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PASSIVITY ANALYSIS OF UNCERTAIN STOCHASTIC NEURAL NETWORK WITH LEAKAGE AND DISTRIBUTED DELAYS UNDER IMPULSIVE PERTURBATIONS

Senthil Raj, Raja Ramachandran, Samidurai Rajendiran, Jinde Cao and Xiaodi Li

In this paper, the problem of passivity analysis for a class of uncertain stochastic neural networks with mixed delays and impulsive control is investigated. The mixed delays include constant delay in the leakage term, discrete and distributed delays. The discrete delays are assumed to be time-varying and belong to a given interval, which means that the lower and upper bounds of interval time-varying delays are available. By using Lyapunov stability theory, stochastic analysis, linear matrix inequality techniques and introducing some free-weighting matrices, several novel sufficient conditions are derived to guarantee the passivity of the suggested system in the sense of mean square under two cases: with known or unknown parameters. It is believed that these results are significant and useful for the design and applications of impulsive stochastic neural networks. Finally, two numerical examples are provided to show the effectiveness of the theoretical results.

Keywords: distributed delays, leakage delay, passivity impulses, stochastic disturbances

Classification: 34Dxx

1. INTRODUCTION

Neural networks can imitate the human brain, and they have been used for a wide variety of applications, for example, target tracking, machine learning, system identification and so on [7, 25, 30]. Moreover, as we know, the applications of neural networks heavily depend on their dynamic behaviors. On the other hand, time delays are always unavoidably encountered in the implementation of neural networks due to the finite switching speed of neurons and amplifiers. Therefore, increasing attention has been paid to the problem of neural networks with various delays have been reported in [3, 4, 18, 12, 33]. Some criteria have been proposed to ensure the fixed-time synchronization for memristive neural network [3]. The H_∞ filtering problem for delayed discrete-time switched neural networks has been considered in [4].

On the other hand, due to modeling and measurement errors, neural network is often disturbed by stochastic factors and the parameter uncertainties. Because in real nervous

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system, synaptic transmission is a noisy process and the connection weights of the neurons depend on certain resistance and capacitance values which include uncertainties. Hence, their presence must be considered in realistic dynamics and some results related to this problem have recently published in [8, 11, 14, 29].

The concept of passivity has played an important role in the analysis of the stability of dynamical systems, nonlinear control, and other research areas. The essence of the passivity theory is that the passive properties of a system can keep the system internal stability. So it gives a way to study nonlinear system only by means of the general characteristics of the input-output dynamics. Recently, passivity analysis problem for various neural networks was widely investigated in the literature [5, 15, 20, 34, 37]. Cao and Li have investigated the stability of memristive neural networks with leakage delay, and the uncertainties was also considered [15]. In [5], Chen et al. presented, both delay-independent and delay-dependent passivity conditions for stochastic neural network in the sense of mean square. Very recently Raja et al.[28], discussed the passivity analysis for stochastic BAM neural networks with time-varying structured uncertainties.

As we know, time delay in the stabilizing negative feedback term has a tendency to destabilize a system [6, 16]. Like the traditional time delays, the leakage delays also have a great impact on the dynamics of neural networks and many works appeared in the literature, see [13, 17, 22, 31]. Based on this work, [31] pay attention to the passivity analysis of uncertain neural networks. In [22], authors studied the equilibrium point of two classes fuzzy neural networks with delays in leakage terms; By use of the topological degree theory, delay-dependent stability conditions of neural networks of neutral type with time delay in the leakage term was proposed in [13]. Therefore, it is considerable to investigate the passivity analysis of neural networks with time delays in the leakage term and very little existing works appeared in the literature [1, 31, 36]. The passivity properties of uncertain neural networks with leakage delay and time-varying delay has been studied in [31]. In [1], authors investigated the problem for passivity analysis of neutral type neural networks with Markovian jumping parameters and time delays in the leakage term. Unfortunately, in these works, authors neglected the effects of stochastic disturbances, which has also an important effect on the passivity analysis of neural networks. But in [36], Zhao et al. presented the passivity problem for stochastic neural networks with time-varying delays and leakage delay using Lyapunov functional and free-weighting matrix method.

In addition, many physical systems undergo unexpected changes at certain moments due to instantaneous perturbations, which leads to impulsive effects [19, 21]. It is worth pointing out that neural networks are often subject to impulsive perturbations that in turn affect dynamical behaviors of the system. It frequently occurs in fields such as economics, mechanics, electronics, telecommunications, medicine and biology, etc. Therefore, it is necessary to consider the impulsive effects to the passivity problem of stochastic neural networks to reflect more realistic dynamics and several interesting results have been reported for continuous-time and discrete-time neural networks [26, 27, 19, 35]. More recently, in [27], Raja et al. derived the dissipativity results for a class of uncertain discrete-time stochastic neural networks with impulsive parameters. However, to the best of our knowledge, the passivity analysis problem for stochastic neural network with the effects of leakage delays and impulsive perturbations has not been investigated in the previous literature. This motivates our present study.

In this paper, we deal with the passivity problem of impulsive stochastic neural networks with leakage, discrete and distributed delays. Then by using Lyapunov functional, Free weighting matrix method and stochastic analysis techniques, some sufficient conditions that dependent on the delays for passivity are obtained in terms of LMIs, which can be readily verified by using standard numerical software. Finally, two numerical examples are given to illustrate the effectiveness of the proposed criteria.

Notations. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. The notation $X \geq 0$ (respectively, $X > 0$), where X is symmetric matrices, means that X is positive semi-definite (respectively, positive definite). Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ be the complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i. e. the filtration contains all $\mathcal{P}$- null sets and is right continuous). $\omega(t)$ be a scalar Brownian motion defined on the probability space. $\mathbb{E}[\cdot]$ is the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$.

2. PROBLEM FORMULATION

Consider the following uncertain stochastic neural networks with both discrete and distributed time-varying delays described by

$$\begin{aligned} dx(t) &= \Big[-Ax(t-\delta) + Bg(x(t)) + Cg(x(t-\tau(t))) + D\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s + u(t)\Big]\,\mathrm{d}t \\ &\quad +\sigma(t, x(t), x(t-\tau(t)), x(t-d(t)))\,\mathrm{d}\omega(t), \qquad t \neq t_k \\ y(t) &= g(x(t)), \\ \Delta x(t_k) &= -I_k\Big\{x(t_k^-) - A\int_{t_k-\delta}^{t_k} x(s)\,\mathrm{d}s\Big\}, \qquad t = t_k, \quad k \in \mathbb{Z}_+, \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector associated with the neurons, $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T$ is the activation function, $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T$ is the input, $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]$ is the output. $\sigma \in R^{n\times q}$ is the diffusion coefficient vector and $w(t) = (w_1(t), w_1(t), \ldots, w_q(t))^T$ is a q-dimensional Brownian motion defined on a complete probability space
$(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ be the complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i. e.the filtration contains all $\mathcal{P}$- null sets and is right continuous). The matrix A=diag$(a_1, a_2, \ldots, a_n)$ is a diagonal matrix with positive entries $a_i > 0$. B, C, D are the interconnection matrices representing the weight coefficients of the neurons. $I_k \in \mathbb{R}^{n\times n}$, $k \in \mathbb{Z}_+$ denotes the impulsive matrix. The discrete delay $\tau(t)$ and distribute delay $d(t)$ satisfies

$$0 \leq \tau_1 \leq \tau(t) \leq \tau_2, \quad \dot{\tau}(t) \leq \mu, \quad 0 < d(t) \leq d \tag{2}$$

where τ_1, τ_2, d and μ are constants. The initial condition associated with model (1) is given by

$$x(t) = \phi(t), \quad \forall t \in [-max\{\delta, \tau_1, \tau_2, d\}, 0].$$

Throughout this paper, it is assumed that the activation functions satisfy the following assumptions:

(H1) For any $j \in 1, 2, \ldots, n,\ f_j(0) = 0$ and there exist constants F_j^- and F_j^+ such that

$$F_j^- \leq \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \leq F_j^+ \tag{3}$$

for all $\alpha_1 \neq \alpha_2$.

(H2) Assume that $\sigma : \mathcal{R}^n \times \mathcal{R}^n \times \mathcal{R}^+ \times S \to \mathcal{R}^n$ is locally Lipschitz continuous and satisfies the linear growth condition [24]. Moreover, σ satisfies

$$trace[\sigma^T(x_1, x_2, t, i)\sigma(x_1, x_2, t, i)] \leq x_1^T \Sigma_{1i} x_1 + x_2^T \Sigma_{2i} x_2 \tag{4}$$

for all $x_1, x_2 \in \mathcal{R}^n$ and $x(t) = i, i \in S$, where Σ_{1i} and Σ_{2i} are known positive constant matrices with appropriate dimensions.

