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STABILITY ANALYSIS FOR NEUTRAL-TYPE IMPULSIVE NEURAL NETWORKS WITH DELAYS

Bo Du, Yurong Liu and Dan Cao

By using linear matrix inequality (LMI) approach and Lyapunov functional method, we obtain some new sufficient conditions ensuring global asymptotic stability and global exponential stability of a generalized neutral-type impulsive neural networks with delays. A simulation example is provided to demonstrate the usefulness of the main results obtained. The main contribution in this paper is that a new neutral-type impulsive neural networks with variable delays is studied by constructing a novel Lyapunov functional and LMI approach.

Keywords: neutral-type, neural networks, Lyapunov functional method, stability

Classification: 34G20, 35B40

1. INTRODUCTION

In the past few decades, the successful applications of cellular neural networks (CNNs) in a variety of areas (e.g. pattern recognition, associative memory and combinational optimization) have aroused a surge of research interests in the dynamical behaviors of the CNNs, see [2, 16, 32, 45]. We note that for various behaviors, the stability has proven to be the most important one that has received considerable research attention. For example, if a neural network is given to solve some optimization problems, it is highly desirable for the neural network to have a unique globally stable equilibrium, and so, the stability analysis of CNNs has been an ever hot research topic resulting in enormous stability conditions reported in the literature, see e.g. [3, 6, 8, 10, 14, 20, 26, 28, 34, 37, 39, 42, 43, 44, 45, 46].

Neutral functional differential equation (NFDE) is a class of equations depending on past as well as present values, but which involve derivatives with delays as well as the function itself. NFDEs are not only an extension of functional differential equations, but also provide good models in many fields including biology, electronics, mechanics and economics. In practice, a large class of electrical networks containing lossless transmission lines such as automatic control, high speed computers, robotics and etc., these systems can be well described by neutral-type delayed differential equations, see e.g. [7, 22, 29, 30]. Particularly, we note that the time-delays occur not only in the system states (or outputs) but also in the derivatives of system states in engineering systems

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[27]. Accordingly, CNNs with neutral terms have gained extensive research interests due to the fact that the neutral delays could exist during the implementation process of CNNs in VLSI circuits. The stability analysis issue of neutral CNNs has recently received much more research attention and a rich body of results has been obtained, see e. g. [4, 5, 48].

As is well known, the theory of impulsive differential equations has become more important in recent years in some mathematical models of real processes and phenomena. There has been a significant development in impulse theory, in recent years, especially in the area of impulsive differential equations with fixed moments and variable time; see e. g. [17, 18, 19, 23, 33]. We note that study of the existence and stability of the differential equations with delays was initiated by Travis and Webb [35] and Webb [38]. Since such equations are often more realistic to describe natural phenomena than those without delay, they have been investigated in variant aspects by many authors. In addition, an artificial electronic system and neural networks, are often subject to impulsive perturbation which can affect the dynamical behaviors of the system, just as time delays. Furthermore, the research of impulsive neural networks has received much interesting in recent years, see e. g. [3, 11, 12, 13, 15]. Several sufficient conditions ensuring the existence and global exponential stability of a unique equilibrium solution are given, by constructing suitable Lyapunov functional and employing some analysis techniques. In particular, the authors [25, 40, 47] studied the global exponential stability problems for impulsive neural networks with time-varying delays, and some stability results of periodic solutions were obtained in [41].

So far, to the best of the authors' knowledge, there is few results for the stability analysis to neutral-type impulsive neural networks with delays. The major challenges are as follows: (1) in order to construct a feasible Lyapunov-Krasovskii functional, the neutral operator D need exist inverse operator. So, when the neutral operator D is unstable, how can we obtain its inverse operator D^{-1} and some inequalities about D^{-1}; (2) when the impulse exists in CNNs, the corresponding stability analysis becomes more complicated since a new Lyapunov functional is required to reflect impulsive influence; and (3) it is non-trivial to establish a unified framework to handle the impulsive influence, neutral terms and variable delays. It is, therefore, the main purpose of this paper to make the first attempt to handle the listed challenges.

In this paper, we consider the stability analysis problem for a generalized neutral-type impulsive neural networks with variable delays. Note that neural system comprise both the impulsive and variable delays that are all dependent on the properties of neutral operator. We first develop a special matrix inequality to account for the impulse and neutral time-delays, and then a novel Lyapunov-Krasovskii functional is proposed to reflect the nature of impulse and neutral term. A matrix inequality approach is utilized to derive sufficient conditions guaranteeing global asymptotic stability and global exponential stability of the considered neural networks. A numerical example is presented to illustrate the usefulness and effectiveness of the main results obtained.

Throughout the manuscript, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes the matrix transposition. We will use the notation $A > 0$ (or $A < 0$) to denote that A is a symmetric and positive definite (or negative definite) matrix. If A, B are symmetric matrices, $A > B$ ($A \geq B$), then $A - B$ is a positive definite (positive semi-definite). $\lambda_M(A)$ denotes the largest eigenvalue of the matrix A. $||z||$ denotes the Euclidean norm of a vector z and $||A||$ denotes the induced norm of the matrix A, that is $||A|| = \sqrt{\lambda_M(A^\top A)}$ In symmetric block matrices, an asterisk "$*$" is used to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. PROBLEM FORMULATION

