h_3)) \\ &\quad +396\xi_{31}(t) + 360\xi_{31}(t - h_3) + 99\xi_{32}(t) + 10\xi_{32}(t - h_1) \\ &\quad +85\xi_{32}(t - h_3) + 10\xi_{32}(t - h_1 - h_3) + u_m(t), \\ u_m(t) &= r_m(t) - 26\xi_{41}(t) - 5\xi_{42}(t). \end{aligned}$$

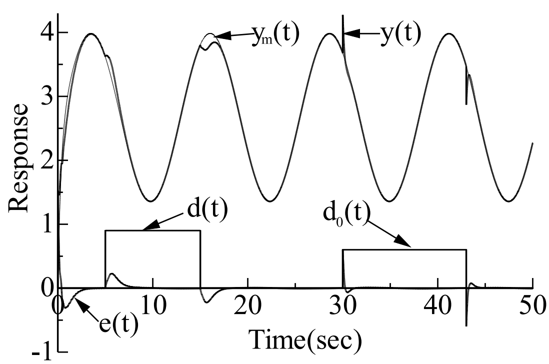

Fig. 1. Responses of the system for nonlinear system with time delays.

The output responses for the above case are shown in Figure 1. From the simulation results, it can be seen that the control systems are efficient for the disturbances, and the output error converges to zero.

6. CONCLUSIONS

We have presented a new design of the MFCS with time delays. The illustrative example and the simulation results show the benefits of this proposed design method. Topics for future study including the nonlinear discrete control system with time delays.

Appendix A. $|pE - A_s(\sigma)|$

From (44) and (45), we have

$$|pE - A_s(\sigma)| = \begin{vmatrix} pI - A(\sigma) & 0 & 0 & 0 & -B(\sigma) \\ 0 & pI - F_1 & 0 & 0 & -G_1 \\ -G_2C(\sigma) & 0 & pI - F_2 & 0 & 0 \\ 0 & 0 & 0 & pI - F_3 & 0 \\ E_2(\sigma)C(\sigma) & H_1(\sigma) & H_2(\sigma) & H_3(\sigma) & I + E_0(\sigma) \end{vmatrix}.$$

Recall ($|Z| \neq 0$):

$$\begin{aligned} &\begin{vmatrix} X & Y \\ W & Z \end{vmatrix} = |Z||X - YZ^{-1}W|, \\ &I - X(I + YX)^{-1}Y = (I + XY)^{-1}, \\ &|I + XY| = |I + YX|, \\ &\begin{vmatrix} X_n & Y \\ W & Z_n \end{vmatrix} = (-1)^{mn} \begin{vmatrix} W & Z_m \\ X_n & Y \end{vmatrix} = \begin{vmatrix} Z_m & W \\ Y & X_n \end{vmatrix}. \end{aligned}$$

We can conclude that

$$\begin{aligned} &|pE - A_s(\sigma)| \\ &\quad = |pI - F_1||pI - F_2||pI - A(\sigma)||I + E_0(\sigma) + H_1(\sigma)[pI - F_1]^{-1}G_1| \\ &\quad .|I + \{E_2(\sigma) + H_2(\sigma)[pI - F_2]^{-1}G_2\}C(\sigma)[pI - F_1]^{-1}B(\sigma)\{I + E_0(\sigma) \\ &\quad +H_1(\sigma)[pI - F_1]^{-1}G_1\}^{-1}|. \end{aligned}$$

where $|pI - F_1||pI - F_2| = Q(p)$, $|pI - A(\sigma)| = D(\sigma, p)$.