(H3) The impulsive time instant t_k satisfy $0 = t_0 < t_1 < \cdots < t_k \to \infty$ and $\inf_{k \in z_+}\{t_k - t_{k-1}\} > 0$.

Definition 1. The stochastic neural networks (1) is said to be stochastically passive from input $u(t)$ to output $y(t)$, if there exists a scalar $\gamma \geq 0$ such that the following inequality holds:

$$2\mathbb{E}\left[\int_0^{t_f} y^T(s)u(s)\,\mathrm{d}s\right] \geq -\gamma\mathbb{E}\left[\int_0^{t_f} u^T(s)u(s)\,\mathrm{d}s\right] \tag{5}$$

for the solution of (1) with $x(0) = 0$. We introducing the following lemmas which are useful in the proof of the main results.

Lemma 2.1. (Gu [9]) For any positive definite matrix $M \in \mathbb{R}^{n\times n}$, scalars $h_2 > h_1 > 0$, vector function $w : [h_1, h_2] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds:

$$-(h_2 - h_1)\int_{t-h_2}^{t-h_1} w^T(s)Mw(s)\,\mathrm{d}s \leq -\left(\int_{t-h_2}^{t-h_1} w(s)\,\mathrm{d}s\right)^T M\left(\int_{t-h_2}^{t-h_1} w(s)\,\mathrm{d}s\right)$$

$$-\frac{1}{2}(h_2^2 - h_1^2)\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} w^T(s)Mw(s)\,\mathrm{d}s\mathrm{d}\theta \leq -\left(\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} w^T(s)\,\mathrm{d}s\mathrm{d}\theta\right)M \times\left(\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} w(s)\,\mathrm{d}s\mathrm{d}\theta\right)$$

Lemma 2.2. (Schur complement Boyd et al. [2]) For a symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, the following conditions are equivalent:

(1) $S < 0$,

(2) $S_{11} < 0$, and $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,

(3) $S_{22} < 0$, and $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2.3. (Boyd et al. [2]) For any matrices X, Y, the following matrix inequality holds:

$$X^T Y + Y^T X \leq X^T X + Y^T Y.$$

3. MAIN RESULTS

In this section, we will present passivity criteria for stochastic neural networks with both discrete and distributed time delays in (1). Based on Lyapunov function and stochastic analysis approach, delay-dependent passivity condition with impulsive perturbations is presented in the following theorem. For presentation convenience, we denote

$$F_1 = diag(F_1^- F_1^+, F_2^- F_2^+, \dots, F_n^- F_n^+), F_2 = diag\left(\frac{F_1^- + F_1^+}{2}, \frac{F_2^- + F_2^+}{2}, \dots, \frac{F_n^- + F_n^+}{2}\right).$$

Theorem 3.1. Assume that assumptions (H1) – (H3) hold. For given scalars τ_1, τ_2, d, δ and μ the stochastic neural network described by (1) is stochastically passive in the sense of Definition 1, for any time varying delay $\tau(t)$ and $d(t)$ satisfying (2), if there exist constant scalars $\lambda_i > 0$ $(i = 1, 2, 3)$, $\gamma > 0$, positive definite symmetric matrices P_i $(i = 1, 2, \dots, 5), Q_i, T_i$ $(i = 1, 2, \dots, 4), X_1, X_2$, $\angle = \begin{bmatrix} L_1 & L_2 \\ L_2^T & L_3 \end{bmatrix}$, $\Re = \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix}$, $\Im = \begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix}$, positive diagonal matrices L, S, real matrices Z_1, Z_2, M_i, N_i, U_i $(i = 1, 2, \dots, 8)$ such that the following LMI's holds:

$$P_1 < \lambda_1 I, \tag{6}$$
$$X_1 < \lambda_2 I, \tag{7}$$
$$X_2 < \lambda_3 I, \tag{8}$$

$$\begin{bmatrix} P_1 & (I - I_k)^T P_1 \\ * & P_1 \end{bmatrix} \geq 0, \quad k \in \mathbb{Z}_+, \tag{9}$$

and

$$\Psi = \begin{bmatrix} \Psi_1 & \sqrt{\tau_1} M & \sqrt{\tau_2 - \tau_1} N & \sqrt{\tau_2 - \tau_1} U & M & N & U \\ * & -T_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -T_2 & 0 & 0 & 0 & 0 \\ * & * & * & -T_2 & 0 & 0 & 0 \\ * & * & * & * & -X_1 & 0 & 0 \\ * & * & * & * & * & -X_2 & 0 \\ * & * & * & * & * & * & -X_2 \end{bmatrix} < 0, \tag{10}$$

where

$$
\begin{aligned}
\Psi_1 &= (\Psi_{ij})_{20\times 20}\\
\Psi_{11} &= -P_1A - A^TP_1 + P_2 + \delta^2P_3 + P_4 + L_1 + \tau_1^2Q_1 + (\tau_2-\tau_1)^2Q_2 - F_1L + M_1\\
&\quad + M_1^T + [\lambda_1 + \tau_1\lambda_2 + (\tau_2-\tau_1)\lambda_3]\Sigma_1^T\Sigma_1, \quad \Psi_{12} = M_2^T - N_1 + U_1,\\
\Psi_{13} &= M_3^T - M_1 + N_1, \quad \Psi_{14} = M_4^T - U_1, \quad \Psi_{15} = P_1B + L_2 + F_2L + Z_1B + M_5^T,\\
\Psi_{16} &= P_1C + Z_1C + M_6^T,\ \Psi_{17} = M_7^T,\ \Psi_{18} = M_8^T,\ \Psi_{19} = -Z_1A,\ \Psi_{1_{10}} = A^TP_1A,\\
\Psi_{1_{12}} &= -Z_1, \quad \Psi_{1_{13}} = P_1 + Z_1, \quad \Psi_{1_{14}} = P_1D + Z_1D,\\
\Psi_{22} &= -(1-\mu)R_1 - F_1S - N_2 - N_2^T + U_2 + U_2^T + [\lambda_1 + \tau_1\lambda_2 + (\tau_2-\tau_1)\lambda_3]\Sigma_2^T\Sigma_2,\\
\Psi_{23} &= -M_2 + N_2 - N_3^T + U_3^T, \quad \Psi_{24} = -N_4^T + U_4^T - U_2, \quad \Psi_{25} = -N_5^T + U_5^T,\\
\Psi_{26} &= -(1-\mu)R_2 + F_2S - N_6^T + U_6^T, \quad \Psi_{27} = -N_7^T + U_7^T, \quad \Psi_{28} = -N_8^T + U_8^T,\\
\Psi_{33} &= -L_1 + R_1 + S_1 - M_3 - M_3^T + N_3 + N_3^T, \quad \Psi_{34} = -M_4^T + N_4^T - U_3,\\
\Psi_{35} &= -M_5^T + N_5^T, \quad \Psi_{36} = -M_6^T + N_6^T, \quad \Psi_{37} = -L_2 + R_2 + S_2 - M_7^T + N_7^T,\\
\Psi_{38} &= -M_8^T + N_8^T, \Psi_{44} = -S_1 - U_4 - U_4^T,\ \Psi_{45} = -U_5^T,\ \Psi_{46} = -U_6^T,\ \Psi_{47} = -U_7^T,\\
\Psi_{48} &= -S_2 - U_8^T,\ \Psi_{55} = d^2P_5 + L_3 + \tau_1^2Q_3 + (\tau_2-\tau_1)^2Q_4 - L,\ \Psi_{5_{10}} = -B^TP_1A,\\
\Psi_{5_{12}} &= B^TZ_2^T,\ \Psi_{5_{13}} = -I,\ \Psi_{66} = -(1-\mu)R_3 - S,\ \Psi_{6_{10}} = -C^TP_1A,\ \Psi_{6_{12}} = C^TZ_2^T,\\
\Psi_{77} &= -L_3 + R_3 + S_3,\ \Psi_{88} = -S_3,\ \Psi_{99} = -P_2,\ \Psi_{9_{12}} = -A^TZ_2^T,\ \Psi_{10_{10}} = -P_3,\\
\Psi_{10_{13}} &= -A^TP_1, \quad \Psi_{10_{14}} = -A^TP_1D, \quad \Psi_{11_{11}} = -P_4 + [\lambda_1 + \tau_1\lambda_2 + (\tau_2-\tau_1)\lambda_3]\Sigma_3^T\Sigma_3,\\
\Psi_{12_{12}} &= \tau_1T_1 + (\tau_2-\tau_1)T_2 + \frac{\tau_1^4}{4}T_3 + \frac{(\tau_2^2-\tau_1^2)^2}{4}T_4 - Z_2 - Z_2^T, \quad \Psi_{12_{13}} = Z_2,\\
\Psi_{12_{14}} &= Z_2D, \quad \Psi_{13_{13}} = -\gamma I, \quad \Psi_{14_{14}} = -P_5, \quad \Psi_{15_{15}} = -Q_1, \quad \Psi_{16_{16}} = -Q_2,\\
\Psi_{17_{17}} &= -Q_3, \quad \Psi_{18_{18}} = -Q_4, \quad \Psi_{19_{19}} = -T_3, \quad \Psi_{20_{20}} = -T_4
\end{aligned}
$$

$$
\begin{aligned}
M^T &= \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T & M_5^T & M_6^T & M_7^T & M_8^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\\
N^T &= \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T & N_5^T & N_6^T & N_7^T & N_8^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\\
U^T &= \begin{bmatrix} U_1^T & U_2^T & U_3^T & U_4^T & U_5^T & U_6^T & U_7^T & U_8^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