Let $C([-\tau,0],\mathbb{R}^n)$ is the Banach space of continuous functions on $[-\tau,0]$, where $\tau > 0$. $PC[J,\mathbb{R}^n] = \{\psi : J \to \mathbb{R}^n | \psi$ is continuous for all but, at most, a finite number of points $s \in J$ and at these ponits $s \in J$, $\psi(s^+)$ and $\psi(s^-)$ exist and $\psi(s^+) = \psi(s)\}$, where $J \subset \mathbb{R}$ is a bounded interval, $\psi(s^+)$ and $\psi(s^-)$ denote the right-hand and left-hand limits of the functions ψ, respectively. $PC^1[J,\mathbb{R}^n] = \{\psi : J \to \mathbb{R}^n | \psi$ is continuous differentiable for all but, at most, a finite number of points $s \in J$ and at these points $s \in J$, $\psi'(s^+)$ and $\psi'(s^-)$ exist and $\psi'(s^+) = \psi'(s)\}$. In particular, let $PC^1 = PC^1([-h,0],\mathbb{R}^n)$. For $\phi \in PC$ or $\phi \in PC^1$, denote the following norm:

$$||\phi||_\rho = \max_{1\leq i\leq n}\left\{ \max_{-\tau\leq s\leq 0}|\phi_i(s)|, \quad \max_{-h\leq s\leq 0}|\phi_i'(s)| \right\}.$$

Consider the following impulse neural networks of neutral-type:

$$\begin{aligned} &(Dy)'(t) = -Ay(t) + Bg(y(t)) + Cg(y(t-\tau(t))), \quad t \neq t_k, \\ &\Delta y(t) = I_k(y(t)), \quad t = t_k, \\ &y(t_0^+ + s) = \phi(s), \quad s \in [t_0 - \rho, t_0], \ k \in \mathbb{N}, \end{aligned} \tag{2.1}$$

where D is a difference operator defined by

$$(Dy)(t) = y(t) - V(t)y(t - \delta(t)), \tag{2.2}$$

where $y = (y_1, y_2, \dots, y_n)^\top$ is the neuron vector, $A = \mathrm{diag}(a_i)$ is a positive diagonal matrix, $B = (b_{ij})_{n\times n}$ and $C = (c_{ij})_{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons, $V(t)$ is a $n \times n$ real positive definite symmetric matrix, $g(y(t)) = (g_1(y_1(t)), g_2(y_2(t)), \dots, g_n(y_n(t)))^\top$ denotes the neuron activations, $\tau(t)$ and $\delta(t)$ are nonnegative, bounded, and differentiable delays satisfying

$$0 < \tau(t) \leq \tau_0, \quad \tau'(t) \leq \tau_1 < 1, \quad 0 < \delta(t) \leq \delta_0.$$

Let $\rho = \max\{\tau_0, \delta_0\}$. $\phi(\cdot)$ is the given piecewise continuously differentiable function on $[t_0 - \rho, t_0]$, the fixed moments t_k satisfy $t_1 < t_2 < \cdots, \lim_{k\to\infty} t_k = \infty$, $k = 1, 2, \dots$. At time instants t_k, jumps in the state variable are denoted by

$$\Delta y(t)|_{t=t_k} = y(t_k) - y(t_k^-),$$

where $y(t_k^-) = \lim_{t\to t_k^-} y(t)$, $I_k \in C(\mathbb{R}^n,\mathbb{R}^n)$ denotes the incremental change of the state at time t_k.

Remark 2.1. The neural network model (2.1) shows the neutral character by the D operator, which is different from other papers. For example, in [7] and [31], the authors studied the following neutral type neural system respectively,

$$\begin{cases} (x_i)'(t) = -a_i(t)x_i(t) + \sum_{j=1}^n [b_{ij}(t)f_j(t, x_j(t)) + d_{ij}(t)g_j(t, x_j'(t-\tau_{ij}(t)))] + I_i(t), \\ x_i(t) = \phi_i(t),\ t \in [-\tau, 0],\ i = 1, 2, \ldots, n \end{cases}$$

and

$$\begin{aligned} &y'(t) = -Ay(t) + Bg(y(t)) + Cg(y(t-\tau(t))) + Dy'(t-h(t)), \quad t \neq t_k, \\ &\Delta y(t) = I_k(y(t)), \quad t = t_k, \\ &y(t_0^+ + s) = \phi(s), \quad s \in [t_0 - \rho, t_0],\ k \in \mathbb{N}, \end{aligned}$$

their neutral terms are $g_j(t, x_j'(t-\tau_{ij}(t)))$ and $y'(t-h(t))$. As was point by Hale [9] that the properties of D operator are important for studying NFDEs. Hence model (2.1) has significant theoretical value for study of functional differential equation and neural networks.

Next we introduce some basic definitions and lemmas for deriving our main results.

Definition 2.2. The zero solution of (2.1) is said to be globally asymptotic stable in PC or PC^1, if for any solution $y(t, t_0, y(t_0))$ with the initial condition $y(t_0) \in PC$ or PC^1,

$$\lim_{t\to\infty} ||y(t, t_0, y(t_0))|| = 0.$$

Definition 2.3. The zero solution of (2.1) is said to be globally exponentially stable in PC or PC^1, if there exist $\alpha > 0$ and $K \geq 0$ such that for any solution $y(t, t_0, \phi)$ with the initial condition $\phi \in PC$ or PC^1,

$$||y(t, t_0, y(t_0))|| \leq K||\phi||_\rho e^{-\alpha(t-t_0)}, \quad t \geq t_0.$$

Definition 2.4. Letting $V : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+$ for $(t, y) \in [t_k, t_{k+1}) \times \mathbb{R}^n$, we define

$$D^+V(t, y(t)) = \lim_{h\to 0^+} \sup \frac{1}{h}\big(V(t+h, y+hy'(t) - V(t, y))\big).$$

Lemma 2.5. Suppose that $\lambda_1(t), \lambda_2(t), \ldots, \lambda_n(t)$ are eigenvalues of $V(t)$ and

$$\lambda_L = \max\{\lambda_i(t),\ i = 1, 2, \ldots, n,\ t \in \mathbb{R},\ t \neq t_k\},$$

$$\lambda_l = \min\{\lambda_i(t),\ i = 1, 2, \ldots, n,\ t \in \mathbb{R},\ t \neq t_k\}.$$

If $\lambda_L < 1$, then operator D has continuous inverse operator D^{-1}, satisfying $\frac{1}{1-\lambda_l} \leq ||D^{-1}|| \leq \frac{1}{1-\lambda_L}$.