From (35) – (38), we have

$$\begin{aligned} &|pE - A_s(\sigma)| \\ &\quad = |Q(p)|^2 D(\sigma, p)|\hat{N}_r^{-1}||\hat{N}_r + \bar{N}_r(\sigma) + Q^{-1}(p)D_d(p)R(\sigma, p)N(\sigma, p) \\ &\quad -N_r(\sigma) + Q^{-1}(p)S(\sigma, p)N(\sigma, p)\diagup D(\sigma, p)|. \end{aligned}$$

From $N_r(\sigma) = \bar{N}_r(\sigma) + \hat{N}_r$, we have

$$
\begin{aligned}
&|pE - A_s(\sigma)| \\
&\quad = |Q(p)|^2 D(\sigma,p)|\hat{N}_r|^{-1} Q^{-1}(p) D_d(p) R(\sigma,p) N(\sigma,p) + Q^{-1}(p) S(\sigma,p) N(\sigma,p)/D(\sigma,p) \\
&\quad = |\hat{N}_r|^{-1} |Q(p)|^2 D(\sigma,p) |Q^{-1}(p) N(\sigma,p)[D_d(p) D(\sigma,p) + S(\sigma,p)]/D(\sigma,p)|,
\end{aligned}
$$

where $T(p)D_m(p) = D_d(p)D(\sigma,p)R(\sigma,p) + S(\sigma,p)$.

Then

$$|pE - A_s(\sigma)| = |Q(p)|^2 D(\sigma,p)|\hat{N}|_r^{-1} N(\sigma,p) D_m^l(p) T^l(p)/D^l(\sigma,p)$$

Using $V_s(\sigma,p) = |N(\sigma,p)|/D(\sigma,p)^{l-1}$, we can get that

$$|pE - A_s(\sigma)| = |\hat{N}_r|^{-1} T(p)^l D_m(p)^l |Q(p)|^2 V_s(\sigma,p).$$

□

Appendix B. Proof of Lemma 1

Using the (47), we have

$$
\begin{aligned}
&deg|pE - A_s(\sigma)| \\
&\quad = l(deg(T(p)) + degD_m(p)) + 2deg|Q(p)| + degV_s(\sigma,p) \\
&\quad = l(\rho + n_m) + 2\sum_{i=1}^{l}(\rho + n_m - n + \eta_i) + degV_s(\sigma,p).
\end{aligned}
$$

From (48), we obtain $degV_s(\sigma,p) = \sum_{i=1}^{l} \eta_i - (l-1)n$. Then

$$
\begin{aligned}
&deg|pE - A_s(\sigma)| \\
&\quad = l(\rho + n_m) + 2\sum_{i=1}^{l}(\rho + n_m - n + \eta_i) + \sum_{i=1}^{l}\eta_i - (l-1)n \\
&\quad = \sum_{i=1}^{l}(\rho + n_m - n + \eta_i) + n + 2\sum_{i=1}^{l}(\rho + n_m - n + \eta_i) \\
&\quad = 3\sum_{i=1}^{l}(\rho + n_m - n + \eta_i) + n.
\end{aligned}
$$

Hence, we get $rankE = rank|pE - A_s(\sigma)| = 3\sum_{i=1}^{l}(\rho + n_m - n + \eta_i) + n$. Therefore, $[pE - A_s(\sigma)]^{-1}$ is proper, namely, no Impulse mode, $|pE - A_s(\sigma)|$ of the (47) is stable polynomial,

$$d_v(t) = C_s(\sigma)\frac{adj[pE - A_s(\sigma)]}{|pE - A_s(\sigma)|} d_s(t).$$

This implies the desired result. □

Appendix C. Proof of Lemma 3

It is easy to see that $r \geq 1$, next we give a proof of $0 < r < 1$. Let $f(x) = (K_3 + \beta_3 x)^\gamma - (K_2 + \beta_2 x^\gamma)$, we have

$$\begin{aligned} f'(x) &= \gamma\beta_3(K_3+\beta_3 x)^{\gamma-1} - \beta_2\gamma x^{\gamma-1} \\ &= \gamma\{\beta_3 x^{1-\gamma} - \beta_2(K_3+\beta_3 x)^{1-\gamma}\}/x^{1-\gamma}(K_3+\beta_3 x)^{1-\gamma}. \end{aligned}$$