P r o o f. For simplicity, we denote

$$
\begin{aligned}
g(t) &= -Ax(t-\delta) + Bg(x(t)) + Cg(x(t-\tau(t))) + D\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s + u(t), \quad (11)\\
\alpha(t) &= \sigma(t, x(t), x(t-\tau(t)), x(t-d(t))), \quad (12)
\end{aligned}
$$

then system (1) can be rewritten as

$$
dx(t) = g(t)dt + \alpha(t)dw(t). \quad (13)
$$

Choose a Lyapunov functional candidate for the system (1) to be

$$V(x_t) = \sum_{i=1}^{9} V_i(x_t) \tag{14}$$

where

$$\begin{aligned}
V_1(x_t) &= \left[x(t) - A\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right]^T P_1 \left[x(t) - A\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right] \\
V_2(x_t) &= \int_{t-\delta}^{t} x^T(s)P_2x(s)\,\mathrm{d}s + \delta\int_{-\delta}^{0}\int_{t+\theta}^{t} x^T(s)P_3x(s)\,\mathrm{d}s\mathrm{d}\theta \\
V_3(x_t) &= \int_{t-d(t)}^{t} x^T(s)P_4x(s)\,\mathrm{d}s + d\int_{-d}^{0}\int_{t+\theta}^{t} g^T(x(s))P_5g(x(s))\,\mathrm{d}s\mathrm{d}\theta \\
V_4(x_t) &= \int_{t-\tau_1}^{t} \varphi^T(s)\angle\varphi(s)\,\mathrm{d}s + \int_{t-\tau(t)}^{t-\tau_1} \varphi^T(s)\,\Re\,\varphi(s)\,\mathrm{d}s \\
&\quad + \int_{t-\tau_2}^{t-\tau_1} \varphi^T(s)\,\Im\,\varphi(s)\,\mathrm{d}s \\
V_5(x_t) &= \tau_1\int_{-\tau_1}^{0}\int_{t+\theta}^{t} x^T(s)Q_1x(s)\,\mathrm{d}s\mathrm{d}\theta + (\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} x^T(s)Q_2x(s)\,\mathrm{d}s\mathrm{d}\theta \\
V_6(x_t) &= \tau_1\int_{-\tau_1}^{0}\int_{t+\theta}^{t} g^T(x(s))Q_3g(x(s))\,\mathrm{d}s\mathrm{d}\theta \\
&\quad +(\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} g^T(x(s))Q_4g(x(s))\,\mathrm{d}s\mathrm{d}\theta \\
V_7(x_t) &= \int_{-\tau_1}^{0}\int_{t+\theta}^{t} g^T(s)T_1g(s)\,\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} g^T(s)T_2g(s)\,\mathrm{d}s\mathrm{d}\theta \\
V_8(x_t) &= \frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} g^T(s)T_3g(s)\,\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta \\
&\quad +\frac{\tau_2^2-\tau_1^2}{2}\int_{-\tau_2}^{-\tau_1}\int_{\theta}^{0}\int_{t+\lambda}^{t} g^T(s)T_4g(s)\,\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta \\
V_9(x_t) &= \int_{-\tau_1}^{0}\int_{t+\theta}^{t} tr(\alpha^T(s)X_1\alpha(s))\,\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} tr(\alpha^T(s)X_2\alpha(s))\,\mathrm{d}s\mathrm{d}\theta
\end{aligned}$$

and

$$\varphi(s) = \begin{bmatrix} x(s) \\ g(x(s)) \end{bmatrix}.$$

Then, it can be obtained by *Itô*'s formula that

$$dV(x_t,t) = \mathcal{L}V(x_t,t)dt + 2x^T(t)P_1\alpha(t)\,\mathrm{d}\omega(t) \tag{15}$$

where

$$\begin{aligned}
\mathcal{L}V_1(x_t,t) &= 2\Big[x(t) - A\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\Big]^T P_1\Big[-Ax(t) + Bg(x(t)) + Cg(x(t-\tau(t))) \\
&\quad + D\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s + u(t)\Big] + tr(\alpha^T(t)P_1\alpha(t)) \qquad (16)
\end{aligned}$$

$$\mathcal{L}V_2(x_t,t) = x^T(t)[P_2 + \delta^2 P_3]x(t) - x^T(t-\delta)P_2x(t-\delta) - \delta\int_{t-\delta}^{t} x^T(s)P_3x(s)\,\mathrm{d}s \qquad (17)$$

$$\begin{aligned}
\mathcal{L}V_3(x_t,t) &= x^T(t)P_4x(t) - x^T(t-d(t))P_4x(t-d(t)) + d^2g^T(x(t))P_5g(x(t)) \\
&\quad - d\int_{t-d}^{t} g^T(x(s))P_5g(x(s))\,\mathrm{d}s \qquad (18)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}V_4(x_t,t) &= \varphi^T(t)\angle\varphi(t) - \varphi^T(t-\tau_1)\angle\varphi(t-\tau_1) + \varphi^T(t-\tau_1)\Re\varphi(t-\tau_1) \\
&\quad -(1-\mu)\varphi^T(t-\tau(t))\Re\varphi(t-\tau(t)) + \varphi^T(t-\tau_1)\Im\varphi(t-\tau_1) \\
&\quad -\varphi^T(t-\tau_2)\Im\varphi(t-\tau_2) \\
&\leq \begin{bmatrix} x(t) \\ g(x(t)) \end{bmatrix}^T \begin{bmatrix} L_1 & L_2 \\ L_2^T & L_3 \end{bmatrix} \begin{bmatrix} x(t) \\ g(x(t)) \end{bmatrix} - \begin{bmatrix} x(t-\tau_1) \\ g(x(t-\tau_1)) \end{bmatrix}^T \begin{bmatrix} L_1 & L_2 \\ L_2^T & L_3 \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ g(x(t-\tau_1)) \end{bmatrix} \\
&\quad + \begin{bmatrix} x(t-\tau_1) \\ g(x(t-\tau_1)) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ g(x(t-\tau_1)) \end{bmatrix} \\
&\quad -(1-\mu) \begin{bmatrix} x(t-\tau(t)) \\ g(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ g(x(t-\tau(t))) \end{bmatrix} \\
&\quad + \begin{bmatrix} x(t-\tau_1) \\ g(x(t-\tau_1)) \end{bmatrix}^T \begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ g(x(t-\tau_1)) \end{bmatrix} \\
&\quad - \begin{bmatrix} x(t-\tau_2) \\ g(x(t-\tau_2)) \end{bmatrix}^T \begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix} \begin{bmatrix} x(t-\tau_2) \\ g(x(t-\tau_2)) \end{bmatrix} \qquad (19)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}V_5(x_t,t) &= x^T(t)[\tau_1^2Q_1 + (\tau_2-\tau_1)^2Q_2]x(t) - \tau_1\int_{t-\tau_1}^{t} x^T(s)Q_1x(s)\,\mathrm{d}s \\
&\quad -(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1} x^T(s)Q_2x(s)\,\mathrm{d}s \qquad (20)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}V_6(x_t,t) &= g^T(x(t))[\tau_1^2Q_3 + (\tau_2-\tau_1)^2Q_4]g(x(t)) - \tau_1\int_{t-\tau_1}^{t} g^T(x(s))Q_3g(x(s))\,\mathrm{d}s \\
&\quad -(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1} g^T(x(s))Q_4g(x(s))\,\mathrm{d}s \qquad (21)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}V_7(x_t,t) &= g^T(t)[\tau_1T_1 + (\tau_2-\tau_1)T_2]g(t) - \int_{t-\tau_1}^{t} g^T(s)T_1g(s)\,\mathrm{d}s - \int_{t-\tau_2}^{t-\tau_1} g^T(s)T_2g(s)\,\mathrm{d}s \\
&= g^T(t)[\tau_1T_1 + (\tau_2-\tau_1)T_2]g(t) - \int_{t-\tau_1}^{t} g^T(s)T_1g(s)\,\mathrm{d}s \\
&\quad - \int_{t-\tau(t)}^{t-\tau_1} g^T(s)T_2g(s)\,\mathrm{d}s - \int_{t-\tau_2}^{t-\tau(t)} g^T(s)T_2g(s)\,\mathrm{d}s \qquad (22)
\end{aligned}$$