P r o o f. Since $V(t)$ is a real symmetric positive definite matrix, there exists orthogonal matrix $U(t)$ such that

$$U(t)V(t)U^T(t) = E_\lambda(t) = diag(\lambda_1(t), \lambda_2(t), \ldots, \lambda_n(t)).$$

Consider the system

$$Dy(t) = y(t) - U^T(t)E_\lambda(t)U(t)y(t-\delta(t)).$$

Let $By(t) = U^T(t)E_\lambda(t)U(t)y(t-\delta(t))$, then $||B|| = \lambda_L < 1$. Thus, $D^{-1} = (I-B)^{-1}$ exists and $||D^{-1}|| = ||(I-B)^{-1}|| \leq \frac{1}{1-\lambda_L}$. Obviously, $||D^{-1}|| \geq \frac{1}{1-\lambda_l}$. □

Lemma 2.6. (Berman and Plemmons [1]) Let $A \in \mathbb{R}^{n\times n}$, then

$$\lambda_m(A)y^\top Ay \leq y^\top Ay \leq \lambda_M(A)y^\top Ay$$

for any $y \in \mathbb{R}^n$ and A is a real symmetric matrix.

For convenience of proof, we list the following conditions:

(H_0) $I_k(0) = 0$ for all $k \in \mathbb{N}$, $g(0) = 0$.

(H_1) $0 \leq \frac{g_j(d_j^{-1}y_1)-g_j(d_j^{-1}y_2)}{y_1-y_2} \leq l_j$, where l_j $(j = 1, 2, \dots, n)$ are positive constants. Note that $L = \mathrm{diag}\{l_1, l_2, \dots, l_n\}$, $l_M = \max\{l_i,\ i = 1, 2, \dots, n\}$ and $l_m = \min\{l_i,\ i = 1, 2, \dots, n\}$.

(H_2) There exist a positive constant γ, symmetric positive definite matrices P, Q and a positive diagonal matrix $M = \mathrm{diag}\{m_1, m_2, \dots, m_n\}$ such that

$$\Xi = \begin{pmatrix} \Psi_1 & PB + B^\top P & PC + C^\top P \\ * & \Psi_2 & MC \\ * & * & \Psi_3 \end{pmatrix} < 0,$$

where

$$\Psi_1 = -2(1-\lambda_l)^{-1}PA + \gamma P,$$

$$\Psi_2 = -MAL^{-1} + Q + MB, \quad \Psi_3 = -(1-\tau_1)Q.$$

(H_3) $||I_k(Dy(t_k^-))|| \leq \mu_k||Dy(t_k^-)||$ for $\mu_k \geq 0$, $k \in \mathbb{N}$.

(H_4) There exist $\mu > 1$ and $\gamma > 0$ satisfying

$$\mu\rho \leq \inf\{t_k - t_{k-1}\}, \quad \gamma\mu\rho > 1.$$

(H_5) $\max\{\nu_k\} \leq G$, $k \in \mathbb{N}$, where $\nu_k = 1 + \frac{(2||P||+||ML||)\mu_k + (||P||+0.5||ML||)\mu_k^2}{\lambda_m(P)}$, G is a constant satisfying $G > e$.

(H_6) There exist a positive constant $\tilde{\gamma} > 0$, symmetric positive definite matrices P, Q and a positive diagonal matrix $M = \mathrm{diag}\{m_1, m_2, \dots, m_n\}$ such that

$$\tilde{\Xi} = \begin{pmatrix} \tilde{\Psi}_1 & PB + B^\top P & PC + C^\top P \\ * & \tilde{\Psi}_2 & MC \\ * & * & \tilde{\Psi}_3 \end{pmatrix} < 0,$$

where

$$\tilde{\Psi}_1 = 2\tilde{\gamma}P + \tilde{\gamma}LM - 2(1-\lambda_l)^{-1}PA,\ \Psi_2 = -MAL^{-1} + Q + MB,$$

$$\tilde{\Psi}_3 = -(1-\tau_1)e^{-\tilde{\gamma}\rho}Q.$$

(H_7) There exists $\tilde{\mu} > 1$ satisfying

$$\tilde{\mu}\rho \le \inf\{t_k - t_{k-1}\}.$$

(H_8) $\max\{\tilde{\nu}_k\} \le G < e^{2\tilde{\gamma}\tilde{\mu}\rho},\ k \in \mathbb{N}$, where $\tilde{\nu}_k = 1 + \frac{(||P||+||ML||)\mu_k + (||P||+0.5||ML||)\mu_k^2}{\lambda_m(P)}$, G is a constant.

Remark 2.7. Under the condition (H_0), it is obvious that the origin $y = \mathbf{0}$ is the equilibrium point of (2.1). Usually, the existence of equilibrium points of a neural network can be guaranteed by boundedness of the activation functions (see [24]), and there is a standard way to shift the equilibrium point to the origin. Therefore, condition (H_0) is made here without loss of any generality. In addition, for the uniqueness of the equilibrium, see assumption (H_1)(Lipschiz condition).

The following sections are organized as follows: In Section 3, sufficient conditions are established for global asymptotic stability of (2.1). The global exponential stability of (2.1) are studied in Sections 4. In Section 5, an example is given to show the feasibility of our results.