Let $f'(x) = 0$. Then, we have $x^* = \frac{K_3\beta_2^{\frac{1}{1-\gamma}}}{\beta_3^{\frac{1}{1-\gamma}} - \beta_3\beta_2^{\frac{1}{1-\gamma}}}$, x^* is minimum point of $f(x)$, and $\beta_3^\gamma > \beta_2$, we have

$$f(x^*) = \frac{K_3^\gamma \beta_3^{\frac{\gamma}{1-\gamma}}}{(\beta_3^{\frac{1}{1-\gamma}} - \beta_3\beta_2^{\frac{1}{1-\gamma}})^\gamma} - (K_2 + \frac{\beta_2 K_3 \beta_2^{\frac{1}{1-\gamma}}}{\beta_3^{\frac{1}{1-\gamma}} - \beta_3\beta_2^{\frac{1}{1-\gamma}}}) > 0.$$

Then

$$K_3^\gamma \beta_3^{\frac{\gamma}{1-\gamma}} - K_2(\beta_3^{\frac{\gamma}{1-\gamma}} - \beta_3\beta_2^{\frac{\gamma}{1-\gamma}})^\gamma - \beta_2 K_3 \beta_2^{\frac{\gamma}{1-\gamma}}(\beta_3^{\frac{\gamma}{1-\gamma}} - \beta_3\beta_2^{\frac{\gamma}{1-\gamma}})^{\gamma-1} > 0.$$

Let

$$A = \beta_3^{\frac{\gamma}{1-\gamma}},\ B = K_2(\beta_3^{\frac{\gamma}{1-\gamma}} - \beta_3\beta_2^{\frac{\gamma}{1-\gamma}})^\gamma,\ C = \beta_2\beta_2^{\frac{\gamma}{1-\gamma}}(\beta_3^{\frac{\gamma}{1-\gamma}} - \beta_3\beta_2^{\frac{\gamma}{1-\gamma}})^{\gamma-1}.$$

Thus, the inequality can represented as $AK_3^\gamma - B - K_3 C > 0$. If K_3 can seek to get, then the problem is solved. Let $g(K_3) = AK_3^\gamma - B - K_3 C$. From $g'(K_3) = \frac{A\gamma}{K_3^{1-\gamma}} - C = 0$, we have $K_3 = (\frac{C}{A\gamma})^{\frac{1}{\gamma-1}}$. It follows that $g(K_3) = C(\frac{1}{\gamma} - 1)(\frac{C}{A\gamma})^{\frac{1}{\gamma-1}} - B > 0$. We can conclude that

$$\gamma^{\frac{1}{1-\gamma}} A^{\frac{1}{1-\gamma}}(\frac{1-\gamma}{\gamma}) > BC^{\frac{\gamma}{1-\gamma}},$$

namely,

$$\gamma^{\frac{1}{1-\gamma}} \beta_3^{\frac{\gamma}{(1-\gamma)^2}}(\frac{1-\gamma}{\gamma}) > K_2\beta_2^{\frac{2\gamma-\gamma^2}{(1-\gamma)^2}}.$$

That is to say, $\beta_3 > K_2^{\frac{(1-\gamma)^2}{\gamma}} \beta_2^{2-\gamma}/\gamma^{\frac{1-\gamma}{\gamma}}(\frac{1-\gamma}{\gamma})^{\frac{(1-\gamma)^2}{\gamma}}$. Thus, we have

$$\beta_3 > \max\left\{ \beta_2^{\frac{1}{\gamma}}, \frac{K_2^{\frac{(1-\gamma)^2}{\gamma}}\beta_2^{2-\gamma}}{\gamma^{\frac{1-\gamma}{\gamma}}(\frac{1-\gamma}{\gamma})^{\frac{(1-\gamma)^2}{\gamma}}} \right\}.$$

It follows that

$$K_3 = \frac{\gamma^{\frac{1}{1-\gamma}}\beta_3^{\gamma+1}(\beta_3^{\frac{\gamma}{1-\gamma}}\beta_2^{\frac{1}{1-\gamma}})}{\beta_2^{\frac{2-\gamma}{1-\gamma}}}.$$

This completes the proof of Lemma 3. $\square$

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