$$
\begin{aligned}
\mathcal{L}V_8(x_t,t) &= \frac{\tau_1^4}{4}g^T(t)T_3g(t) - \frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t} g^T(s)T_3g(s)\,\mathrm{d}s\mathrm{d}\theta + \frac{(\tau_2^2-\tau_1^2)^2}{4}g^T(t)T_4g(t) \\
&\quad -\Big(\frac{\tau_2^2-\tau_1^2}{2}\Big)\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} g^T(s)T_4g(s)\,\mathrm{d}s\mathrm{d}\theta \qquad (23)\\
\mathcal{L}V_9(x_t,t) &= \tau_1\ tr(\alpha^T(t)X_1\alpha(t)) + (\tau_2-\tau_1)\ tr(\alpha^T(t)X_2\alpha(t)) \\
&\quad -\int_{t-\tau_1}^{t} tr(\alpha^T(s)X_1\alpha(s))\,\mathrm{d}s - \int_{t-\tau_2}^{t-\tau_1} tr(\alpha^T(s)X_2\alpha(s))\,\mathrm{d}s \\
&\le \tau_1\lambda_2\ tr(\alpha^T(t)X_1\alpha(t)) + (\tau_2-\tau_1)\lambda_3\ tr(\alpha^T(t)X_2\alpha(t)) \\
&\quad -\int_{t-\tau_1}^{t} tr(\alpha^T(s)X_1\alpha(s))\,\mathrm{d}s \\
&\quad -\int_{t-\tau(t)}^{t-\tau_1} tr(\alpha^T(s)X_2\alpha(s))\,\mathrm{d}s - \int_{t-\tau_2}^{t-\tau(t)} tr(\alpha^T(s)X_2\alpha(s))\,\mathrm{d}s. \qquad (24)
\end{aligned}
$$

From Lemma 2.1, one can obtain

$$
\begin{aligned}
-\,\delta\int_{t-\delta}^{t} x^T(s)P_3x(s)\,\mathrm{d}s &\le -\Big(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\Big)^T P_3\Big(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\Big) \qquad (25)\\
-d\int_{t-d}^{t} g^T(x(s))P_5g(x(s)))\,\mathrm{d}s &\le -\Big(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\Big)^T P_5\Big(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\Big) \qquad (26)\\
-\tau_1\int_{t-\tau_1}^{t} x^T(s)Q_1x(s)\,\mathrm{d}s &\le -\Big(\int_{t-\tau_1}^{t} x(s)\,\mathrm{d}s\Big)^T Q_1\Big(\int_{t-\tau_1}^{t} x(s)\,\mathrm{d}s\Big) \qquad (27)\\
-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1} x^T(s)Q_2x(s)\,\mathrm{d}s &\le -\Big(\int_{t-\tau_2}^{t-\tau_1} x(s)\,\mathrm{d}s\Big)^T Q_2\Big(\int_{t-\tau_2}^{t-\tau_1} x(s)\,\mathrm{d}s\Big) \qquad (28)\\
-\tau_1\int_{t-\tau_1}^{t} g^T(x(s))Q_3g(x(s))\,\mathrm{d}s &\le -\Big(\int_{t-\tau_1}^{t} g(x(s))\,\mathrm{d}s\Big)^T Q_3\Big(\int_{t-\tau_1}^{t} g(x(s))\,\mathrm{d}s\Big) \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
-\,(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1} g^T(x(s))Q_4g(x(s))\,\mathrm{d}s &\le -\Big(\int_{t-\tau_2}^{t-\tau_1} g(x(s))\,\mathrm{d}s\Big)^T Q_4 \\
&\quad \times\Big(\int_{t-\tau_2}^{t-\tau_1} g(x(s))\,\mathrm{d}s\Big) \qquad (30)\\
-\frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t} g^T(s)T_3g(s)\,\mathrm{d}s\,\mathrm{d}\theta &\le -\Big(\int_{-\tau_1}^{0}\int_{t+\theta}^{t} g(s)\,\mathrm{d}s\mathrm{d}\theta\Big)^T \\
&\quad \times T_3\ \Big(\int_{-\tau_1}^{0}\int_{t+\theta}^{t} g(s)\,\mathrm{d}s\mathrm{d}\theta\Big) \qquad (31)\\
-\Big(\frac{\tau_2^2-\tau_1^2}{2}\Big)\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} g^T(s)T_4g(s)\,\mathrm{d}s\mathrm{d}\theta &\le -\Big(\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} g(s)\,\mathrm{d}s\mathrm{d}\theta\Big)^T \\
&\quad \times T_4\ \Big(\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} g(s)\,\mathrm{d}s\mathrm{d}\theta\Big). \qquad (32)
\end{aligned}
$$

For positive diagonal matrices L and S, we can get from Assumption (H1) that

$$0 \leq \begin{bmatrix} x(t) \\ g(x(t)) \end{bmatrix}^T \begin{bmatrix} -F_1L & F_2L \\ F_2L & -L \end{bmatrix} \begin{bmatrix} x(t) \\ g(x(t)) \end{bmatrix}, \tag{33}$$

$$0 \leq \begin{bmatrix} x(t-\tau(t)) \\ g(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} -F_1S & F_2S \\ F_2S & -S \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ g(x(t-\tau(t))) \end{bmatrix}. \tag{34}$$

From (11) – (13) the following equations are true for any matrices M, N, U, Z_i (i = 1,2,3,4) we have the following equations:

$$0 = 2\zeta^T(t)M\Big[x(t) - x(t-\tau_1) - \int_{t-\tau_1}^{t} f(s)\,\mathrm{d}s - \int_{t-\tau_1}^{t} \alpha(s)\,\mathrm{d}\omega(s)\Big] \tag{35}$$

$$0 = 2\zeta^T(t)N\Big[x(t-\tau_1) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_1} f(s)\,\mathrm{d}s - \int_{t-\tau(t)}^{t-\tau_1} \alpha(s)\,\mathrm{d}\omega(s)\Big] \tag{36}$$

$$0 = 2\zeta^T(t)U\Big[x(t-\tau(t)) - x(t-\tau_2) - \int_{t-\tau_2}^{t-\tau(t)} f(s)\,\mathrm{d}s - \int_{t-\tau_2}^{t-\tau(t)} \alpha(s)\,\mathrm{d}\omega(s)\Big] \tag{37}$$

$$\begin{aligned} 0 = {} & 2[x^T(t)Z_1 + g^T(t)Z_2] \times [-Ax(t-\delta) + Bg(x(t)) + Cg(x(t-\tau(t)) \\ & +D\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s + u(t) - g(t)] \end{aligned} \tag{38}$$

where

$$\begin{aligned} \zeta^T(t) = \Big[& x^T(t) \quad x^T(t-\tau(t)) \quad x^T(t-\tau_1) \quad x^T(t-\tau_2) \quad g^T(x(t)) \quad g^T(x(t-\tau(t))) \\ & g^T(x(t-\tau_1)) \quad g^T(x(t-\tau_2)) \quad x^T(t-\delta) \quad \int_{t-\delta}^{t} x^T(s)\,\mathrm{d}s \quad x^T(t-d(t)) \\ & g^T(t) \quad u^T(t)\Big(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\Big)^T \quad \Big(\int_{t-\tau_1}^{t} x(s)\,\mathrm{d}s\Big)^T \quad \Big(\int_{t-\tau_2}^{t-\tau_1} x(s)\,\mathrm{d}s\Big)^T \\ & \Big(\int_{t-\tau_1}^{t} g(x(s))\,\mathrm{d}s\Big)^T \Big(\int_{t-\tau_2}^{t-\tau_1} g(x(s))\,\mathrm{d}s\Big)^T \quad \Big(\int_{-\tau_1}^{0}\int_{t+\theta}^{t} g(s)\,\mathrm{d}s\mathrm{d}\theta\Big)^T \\ & \Big(\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} g(s)\,\mathrm{d}s\mathrm{d}\theta\Big)^T\Big] \end{aligned}$$

From the formula (35) – (37), we have

$$\begin{aligned} -2\zeta^T(t)M\int_{t-\tau_1}^{t} \alpha(s)\,\mathrm{d}\omega(s) \leq {} & \zeta^T(t)MX_1^{-1}M^T\zeta(t) + \Big(\int_{t-\tau_1}^{t} \alpha(s)\,\mathrm{d}\omega(s)\Big)^T X_1 \\ & \times\Big(\int_{t-\tau_1}^{t} \alpha(s)\,\mathrm{d}\omega(s)\Big) \end{aligned} \tag{39}$$

$$-2\zeta^T(t)N\int_{t-\tau(t)}^{t-\tau_1}\alpha(s)\,\mathrm{d}\omega(s) \le \zeta^T(t)NX_2^{-1}N^T\zeta(t)+\Big(\int_{t-\tau(t)}^{t-\tau_1}\alpha(s)\,\mathrm{d}\omega(s)\Big)^T X_2 \times\Big(\int_{t-\tau(t)}^{t-\tau_1}\alpha(s)\,\mathrm{d}\omega(s)\Big) \tag{40}$$

$$-2\zeta^T(t)U\int_{t-\tau_2}^{t-\tau(t)}\alpha(s)\,\mathrm{d}\omega(s) \le \zeta^T(t)UX_2^{-1}U^T\zeta(t)+\Big(\int_{t-\tau_2}^{t-\tau(t)}\alpha(s)\,\mathrm{d}\omega(s)\Big)^T X_2 \times\Big(\int_{t-\tau_2}^{t-\tau(t)}\alpha(s)\,\mathrm{d}\omega(s)\Big). \tag{41}$$