3. GLOBAL ASYMPTOTIC STABILITY

Theorem 3.1. Assume that conditions (H_0)-(H_5) hold. Then the equilibrium point of (2.1) is globally asymptotic stable.

Proof. We use the following Lyapunov-Krasovskii functional to derive global asymptotic stability result,

$$V(t, y(t)) = e^{\gamma t}(Dy(t))^\top PDy(t) + \sum_{i=1}^{n} m_i \int_0^{d_i y_i(t)} g_i(d_i^{-1}s)\,\mathrm{d}s + \int_{-\tau(t)}^{0} g^\top(y(t+s))Qg(y(t+s))\,\mathrm{d}s,$$

where P and Q are positive definite matrices. Now, if we can calculate the time derivative of $V(t,y(t))$ along the trajectories of (2.1), then we have

$$\begin{aligned}
D^+V(t,y(t)) &= 2e^{\gamma t}(Dy(t))^\top P[-Ay(t)+Bg(y(t))+Cg(y(t-\tau(t)))] \\
&\quad + \gamma e^{\gamma t}(Dy(t))^\top PDy(t) \\
&\quad + g^\top(y(t))M[-Ay(t)+Bg(y(t))+Cg(y(t-\tau(t)))] \\
&\quad + g^\top(y(t))Qg(y(t)) \\
&\quad - (1-\tau'(t))g^\top(y(t-\tau(t)))Qg(y(t-\tau(t))) \\
&= e^{\gamma t}(Dy(t))^\top[-2PA]y(t) \\
&\quad + e^{\gamma t}(Dy(t))^\top 2PBg(y(t)) + e^{\gamma t}(Dy(t))^\top \gamma PDy(t) \\
&\quad + e^{\gamma t}(Dy(t))^\top 2PCg(y(t-\tau(t)))
\end{aligned}$$

$$\begin{aligned}
&- g^\top(y(t))MAy(t) + g^\top(y(t))(MB+Q)g(y(t)) \\
&+ g^\top(y(t))MCg(y(t-\tau(t))) - (1-\tau'(t))g^\top(y(t-\tau(t)))Qg(y(t-\tau(t))).
\end{aligned} \tag{3.1}$$

By condition (H_1)

$$-g^\top(y(t))(MA)y(t) \le -g^\top(y(t))(MAL^{-1})g(y(t)). \tag{3.2}$$

Using Lemma 2.5, we have

$$||y(t)|| = ||D^{-1}Dy(t)|| \ge (1-\lambda_l)^{-1}||Dy(t)||. \tag{3.3}$$

From (3.1)-(3.3) and condition (H_2), we have

$$\begin{aligned}
D^+V(t,y(t)) &\le e^{\gamma t}(Dy(t))^\top[-2(1-\lambda_l)^{-1}PA+\gamma P]Dy(t) \\
&\quad + e^{\gamma t}(Dy(t))^\top[PB+B^\top P]g(y(t)) + e^{\gamma t}(Dy(t))^\top[PC+C^\top P]g(y(t-\tau(t))) \\
&\quad + g^\top(y(t))[-MAL^{-1}+Q+MB]g(y(t)) \\
&\quad + g^\top(y(t))[MC]g(y(t-\tau(t))) \\
&\quad + g^\top(y(t-\tau(t)))[-(1-\tau_1)Q]g(y(t-\tau(t))) \\
&= \begin{pmatrix} Y(t) \\ g(y(t)) \\ g(y(t-\tau(t))) \end{pmatrix}^\top \Xi \begin{pmatrix} Y(t) \\ g(y(t)) \\ g(y(t-\tau(t))) \end{pmatrix} < 0,
\end{aligned} \tag{3.4}$$

where

$$Y(t) = e^{\gamma t}Dy(t).$$

In addition, when $t=t_k$, by using (3.4) and condition (H_3), we have

$$\begin{aligned}
V(t_k,y(t_k)) &= V(t_k, y(t_k^-)+I_k(y(t_k^-))) \\
&= e^{\gamma t_k^-}(D(y(t_k^-)+I_k(y(t_k^-))))^\top PD(y(t_k^-)+I_k(y(t_k^-))) \\
&\quad + \sum_{i=1}^n m_i \int_0^{d_i(y_i(t_k^-)+I_k(y_i(t_k^-)))} g_i(d_i^{-1}s)\,\mathrm{d}s \\
&\quad + \int_{t_k-\tau(t_k)}^{t_k} g^\top(y(s))Qg(y(s))\,\mathrm{d}s
\end{aligned}$$

$$
\begin{aligned}
&= e^{\gamma t_k^-}(D(y(t_k^-))^\top PD(y(t_k^-)) \\
&+ \sum_{i=1}^n m_i \int_0^{d_i(y_i(t_k^-))} g_i(d_i^{-1}s)\,\mathrm{d}s \\
&+ \int_{t_k^- -\tau(t_k^-)}^{t_k^-} g^\top(y(s))Qg(y(s))\,\mathrm{d}s + \sum_{i=1}^n m_i \int_{d_i(y_i(t_k^-))}^{d_i(y_i(t_k^-)+I_k(y_i(t_k^-)))} g_i(d_i^{-1}s)\,\mathrm{d}s \\
&+ e^{\gamma t_k^-}(D(I_k(y(t_k^-))))^\top PD(I_k(y(t_k^-))) \\
&+ e^{\gamma t_k^-}(D(y(t_k^-)))^\top PD(I_k(y(t_k^-))) + e^{\gamma t_k^-}(D(I_k(y(t_k^-))))^\top PD(y(t_k^-)) \\
&\le V(t_k^-, y(t_k^-)) + e^{\gamma t_k^-}(D(I_k(y(t_k^-))))^\top PD(I_k(y(t_k^-))) \\
&+ e^{\gamma t_k^-}(D(y(t_k^-)))^\top PD(I_k(y(t_k^-))) + e^{\gamma t_k^-}(D(I_k(y(t_k^-))))^\top PD(y(t_k^-)) \\
&+ D((y(t_k^-)))^\top MLD(I_k(y(t_k^-))) + D(I_k((y(t_k^-))))^\top 0.5MLD(I_k(y(t_k^-))) \\
&\le V(t_k^-, y(t_k^-)) + \Big((2||P|| + ||ML||)\mu_k + (||P|| + 0.5||ML||)\mu_k^2\Big)e^{\gamma t_k^-}||D(y(t_k^-))||^2 \\
&\le \left(1 + \frac{(2||P|| + ||ML||)\mu_k + (||P|| + 0.5||ML||)\mu_k^2}{\lambda_m(P)}\right)V(t_k^-, y(t_k^-)) \\
&= \nu_k V(t_k^-, y(t_k^-)).
\end{aligned}
\tag{3.5}
$$