On the other hand, from the Itô isometry in [24], we can obtain

$$\mathbb{E}\Big\{\Big[\int_{t-\tau_1}^{t}\alpha(s)\,\mathrm{d}\omega(s)\Big]^T X_1\Big[\int_{t-\tau_1}^{t}\alpha(s)\,\mathrm{d}\omega(s)\Big]\Big\} = \mathbb{E}\Big\{\int_{t-\tau_1}^{t} tr[\alpha^T(s)X_1\alpha(s)]\,\mathrm{d}s\Big\} \tag{42}$$

$$\mathbb{E}\Big\{\Big[\int_{t-\tau(t)}^{t-\tau_1}\alpha(s)\,\mathrm{d}\omega(s)\Big]^T X_2\Big[\int_{t-\tau(t)}^{t-\tau_1}\alpha(s)\,\mathrm{d}\omega(s)\Big]\Big\} = \mathbb{E}\Big\{\int_{t-\tau(t)}^{t-\tau_1} tr[\alpha^T(s)X_2\alpha(s)]\,\mathrm{d}s\Big\} \tag{43}$$

$$\mathbb{E}\Big\{\Big[\int_{t-\tau_2}^{t-\tau(t)}\alpha(s)\,\mathrm{d}\omega(s)\Big]^T X_2\Big[\int_{t-\tau_2}^{t-\tau(t)}\alpha(s)\,\mathrm{d}\omega(s)\Big]\Big\} = \mathbb{E}\Big\{\int_{t-\tau_2}^{t-\tau(t)} tr[\alpha^T(s)X_2\alpha(s)]\,\mathrm{d}s\Big\} \tag{44}$$

Substituting (16)−(44) into (15), and by the mathematical expectation

$$\begin{aligned}
&\mathbb{E}\big\{\mathcal{L}V(x_t,t)-2y^T(t)u(t)-\gamma u^T(t)u(t)\big\}\\
&\le \mathbb{E}\zeta^T(t)\big\{\Psi_1+\tau_1 MT_1^{-1}M^T+(\tau_2-\tau_1)NT_2^{-1}N^T+(\tau_2-\tau_1)UT_2^{-1}U^T\\
&\qquad +MX_1^{-1}M^T+NX_2^{-1}N^T+UX_2^{-1}U^T\big\}\zeta(t)\\
&\quad -\int_{t-\tau_1}^{t}\Big[\zeta^T(t)M+g^T(s)T_1\Big]T_1^{-1}\Big[M^T\zeta(t)+T_1g(s)\Big]\,\mathrm{d}s\\
&\quad -\int_{t-\tau(t)}^{t-\tau_1}\Big[\zeta^T(t)N+g^T(s)T_2\Big]T_2^{-1}\Big[N^T\zeta(t)+T_2g(s)\Big]\,\mathrm{d}s\\
&\quad -\int_{t-\tau_2}^{t-\tau(t)}\Big[\zeta^T(t)U+g^T(s)T_2\Big]T_2^{-1}\Big[U^T\zeta(t)+T_2g(s)\Big]\,\mathrm{d}s
\end{aligned} \tag{45}$$

where Ψ_1 is defined in (10).
Since last three terms in (45) are less than 0 , and we can obtain

$$\begin{aligned}
&\mathbb{E}\big\{\mathcal{L}V(x_t,t)-2y^T(t)u(t)-\gamma u^T(t)u(t)\big\}\\
&\le \mathbb{E}\zeta^T(t)\big\{\Psi_1+\tau_1 MT_1^{-1}M^T+(\tau_2-\tau_1)NT_2^{-1}N^T+(\tau_2-\tau_1)UT_2^{-1}U^T\\
&\qquad +MX_1^{-1}M^T+NX_2^{-1}N^T+UX_2^{-1}U^T\big\}\zeta(t).
\end{aligned} \tag{46}$$

Now we consider the change of $V(x_t)$ at impulse time $t=t_k,\ k\in\mathbb{Z}_+$. From (1) we have

$$x(t_k)-A\int_{t_k-\delta}^{t_k}x(s)\,\mathrm{d}s = x(t_k^-)-I_k\Big[x(t_k^-)-A\int_{t_k-\delta}^{t_k}x(s)\,\mathrm{d}s\Big]-A\int_{t_k-\delta}^{t_k}x(s)\,\mathrm{d}s$$

$$= (I - I_k)\Big[x(t_k^-) - A\int_{t_k-\delta}^{t_k} x(s)\,\mathrm{d}s\Big]. \tag{47}$$

Moreover, it follows that from (9) that

$$\begin{bmatrix} P_1 & (I-I_k)^T P_1 \\ * & P_1 \end{bmatrix} \geq 0$$

$$\Leftrightarrow \begin{bmatrix} I & -(I-I_k)^T \\ 0 & I \end{bmatrix} \begin{bmatrix} P_1 & (I-I_k)^T P_1 \\ * & P_1 \end{bmatrix} \begin{bmatrix} I & 0 \\ -(I-I_k) & I \end{bmatrix} \geq 0$$

$$\Leftrightarrow \begin{bmatrix} P_1 - (I-I_k)^T P_1 (I-I_k) & 0 \\ * & P_1 \end{bmatrix} \geq 0$$

$$\Leftrightarrow P_1 - (I-I_k)^T P_1 (I-I_k) \geq 0. \tag{48}$$

Together with (47) and (48), it yields

$$\begin{aligned} V_1(x(t_k)) \;&=\; \Big[x(t_k) - A\int_{t_k-\delta}^{t_k} x(s)\,\mathrm{d}s\Big]^T P_1 \Big[x(t_k) - A\int_{t_k-\delta}^{t_k} x(s)\,\mathrm{d}s\Big] \\ &=\; \Big[x(t_k^-) - A\int_{t_k-\delta}^{t_k} x(s)\,\mathrm{d}s\Big]^T (I-I_k)^T P_1 (I-I_k) \Big[x(t_k^-) - A\int_{t_k-\delta}^{t_k} x(s)\,\mathrm{d}s\Big] \\ &\leq\; \Big[x(t_k^-) - A\int_{t_k-\delta}^{t_k} x(s)\,\mathrm{d}s\Big]^T P_1 \Big[x(t_k^-) - A\int_{t_k-\delta}^{t_k} x(s)\,\mathrm{d}s\Big] \\ V_1(x(t_k)) \;&=\; V_1(x(t_k^-)). \end{aligned}$$

Moreover it is obvious that $V_2(t_k) = V_2(t_k^-)$, $V_3(t_k) = V_3(t_k^-)$, $V_4(t_k) = V_4(t_k^-)$, $V_5(t_k) = V_5(t_k^-)$, $V_6(t_k) = V_6(t_k^-)$, $V_7(t_k) = V_7(t_k^-)$, $V_8(t_k) = V_8(t_k^-)$, $V_9(t_k) = V_9(t_k^-)$.
which implies that

$$V(x(t_k)) \;=\; V(x(t_k^-)),\;\; k \in \mathbb{Z}_+. \tag{49}$$

It follows from (46) that

$$\begin{aligned} &\mathbb{E}dV(x_t,t) - 2\mathbb{E}y^T(t)u(t) - \gamma\mathbb{E}u^T(t)u(t) \\ &\quad = \mathbb{E}\mathcal{L}V(x_t,t) - 2\mathbb{E}y^T(t)u(t) - \gamma\mathbb{E}u^T(t)u(t) \\ &\quad\;\; \leq \mathbb{E}\zeta^T(t)\Big[\Psi_1 + \tau_1 M T_1^{-1} M^T + (\tau_2-\tau_1) N T_2^{-1} N^T + (\tau_2-\tau_1) U T_2^{-1} U^T \\ &\qquad\qquad + M X_1^{-1} M^T + N X_2^{-1} N^T + U X_2^{-1} U^T\Big\}\zeta(t). \end{aligned}$$