For each solution $y(t, t_0, y_0)$ of (2.1), using (3.4) and (3.5), we have

$$V(t, y(t, t_0, y_0)) \le V(t_0, y_0)\Pi_{t_0<t_k<t}\nu_k \le V(t_0, y_0)G^{k-1}, \quad t \in [t_{k-1}, t_k).$$

From condition (H_4), one has $k - 1 \le (t_{k-1} - t_0)/\mu\rho$, thus,

$$G^{k-1} \le G^{(t-t_0)/\mu\rho}, \quad t \in [t_{k-1}, t_k). \tag{3.6}$$

On the other hand,

$$
\begin{aligned}
V(t_0, y_0) &= e^{\gamma t_0}(Dy(t_0))^\top PDy(t_0) + \sum_{i=1}^n m_i \int_0^{d_i y_i(t_0)} g_i(d_i^{-1}s)\,\mathrm{d}s \\
&+ \int_{t_0-\tau(t_0)}^{t_0} g^\top(y(s))Qg(y(s))\,\mathrm{d}s \\
&\le e^{\gamma t_0}\lambda_M(P)(1-\lambda_l)^2||\phi||_\rho^2 + 0.5\lambda_M(LM)(1-\lambda_l)^2||\phi||_\rho^2 \\
&+ \lambda_M(Q)\int_{t_0-\tau(t_0)}^{t_0} g^\top(y(s))g(y(s))\,\mathrm{d}s \\
&\le \Big(e^{\gamma t_0}\lambda_M(P)(1-\lambda_l)^2 + \lambda_M(LM)0.5(1-\lambda_l)^2 + \lambda_M(Q)\tau_1 l_M^2\Big)||\phi||_\rho^2.
\end{aligned}
\tag{3.7}
$$

Therefore, by (3.6) and (3.7)

$$
\begin{aligned}
||y||^2 \le\ & \frac{e^{\gamma t_0}\lambda_M(P)(1-\lambda_l)^2 + \lambda_M(LM)0.5(1-\lambda_l)^2 + \lambda_M(Q)\tau_1 l_M^2}{\lambda_m(P)} \\
& \times ||\phi||_\rho^2 G^{\frac{-(\gamma\mu\rho-1)t-t_0}{\mu\rho}}, \ t \in [t_{k-1}, t_k).
\end{aligned}
$$

Then by Definition 2.3 and $\gamma\mu\rho > 1$, Theorem 3.1 is proved. □

4. EXPONENTIAL STABILITY

Theorem 4.1. Assume that conditions (H_0), (H_1), (H_3) and (H_6)-(H_8) hold. Then the equilibrium point of system (2.1) is globally exponentially stable, if $\tilde{\gamma} > \frac{\ln G}{2\tilde{\mu}\rho}$. Moreover,

$$||y|| \le \mathcal{K}||\phi||_\rho e^{-\beta(t-t_0)},$$

where $\mathcal{K} = \left(\frac{\lambda_M(P)(1-\lambda_l)^2+\lambda_M(LM)(1-\lambda_l)^2+\lambda_M(Q)\frac{1-e^{-2\tilde{\gamma}\tau_0}}{2\tilde{\gamma}}l_M^2}{\lambda_m(P)}\right)^{1/2}$, $\beta = \tilde{\gamma} - \frac{\ln G}{2\tilde{\mu}\rho}$.

Proof. In order to obtain the stability result, construct the following Lyapunov-Krasovskii functional:

$$\begin{aligned}V(t,y(t)) &= e^{2\tilde{\gamma}t}(Dy(t))^\top PDy(t) + e^{2\tilde{\gamma}t}\sum_{i=1}^{n} m_i \int_0^{d_iy_i(t)} g_i(d_i^{-1}s)\,\mathrm{d}s\\ &+ \int_{-\tau(t)}^{0} e^{2\tilde{\gamma}(t+s)} g^\top(y(t+s))Qg(y(t+s))\,\mathrm{d}s,\end{aligned}$$

where P and Q are positive definite matrices. Calculating the time derivative of $V(t,y(t))$ along the trajectories of (2.1), then we have