Let

$$\hat{\Psi} = \Psi_1 + \tau_1 M T_1^{-1} M^T + (\tau_2-\tau_1) N T_2^{-1} N^T + (\tau_2-\tau_1) U T_2^{-1} U^T$$

$$+MX_1^{-1}M^T + NX_2^{-1}N^T + UX_2^{-1}U^T.$$

By applying Schur complement [2], it is easy to see that $\hat{\Psi}$ is equivalent to (10), then we can obtain

$$\mathbb{E}dV(x_t, t) - 2\mathbb{E}y^T(t)u(t) - \gamma u^T(t)u(t) \leq 0. \tag{50}$$

By integrating (50) over the time period from 0 to t_f, we have

$$2\mathbb{E}\int_0^{t_f} y^T(s)u(s)\,\mathrm{d}s \geq \mathbb{E}V(x_0, 0) - \gamma\mathbb{E}\int_0^{t_f} u^T(s)u(s)\,\mathrm{d}s \geq -\gamma\mathbb{E}\int_0^{t_f} u^T(s)u(s)\,\mathrm{d}s. \tag{51}$$

From Definition 1, we know that the stochastic neural network (1) is passive in the sense of expectation. This completes the proof. □

Remark 1. In the proof of Theorem 3.1, we introduce a new estimation on the upper bound of the time derivative of V(t). For this, we introduce the following inequalities,

$$\tau_1\zeta^T(t)MT_1^{-1}M^T\zeta(t) - \int_{t-\tau_1}^{t} \zeta^T(t)MT_1^{-1}M^T\zeta(t)\,\mathrm{d}s \geq 0,$$
$$(\tau_2 - \tau_1)NT_2^{-1}N^T\zeta(t) - \int_{t-\tau(t)}^{t-\tau_1} \zeta^T(t)NT_2^{-1}N^T\zeta(t)\,\mathrm{d}s \geq 0,$$
$$(\tau_2 - \tau_1)UT_2^{-1}U^T\zeta(t) - \int_{t-\tau_2}^{t-\tau(t)} \zeta^T(t)UT_2^{-1}U^T\zeta(t)\,\mathrm{d}s \geq 0,$$

where it is employed in [10]; and $\tau(t) - \tau_1$, $\tau_2 - \tau(t)$ were enlarged to $\tau_2 - \tau_1$. It is easy to see that this treatment is more conservative than the expression in the proof of Theorem 3.1.

Remark 2. It should be pointed out that the range of the time-varying delays in [36] is varying from 0 to upper bounds. However, in many practical cases [5, 36, 37], the time delays may typically exist on intervals, where the lower bounds of the time-varying delays are not restricted to be 0. In this work, the time-varying delays are assumed to be intervals, which means that the lower and upper bounds of interval time-varying delay is available, where $\tau(t) \in [\tau_1, \tau_2]$. On the other hand distributed delays, parameter uncertainties and impulsive perturbations are considered; see Theorem 3.1.

Remark 3. In order to reduce the conservativeness, when obtaining the derivative of $V_7(x_t, t)$, $V_9(x_t, t)$ the integral terms $\int_{t-\tau_2}^{t-\tau_1} g^T(s)T_2g(s)\,\mathrm{d}s$, $\int_{t-\tau_2}^{t-\tau_1} \alpha^T(s)X_2\alpha(s)\,\mathrm{d}s$ is divided into two parts as $\int_{t-\tau(t)}^{t-\tau_1} g^T(s)T_2g(s)\,\mathrm{d}s$, $\int_{t-\tau_2}^{t-\tau(t)} g^T(s)T_2g(s)\,\mathrm{d}s$ and $\int_{t-\tau(t)}^{t-\tau_1} \alpha^T(s)X_2$ $\alpha(s)\,\mathrm{d}s$, $\int_{t-\tau_2}^{t-\tau(t)} \alpha^T(s)X_2\alpha(s)\,\mathrm{d}s$ respectively, which is mainly based on the information about $\tau_1 \leq \tau(t) \leq \tau_2$, which may leads to less conservative results.

Remark 4. In Assumption (H1), the constants F_j^- and F_j^+ $j = 1, 2, \ldots, n$ are allowed to be positive, negative or zero. However, in [5, 26, 35], the Lipschitz constants are only allowed to be positive. Hence, Assumption(H1), first proposed by Liu et al.in [23], is weaker than the assumption in [5, 26, 35].

4. PASSIVITY FOR UNCERTAIN STOCHASTIC NEURAL NETWORKS WITH IMPULSES

In this section, we extend the previous passivity condition to the following uncertain stochastic neural network:

$$\begin{aligned} dx(t) &= \Big[-(A+\Delta A(t))x(t-\delta)+(B+\Delta B(t))g(x(t))+(C+\Delta C(t))g(x(t-\tau(t))) \\ &\quad +(D+\Delta D(t))\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s+u(t)\Big]\,\mathrm{d}t \\ &\quad +\sigma(t,x(t),x(t-\tau(t)),x(t-d(t))\,\mathrm{d}\omega(t), \qquad t\neq t_k \\ y(t) &= g(x(t)) \\ \Delta x(t_k) &= -I_k\Big\{x(t_k^-)-A\int_{t_k-\delta}^{t_k} x(s)\,\mathrm{d}s\Big\}, \qquad t=t_k, \quad k\in\mathbb{Z}_+, \end{aligned} \tag{52}$$

where $\Delta A(t), \Delta B(t), \Delta C(t), \Delta D(t)$ are the time varying uncertainties of the form:

$$\begin{bmatrix}\Delta A(t) & \Delta B(t) & \Delta C(t) & \Delta D(t)\end{bmatrix} = HF(t)\begin{bmatrix}G_1 & G_2 & G_3 & G_4\end{bmatrix} \tag{53}$$

where $H, G_i (i = 1,2,3,4)$ are known real constant matrices, F(t) is the time-varying uncertain matrices, which satisfies $F^T(t)F(t) \le I$.

Theorem 4.1. Assume that assumptions (H1) − (H3) holds. For given scalars τ_1, τ_2, μ and d, the stochastic neural network described by (52) is stochastically passive in the sense of Definition 1, for any time varying delay $\tau(t)$ and $d(t)$ satisfying (2), if there exist constant scalars $\lambda_i > 0$ $(i = 1,2,3)$, $\gamma > 0$, positive definite symmetric matrices P_i $(i = 1,2,\ldots,5), Q_i, T_i$ $(i = 1,2,\ldots,4), X_1, X_2$, $\angle = \begin{bmatrix}L_1 & L_2\\ L_2^T & L_3\end{bmatrix}$, $\Re = \begin{bmatrix}R_1 & R_2\\ R_2^T & R_3\end{bmatrix}$, $\Im = \begin{bmatrix}S_1 & S_2\\ S_2^T & S_3\end{bmatrix}$, positive diagonal matrices L, S, real matrices $Z_1, Z_2, M_i, N_i, U_i (i = 1,2,\ldots,8)$ such that the following LMI holds:

$$P_1 < \lambda_1 I, \tag{54}$$
$$X_1 < \lambda_2 I, \tag{55}$$
$$X_2 < \lambda_3 I, \tag{56}$$

$$\begin{bmatrix}P_1 & (I-I_k)^T P_1\\ * & P_1\end{bmatrix} \ge 0, \quad k\in\mathbb{Z}_+, \tag{57}$$

and

$$\Psi = \begin{pmatrix} \Psi_1 & \sqrt{\tau_1}M & \sqrt{\tau_2-\tau_1}N & \sqrt{\tau_2-\tau_1}U & M & N & U & \Gamma_{d1} & \Gamma_{d2} & \Gamma_{d3}\\ * & -T_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & -T_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -T_2 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -X_1 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & -X_2 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & -X_2 & 0 & 0 & 0\\ * & * & * & * & * & * & * & -\frac{1}{4}I & 0 & 0\\ * & * & * & * & * & * & * & * & -\frac{1}{4}I & 0\\ * & * & * & * & * & * & * & * & * & -\frac{1}{4}I \end{pmatrix} < 0, \tag{58}$$

where

$$\Psi_{55} = d^2P_5 + L_3 + \tau_1^2 Q_3 + (\tau_2 - \tau_1)^2 Q_4 - L + 4G_2^T G_2,$$
$$\Psi_{66} = -(1-\mu)R_3 - S + 4G_3^T G_3,$$
$$\Psi_{99} = -P_2 + 4G_1^T G_1, \quad \Psi_{14_{14}} = -P_5 + 4G_4^T G_4.$$

$$\Gamma_{d1} = col\left[\begin{array}{cccccccccccccccccc} P_1H + Z_1H & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$
$$\Gamma_{d2} = col\left[\begin{array}{cccccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_2H & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$
$$\Gamma_{d3} = col\left[\begin{array}{cccccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A^TP_1H & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

and other terms are same as defined in Theorem 3.1.

Proof. It is not difficult to check that system (52) is equivalent to the following form:

$$\begin{aligned} d\left[x(t) - A\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right] = \Big\{ & -Ax(t) - \Delta A(t)x(t-\delta) + [B + \Delta B(t))g(x(t))] \\ & +[C + \Delta C(t))g(x(t-\tau(t))) + [D + \Delta D(t))] \\ & \times \int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s + u(t)\Big\}\,\mathrm{d}t \\ & +\sigma(t, x(t), x(t-\tau(t), x(t-d(t)))\,\mathrm{d}\omega(t). \end{aligned} \tag{59}$$

Then, from Theorem 3.1, we only need to estimate the following equalities:

$$\begin{aligned} \mathcal{L}V_1(x_t, t) &= 2\Big[x(t) - A\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\Big]^T P_1\Big[-Ax(t) - Ax(t-\delta) \\ &\quad +(B+\Delta B)g(x(t)) + (C+\Delta C)g(x(t-\tau(t)) \\ &\quad +(D+\Delta D)\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s + u(t)\Big] + \alpha^T(t)P_1\alpha(t) \end{aligned} \tag{60}$$

$$\begin{aligned} 0 &= 2[x^T(t)Z_1 + f^T(t)Z_2] \times [-(A+\Delta A)x(t-\delta) + (B+\Delta B)g(x(t)) \\ &\quad +(C+\Delta C)g(x(t-\tau(t)) + (D+\Delta D)\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s \\ &\quad +u(t) - g(t)]. \end{aligned} \tag{61}$$

Replace ΔA, ΔB, ΔC, ΔD with $HF(t)G_1$, $HF(t)G_2$, $HF(t)G_3$, $HF(t)G_4$ respectively, and using Lemma 2.3 in (60) and (61), we can obtain

$$\begin{aligned} -2x^T(t)P_1\Delta A(t)x(t-\delta) &= -2x^T(t)P_1HF(t)G_1x(t-\delta) \\ &\leq x^T(t)P_1HF(t)F^T(t)H^TP_1x(t) \\ &\qquad +x^T(t-\delta)G_1^TG_1x(t-\delta) \\ &\leq x^T(t)P_1HH^TP_1x(t) \\ &\qquad +x^T(t-\delta)G_1^TG_1x(t-\delta) \end{aligned} \tag{62}$$

$$\begin{aligned}
2x^T(t)P_1\Delta B(t)g(x(t)) &= 2x^T(t)P_1HF(t)G_2g(x(t)) \\
&\le x^T(t)P_1HF(t)F^T(t)H^TP_1x(t) \\
&\quad +g^T(x(t))G_2^TG_2g(x(t)) \\
&\le x^T(t)P_1HH^TP_1x(t) \\
&\quad +g^T(x(t))G_2^TG_2g(x(t)) \qquad (63)
\end{aligned}$$

$$\begin{aligned}
2x^T(t)P_1\Delta C(t)g(x(t-\tau(t))) &= 2x^T(t)P_1HF(t)G_3g(x(t-\tau(t))) \\
&\le x^T(t)P_1HF(t)F^T(t)H^TP_1x(t) \\
&\quad +g^T(x(t-\tau(t)))G_3^TG_3g(x(t-\tau(t))) \\
&\le x^T(t)P_1HH^TP_1x(t) \\
&\quad +g^T(x(t-\tau(t)))G_3^TG_3g(x(t-\tau(t))) \qquad (64)
\end{aligned}$$

$$\begin{aligned}
2x^T(t)P_1\Delta D(t)\int_{t-d(t)}^t g(x(s))\,\mathrm{d}s &= 2x^T(t)P_1HF(t)G_4\int_{t-d(t)}^t g(x(s))\,\mathrm{d}s \\
&\le x^T(t)P_1HF(t)F^T(t)H^TP_1x(t) \\
&\quad +\bigg(\int_{t-d(t)}^t g(x(s))\,\mathrm{d}s\bigg)^T G_4^TG_4 \\
&\quad \times\bigg(\int_{t-d(t)}^t g(x(s))\,\mathrm{d}s\bigg) \\
&\le x^T(t)P_1HH^TP_1x(t) \\
&\quad +\bigg(\int_{t-d(t)}^t g(x(s))\,\mathrm{d}s\bigg)^T G_4^TG_4 \\
&\quad \times\bigg(\int_{t-d(t)}^t g(x(s))\,\mathrm{d}s\bigg) \qquad (65)
\end{aligned}$$

$$\begin{aligned}
2\bigg(\int_{t-\delta}^t x(s)\,\mathrm{d}s\bigg)^T A^TP_1\Delta A(t)x(t-\delta) &= 2\bigg(\int_{t-\delta}^t x(s)\,\mathrm{d}s\bigg)^T A^TP_1HF(t)G_1x(t-\delta) \\
&\le \bigg(\int_{t-\delta}^t x(s)\,\mathrm{d}s\bigg)^T A^TP_1HF(t)F^T(t)H^TP_1A \\
&\quad \times\bigg(\int_{t-\delta}^t x(s)\,\mathrm{d}s\bigg) \\
&\quad +x^T(t-\delta)G_1^TG_1x(t-\delta) \\
&\le \bigg(\int_{t-\delta}^t x(s)\,\mathrm{d}s\bigg)^T A^TP_1HH^TP_1A \\
&\quad \times\bigg(\int_{t-\delta}^t x(s)\,\mathrm{d}s\bigg) \\
&\quad +x^T(t-\delta)G_1^TG_1x(t-\delta) \qquad (66)
\end{aligned}$$

$$2\bigg(\int_{t-\delta}^t x(s)\,\mathrm{d}s\bigg)^T A^TP_1\Delta B(t)g(x(t)) = 2\bigg(\int_{t-\delta}^t x(s)\,\mathrm{d}s\bigg)^T A^TP_1HF(t)G_2g(x(t))$$

$$\begin{aligned}
&\leq \left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right)^T A^T P_1 H F(t) F^T(t) H^T P_1 A \\
&\quad \times\left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right) \\
&\quad + g^T(x(t)) G_2^T G_2 g(x(t)) \\
&\leq \left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right)^T A^T P_1 H H^T P_1 A \\
&\quad \times\left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right) \\
&\quad + g^T(x(t)) G_2^T G_2 g(x(t))
\end{aligned} \tag{67}$$

$$\begin{aligned}
&2\left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right)^T A^T P_1 \Delta C(t) g(x(t-\tau(t))) \\
&= 2\left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right)^T A^T P_1 H F(t) G_3 g(x(t-\tau(t))) \\
&\leq \left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right)^T A^T P_1 H F(t) F^T(t) H^T P_1 A \left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right) \\
&\quad + g^T(x(t-\tau(t))) G_3^T G_3 g(x(t-\tau(t))) \\
&\leq \left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right)^T A^T P_1 H H^T P_1 A \left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right) + g^T(x(t-\tau(t))) \\
&\quad \times G_3^T G_3 g(x(t-\tau(t)))
\end{aligned} \tag{68}$$

$$\begin{aligned}
&2\left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right)^T A^T P_1 \Delta D(t) \left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right) \\
&= 2\left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right)^T A^T P_1 H F(t) G_4 \\
&\quad \times\left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right) \\
&\leq \left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right)^T A^T P_1 H F(t) F^T(t) H^T P_1 A \left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right) \\
&\quad + \left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right)^T G_4^T G_4 \left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right) \\
&\leq \left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right)^T A^T P_1 H H^T P_1 A \left(\int_{t-\delta}^{t} x(s)\,\mathrm{d}s\right) \\
&\quad + \left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right)^T G_4^T G_4 \left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right)
\end{aligned} \tag{69}$$

$$-2x^T(t) Z_1 \Delta A(t) x(t-\delta) = -2x^T(t) Z_1 H F(t) G_1 x(t-\delta)$$

$$
\begin{aligned}
&\le x^T(t)Z_1HF(t)F^T(t)H^TZ_1^Tx(t)\\
&\quad +x^T(t-\delta)G_1^TG_1x(t-\delta)\\
&\le x^T(t)Z_1HH^TZ_1^Tx(t)\\
&\quad +x^T(t-\delta)G_1^TG_1x(t-\delta) \qquad (70)
\end{aligned}
$$

$$
\begin{aligned}
2x^T(t)Z_1\Delta B(t)g(x(t)) &= 2x^T(t)Z_1HF(t)G_2g(x(t))\\
&\le x^T(t)Z_1HF(t)F^T(t)H^TZ_1^Tx(t)\\
&\quad \times + g^T(x(t))G_2^TG_2g(x(t))\\
&\le x^T(t)Z_1HH^TZ_1^Tx(t)\\
&\quad \times + g^T(x(t))G_2^TG_2g(x(t)) \qquad (71)
\end{aligned}
$$

$$
\begin{aligned}
2x^T(t)Z_1\Delta C(t)g(x(t-\tau(t))) &= 2x^T(t)Z_1HF(t)G_3g(x(t-\tau(t)))\\
&\le x^T(t)Z_1HF(t)F^T(t)H^TZ_1^Tx(t)\\
&\quad +g^T(x(t-\tau(t)))G_3^TG_3\\
&\quad \times g(x(t-\tau(t)))\\
&\le x^T(t)Z_1HH^TZ_1^Tx(t)\\
&\quad +g^T(x(t-\tau(t)))G_3^TG_3\\
&\quad \times g(x(t-\tau(t))) \qquad (72)
\end{aligned}
$$