$$\begin{aligned}D^+V(t,y(t)) &= 2\tilde{\gamma}e^{2\tilde{\gamma}t}(Dy(t))^\top PDy(t)\\
&+ 2e^{2\tilde{\gamma}t}(Dy(t))^\top P[-Ay(t) + Bg(y(t)) + Cg(y(t-\tau(t)))]\\
&+ e^{2\tilde{\gamma}t}g^\top(y(t))M[-Ay(t) + Bg(y(t)) + Cg(y(t-\tau(t)))]\\
&+ \tilde{\gamma}e^{2\tilde{\gamma}t}(Dy(t))^\top LMDy(t) + e^{2\tilde{\gamma}t}g^\top(y(t))Qg(y(t))\\
&- (1-\tau'(t))e^{2\tilde{\gamma}(t-\tau(t))}g^\top(y(t-\tau(t)))Qg(y(t-\tau(t)))\\
&= 2\tilde{\gamma}e^{2\tilde{\gamma}t}(Dy(t))^\top PDy(t) - e^{2\tilde{\gamma}t}(Dy(t))^\top 2PAy(t)\\
&+ e^{2\tilde{\gamma}t}(Dy(t))^\top\tilde{\gamma}LMDy(t)\\
&+ e^{2\tilde{\gamma}t}(Dy(t))^\top 2PBg(y(t)) + e^{2\tilde{\gamma}t}(Dy(t))^\top 2PCg(y(t-\tau(t)))\\
&- e^{2\tilde{\gamma}t}g^\top(y(t))MAy(t) + e^{2\tilde{\gamma}t}g^\top(y(t))(MB+Q)g(y(t))\\
&+ e^{2\tilde{\gamma}t}g^\top(y(t))MCg(y(t-\tau(t)))\\
&- (1-\tau'(t))e^{2\tilde{\gamma}(t-\tau(t))}g^\top(y(t-\tau(t)))Qg(y(t-\tau(t))).\end{aligned} \tag{4.1}$$

From (3.2), (3.3), (4.1) and condition (H_6), we have

$$\begin{aligned} D^+V(t,y(t)) &\le e^{2\tilde{\gamma}t}(Dy(t))^\top[2\tilde{\gamma}P+\tilde{\gamma}LM-2(1-\lambda_l)^{-1}PA]Dy(t)\\ &+e^{2\tilde{\gamma}t}(Dy(t))^\top[PB+B^\top P]g(y(t))\\ &+e^{2\tilde{\gamma}t}(Dy(t))^\top[PC+C^\top P]g(y(t-\tau(t)))\\ &+e^{2\tilde{\gamma}t}g^\top(y(t))[-MAL^{-1}+Q+MB+B^\top M]g(y(t))\\ &+e^{2\tilde{\gamma}t}g^\top(y(t))[MC+C^\top M]g(y(t-\tau(t)))\\ &+e^{2\tilde{\gamma}t}g^\top(y(t-\tau(t)))[-(1-\tau_1)e^{-\tilde{\gamma}\rho}Q]g(y(t-\tau(t)))\\ &=e^{2\tilde{\gamma}t}\begin{pmatrix} Dy(t)\\ g(y(t))\\ g(y(t-\tau(t)))\end{pmatrix}^\top \tilde{\Xi}\begin{pmatrix} Dy(t)\\ g(y(t))\\ g(y(t-\tau(t)))\end{pmatrix}<0. \end{aligned} \tag{4.2}$$

When $t=t_k$, by using (4.2) and condition (H_3), we have

$$\begin{aligned} V(t_k,y(t_k)) &= V(t_k,y(t_k^-)+I_k(y(t_k^-)))\\ &=e^{2\tilde{\gamma}t_k^-}(D(y(t_k^-)+I_k(y(t_k^-))))^\top PD(y(t_k^-)+I_k(y(t_k^-)))\\ &+e^{2\tilde{\gamma}t_k^-}\sum_{i=1}^n m_i\int_0^{d_i(y_i(t_k^-)+I_k(y_i(t_k^-)))} g_i(d_i^{-1}s)\,\mathrm{d}s\\ &+\int_{t_k-\tau(t_k)}^{t_k} e^{2\tilde{\gamma}s}g^\top(y(s))Qg(y(s))\,\mathrm{d}s\\ &=e^{2\tilde{\gamma}t_k^-}(D(y(t_k^-))^\top PD(y(t_k^-))\\ &+e^{2\tilde{\gamma}t_k^-}\sum_{i=1}^n m_i\int_0^{d_i(y_i(t_k^-))} g_i(d_i^{-1}s)\,\mathrm{d}s\\ &+\int_{t_k^--\tau(t_k^-)}^{t_k^-} e^{2\tilde{\gamma}s}g^\top(y(s))Qg(y(s))\,\mathrm{d}s\\ &+e^{2\tilde{\gamma}t_k^-}\sum_{i=1}^n m_i\int_{d_i(y_i(t_k^-))}^{d_i(y_i(t_k^-)+I_k(y_i(t_k^-)))} g_i(d_i^{-1}s)\,\mathrm{d}s\\ &+e^{2\tilde{\gamma}t_k^-}(D(I_k(y(t_k^-))))^\top PD(I_k(y(t_k^-)))\\ &+e^{2\tilde{\gamma}t_k^-}PD(I_k(y(t_k^-)))+e^{2\tilde{\gamma}t_k^-}(D(I_k(y(t_k^-))))^\top PD(y(t_k^-))\\ &\le V(t_k^-,y(t_k^-))+e^{2\tilde{\gamma}t_k^-}(D(I_k(y(t_k^-))))^\top PD(I_k(y(t_k^-)))\\ &+e^{2\tilde{\gamma}t_k^-}PD(I_k(y(t_k^-)))+e^{2\tilde{\gamma}t_k^-}(D(I_k(y(t_k^-))))^\top PD(y(t_k^-))\\ &+e^{2\tilde{\gamma}t_k^-}D((y(t_k^-)))^\top MLD(I_k(y(t_k^-)))\\ &+e^{2\tilde{\gamma}t_k^-}D(I_k((y(t_k^-))))^\top 0.5MLD(I_k(y(t_k^-))) \end{aligned}$$

$$\begin{aligned}
&\le V(t_k^-, y(t_k^-)) + ((||P|| + ||ML||)\mu_k \\
&+ (||P|| + 0.5||ML||)\mu_k^2) e^{2\tilde{\gamma} t_k^-} ||D(y(t_k^-))|| \\
&\le \left(1 + \frac{(||P|| + ||ML||)\mu_k + (||P|| + 0.5||ML||)\mu_k^2}{\lambda_m(P)}\right) V(t_k^-, y(t_k^-)) \\
&= \tilde{\nu}_k V(t_k^-, y(t_k^-)).
\end{aligned} \tag{4.3}$$

For each solution $y(t, t_0, y_0)$ of (2.1), using (4.2) and (4.3), we have

$$V(t, y(t, t_0, y_0)) \le V(t_0, y_0)\Pi_{t_0 < t_k < t}\tilde{\nu}_k \le V(t_0, y_0) G^{k-1}, \quad t \in [t_{k-1}, t_k).$$

From condition (H$_7$), one has $k - 1 \le (t_{k-1} - t_0)/\tilde{\mu}\rho$, thus,

$$G^{k-1} \le e^{\frac{\ln G}{\tilde{\mu}\rho}(t-t_0)}, \quad t \in [t_{k-1}, t_k).$$

On the other hand,

$$\begin{aligned}
&V(t_0, y_0) = e^{2\tilde{\gamma} t_0}(Dy(t_0))^\top P D y(t_0) + e^{2\tilde{\gamma} t_0} \sum_{i=1}^n m_i \int_0^{d_i y_i(t_0)} g_i(d_i^{-1}s)\,\mathrm{d}s \\
&+ \int_{t_0-\tau(t_0)}^{t_0} e^{2\tilde{\gamma} s} g^\top(y(s)) Q g(y(s))\,\mathrm{d}s \\
&\le e^{2\tilde{\gamma} t_0}\lambda_M(P)(1-\lambda_l)^{-2}||\phi||_\rho^2 + e^{2\tilde{\gamma} t_0}\lambda_M(LM)(1-\lambda_l)^{-2}||\phi||_\rho^2 \\
&+ \lambda_M(Q)\int_{t_0-\tau(t_0)}^{t_0} e^{2\tilde{\gamma} s} g^\top(y(s)) g(y(s))\,\mathrm{d}s \\
&\le e^{2\tilde{\gamma} t_0}\left(\lambda_M(P)(1-\lambda_l)^2 + \lambda_M(LM)(1-\lambda_l)^2 + \lambda_M(Q)\frac{1-e^{-2\tilde{\gamma}\tau_0}}{2\tilde{\gamma}} l_M^2\right)||\phi||_\rho^2
\end{aligned}$$

Therefore,

$$\begin{aligned}
||y||^2 \le\ & \frac{\lambda_M(P)(1-\lambda_l)^2 + \lambda_M(LM)(1-\lambda_l)^2 + \lambda_M(Q)\frac{1-e^{-2\tilde{\gamma}\tau_0}}{2\tilde{\gamma}} l_M^2}{\lambda_m(P)} \\
&\times ||\phi||_\rho^2 e^{-2\tilde{\gamma}(t-t_0)} e^{\frac{\ln G}{\tilde{\mu}\rho}(t-t_0)}, \ t \in [t_{k-1}, t_k),
\end{aligned}$$

i. e.,

$$\begin{aligned}
||y|| \le\ & \left(\frac{\lambda_M(P)(1-\lambda_l)^2 + \lambda_M(LM)(1-\lambda_l)^2 + \lambda_M(Q)\frac{1-e^{-2\tilde{\gamma}\tau_0}}{2\gamma} l_M^2}{\lambda_m(P)}\right)^{1/2} \\
&\times ||\phi||_\rho e^{-(\tilde{\gamma} - \frac{\ln G}{2\tilde{\mu}\rho})(t-t_0)}, \ t \in [t_{k-1}, t_k).
\end{aligned}$$

From $\tilde{\gamma} > \frac{\ln G}{2\tilde{\mu}\rho}$, the proof is completed. □

Remark 4.2. From Definitions 2.1 to 2.2, one knows that the global exponential stability of equilibrium point of (2.1) implies its global asymptotic stability. In fact, $\mathcal{K} > 0$ and $\beta > 0$ yield $||y|| \le \mathcal{K}||\phi||_\rho e^{-\beta(t-t_0)} \to 0, \ t \to 0$. However, by comparing Theorem 3.1 and 4.1, we find that the conditions presented in them are almost same. The obvious difference is the framework of Lyapunov-Krasovskii functional. This shows that choosing of a suitable Lyapunov functional is important for obtaining the stability results.

Remark 4.3. In Theorem 4.1, LMI-based sufficient conditions are derived to guarantee the exponential stability for the (2.1). For some special cases (e. g. $\tau(t)$ =constant, no impulse), the resulting stability criteria can be obtained as immediate consequences. We also point out here that our arguments can be extended to the systems in the presence of non-neutral equations (or systems) without major difficulty.

Remark 4.4. In this paper, the stability analysis problem is tackled for impulsive neural networks of neutral type. The distinctive contributions of this paper are outlined as follows: (1) when the neutral delay term is studied as a neutral D operator, novel analysis technique is developed since the conventional analysis tool no longer applies; (2) a new Lyapunov functional is constructed to reflect the neutral operator and impulse influence; (3) a unified framework is established to handle the discontinuous parameters, neutral terms and time-varying delays.

5. NUMERICAL EXAMPLE

To illustrate the validity of our results, the following example will be discussed:

$$\begin{aligned} &(y(t)-V(t)y(t-\delta(t)))' = -Ay(t)+Bg(y(t))+Cg(y(t-\tau(t))), \quad t\neq t_k,\\ &y(t_k)=W^k y(t_k^-), \quad t=t_k, \end{aligned} \tag{5.1}$$

with the initial conditions $y_1(s)=\cos s,\ y_2(s)=\sin s,\ -\rho\le s<0$. Let

$$V(t)=\begin{pmatrix} \frac{53}{55} & 0\\ 0 & \frac{54}{55}\end{pmatrix},\ \tau(t)=\delta(t)=1.5+0.5\sin t,\ \rho=\max_{t\in[0,2\pi]}\{\tau(t),\delta(t)\}=2,\ t_k=t_{k-1}+1,$$

$$w_{11}^k=w_{22}^k=(-1)^k(e-1)^{1/2},\quad w_{12}^k=w_{23}^k=0,\quad k\in\mathbb{N},\ \tilde\gamma=1,\ \tilde\mu=2,$$

$$g_i(x)=\frac{1}{4}(|x+2|-|x-2|)),\quad i=1,2.$$

The constant matrices are

$$A=\begin{pmatrix} 2 & 0\\ 0 & 1\end{pmatrix},\quad B=\begin{pmatrix} 2 & 0.1\\ 0 & 0.1\end{pmatrix},\quad C=\begin{pmatrix} 0.1 & 0.2\\ 0.05 & 0.1\end{pmatrix}.$$

With the parameters given above, by using the Matlab LMI toolbox, we solve the $\tilde\Xi$ in assumption (H_6) and obtain the feasible solutions as follows:

$$P=10^{-7}\begin{pmatrix} 0.0007 & -0.0070\\ -0.0070 & 0.1163\end{pmatrix},\quad Q=10^{-10}\begin{pmatrix} 0.0277 & 0.0216\\ 0.0216 & 0.1410\end{pmatrix},$$

$$M=10^{-10}\begin{pmatrix} 0.5565 & 0\\ 0 & 0.5565\end{pmatrix}.$$

We have

$$||y(t_k^-+I_k(y(t_k)))||=||(W^k-I)y(t_k^-)+Iy(t_k^-)||\le(1+||W^k-I||)||y(t_k^-)||.$$

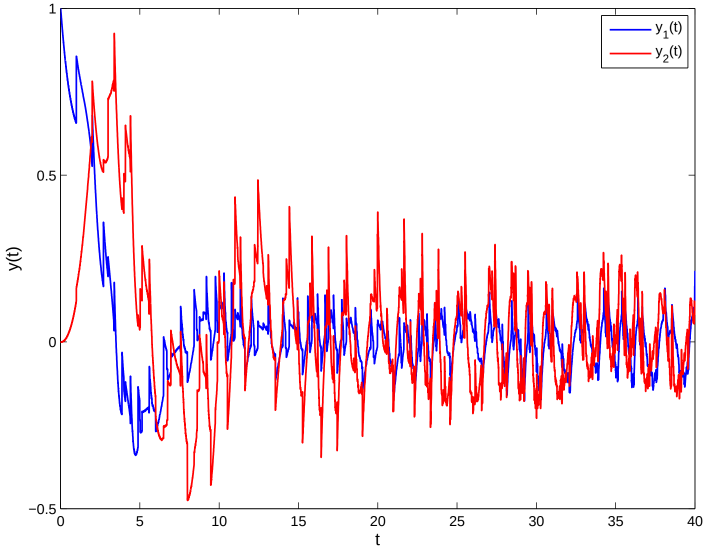

Fig. 1. The States' Evolution of the System (5.1).

Thus, by Lemma 2.1, we have

$$||I_k(Dy(t_k^-))|| \leq \frac{1-\lambda_l}{1-\lambda_L}(1+||W^k - I||)||Dy(t_k^-)||,$$

that is $\mu_k = \frac{1-\lambda_l}{1-\lambda_L}(1+||W^k - I||)$. Thus,

$$\tilde{\nu}_k = 1 + \frac{(||P|| + ||ML||)\mu_k + (||P|| + 0.5||ML||)\mu_k^2}{\lambda_m(P)} = 27.77.$$

Let $G = 27.77$, then $\frac{\ln G}{2\tilde{\mu}\rho} = 0.4155$. Hence $\tilde{\gamma} > \frac{\ln G}{2\tilde{\mu}\rho}$. Therefore, it follows from Theorem 4.1 that the system (5.1) with given parameters is globally exponentially stable. This is further confirmed by our numerical simulation. In fact, Figure 1 shows the evolution of states of the system (5.1) with the above parameters, and the simulation results show the state of the system indeed approach zero, which support the proposed methods.

Remark 5.1. We'd like to give the answer to the circuit diagram of model (2.1). However, model (2.1) contains neutral operators and impulse influence, is more complicated. So far, we can not obtain the circuit diagram of model (2.1). We hope that some authors research this subject in the future.

6. CONCLUSIONS

In this paper, we have investigated stability problems for a class of neutral-type neural networks with time-varying delays and impulse. By utilizing novel Lyapunov-Krasovskii functionals, the sufficient conditions are derived to guarantee global asymptotic stability and exponential stability for the involved systems. The criteria are expressed in the form of LMIs, which can be solved effectively by using the matlab LMI toolbox, and no turning of parameters will be needed. A simulation example has been provided to show the usefulness of the proposed global exponential stability conditions.

We mention here that some finer approaches to deal with time delays would be the delay-slope-dependent method [21] and the delay-fraction approach [36], which could be the further work to reduce the possible conservatism in the dynamical analysis. And another future research topics would be the extension of the present results to more general cases, for example, the case that there exist multiply variable delays, and the case that the neural network of neutral-type is a difference system. The results will appear in the near future.

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