$$
\begin{aligned}
2x^T(t)Z_1\Delta D(t)\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s &= 2x^T(t)Z_1HF(t)G_4\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\\
&\le x^T(t)Z_1HF(t)F^T(t)H^TZ_1^Tx(t)\\
&\quad +\left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right)^T G_4^TG_4\\
&\quad \times\left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right)
\end{aligned}
$$

$$
\begin{aligned}
&\le x^T(t)Z_1HH^TZ_1^Tx(t)+\left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right)^T\\
&\quad \times G_4^TG_4\left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right) \qquad (73)
\end{aligned}
$$

$$
\begin{aligned}
-2f^T(t)Z_2\Delta A(t)x(t-\delta) &= -2f^T(t)Z_2HF(t)G_1x(t-\delta)\\
&\le f^T(t)Z_2HF(t)F^T(t)H^TZ_2^Tf(t)\\
&\quad \times + x^T(t-\delta)G_1^TG_1x(t-\delta)\\
&\le f^T(t)Z_2HH^TZ_2^Tf(t)\\
&\quad +x^T(t-\delta)G_1^TG_1x(t-\delta) \qquad (74)
\end{aligned}
$$

$$
\begin{aligned}
2f^T(t)Z_2\Delta B(t)g(x(t)) &= 2f^T(t)Z_2HF(t)G_2g(x(t))\\
&\le f^T(t)Z_2HF(t)F^T(t)H^TZ_2^Tf(t)\\
&\quad +g^T(x(t))G_2^TG_2g(x(t))\\
&\le f^T(t)Z_2HH^TZ_2^Tf(t)
\end{aligned}
$$

$$
\begin{aligned}
&\qquad\qquad\qquad\qquad \times + g^T(x(t))G_2^T G_2 g(x(t)) && (75)\\
2f^T(t)Z_2\Delta C(t)g(x(t-\tau(t))) &= 2f^T(t)Z_2HF(t)G_3g(x(t-\tau(t)))\\
&\le f^T(t)Z_2HF(t)F^T(t)H^TZ_2^Tf(t)\\
&\quad +g^T(x(t-\tau(t)))G_3^TG_3g(x(t-\tau(t)))\\
&\le f^T(t)Z_2HH^TZ_2^Tf(t)\\
&\quad +g^T(x(t-\tau(t)))G_3^TG_3g(x(t-\tau(t))) && (76)\\
2f^T(t)Z_2\Delta D(t)\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s &= 2f^T(t)Z_2HF(t)G_4\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\\
&\le f^T(t)Z_2HF(t)F^T(t)H^TZ_2^Tf(t)\\
&\quad +\left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right)^T G_4^TG_4\\
&\quad \times\left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right)\\
&\le f^T(t)Z_2HH^TZ_2^Tf(t)\\
&\quad +\left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right)^T G_4^TG_4\\
&\quad \times\left(\int_{t-d(t)}^{t} g(x(s))\,\mathrm{d}s\right). && (77)
\end{aligned}
$$

Then along the same line as for Theorem 3.1, we can obtain the desired result by applying Lemma 2.2 and (62) − (77). This completes the proof of Theorem 4.1. □

Remark 5. From proof of above Theorems 3.1 and 4.1, we can see that the novelty of the Lyapunov functional contains, quadratic Lyapunov–Krasovskii functional terms in $V_4(x_t,t)$, and triple-integral terms in $V_8(x_t,t)$ are introduced, which can be expected to reduce the conservatism. More specifically to improve the feasible region for the corresponding system, by taking the states $\int_{t-\tau_1}^{t} x^T(s)\,\mathrm{d}s$, $\int_{t-\tau_2}^{t-\tau_1} x^T(s)\,\mathrm{d}s$, $\int_{t-\tau_1}^{t} g^T(x(s))\,\mathrm{d}s$, $\int_{t-\tau_2}^{t-\tau_1} g^T(x(s))\,\mathrm{d}s$, $\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} g(s)\,\mathrm{d}s\mathrm{d}\theta$, $\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \alpha^T(s)\,\mathrm{d}s\mathrm{d}\theta$, the passivity conditions in Theorems 3.1 and 4.1 sufficiently utilize more information on state variables, which can yield less conservatism.

Remark 6. In [5, 20], the authors discussed the passivity analysis of stochastic neural networks with time varying delay. But in this paper, we have studied passivity analysis of stochastic neural networks with leakage and distributed delays using impulse control. Moreover, different from the previous literature, our results are derived by constructing a new Lyapunov–Krasovskii functional with triple integral terms with bounding technique. In addition, some free weighting matrices are introduced in Theorem 3.1 for getting feasible solution. To ascertain the passivity for the stochastic neural network with time delays in the leakage terms, Theorem 4.1 further presents sufficient conditions for the correspondent system with unknown parameters (58). Hence, in our knowledge, passivity problem of stochastic interval neural network with distributed delays in the leakage terms using impulsive perturbations has never been studied in the previous literature and it is essentially new.

5. NUMERICAL EXAMPLES

In this section, we are analyzing examples showing the effectiveness of the proposed methods.

Example 1. Consider the stochastic neural network with time varying delays and impulses in (1). The parametric coefficients are

$$A = \begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 0.3 & -4 \\ 0.1 & 0.3 \end{bmatrix}, \quad C = \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.6 \end{bmatrix}, \quad D = \begin{bmatrix} 0.4 & -0.3 \\ 0.1 & 0.6 \end{bmatrix},$$

$$\Sigma_1 = \begin{bmatrix} 0.3 & 0.1 \\ 0.5 & -0.1 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} -0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix}, \quad \Sigma_3 = \begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0 \end{bmatrix}, \quad I_k = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

The activation functions are taken as follows:

$$g_1(x) = \frac{1}{20}(\mid x+1 \mid + \mid x-1 \mid), \; g_2(x) = \frac{1}{10}(\mid x+1 \mid + \mid x-1 \mid).$$

It can be verified that Assumption (H1) is satisfied with $F_1^- = -0.1$, $F_1^+ = 0.1$, $F_2^- = -0.2$, $F_2^+ = 0.2$. Thus

$$F_1 = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.04 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

Our main purpose in this example is to estimate the maximum allowable upper bound delay τ_2, d for given lower bound τ_1. For $\tau_1 = 1.6, \mu = 0.5, \delta = 0.1$, by solving LMIs (6) − (10) in Theorem 3.1 using MATLAB LMI toolbox, one can obtain the feasible solution for any time delay satisfying $0 < d(t) \leq 1.5243$ and $0 \leq 1.6 < \tau(t) \leq 3.2458$. For example, if we take $\mu = 0.5, \delta = 0.1, d = 0.8, \tau_1 = 1.6$ and $\tau_2 = 2.8$ we obtain the following feasible solutions

$$P_1 = \begin{bmatrix} 175.1815 & 0.1912 \\ 0.1912 & 220.2178 \end{bmatrix}, P_2 = \begin{bmatrix} 101.2050 & 0.0368 \\ 0.0368 & 138.3154 \end{bmatrix}, P_3 = 10^4 \times \begin{bmatrix} 2.2897 & 0.0987 \\ 0.0987 & 7.5980 \end{bmatrix},$$

$$P_4 = \begin{bmatrix} 54.1964 & 22.1242 \\ 22.1242 & 33.6705 \end{bmatrix}, P_5 = 10^3 \times \begin{bmatrix} 1.1855 & -0.0208 \\ -0.0208 & 0.6355 \end{bmatrix}, L_1 = \begin{bmatrix} 72.6224 & -5.3054 \\ -5.3054 & 93.0195 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 110.0120 & 36.3356 \\ 36.3356 & 106.1753 \end{bmatrix}, L_3 = 10^3 \times \begin{bmatrix} 2.1376 & 0.2181 \\ 0.2181 & 1.0399 \end{bmatrix}, \; R_1 = \begin{bmatrix} 38.0286 & -5.9779 \\ -5.9779 & 59.6507 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 71.5361 & 13.2683 \\ 13.2683 & 55.1780 \end{bmatrix}, R_3 = \begin{bmatrix} 659.6061 & 116.7109 \\ 116.7109 & 362.1591 \end{bmatrix}, S_1 = \begin{bmatrix} 23.0397 & 1.2640 \\ 1.2640 & 23.5236 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 63.7244 & 14.1845 \\ 14.1845 & 55.4936 \end{bmatrix}, S_3 = \begin{bmatrix} 909.2693 & 88.9898 \\ 88.9898 & 459.1695 \end{bmatrix}, Q_1 = \begin{bmatrix} 7.1802 & -0.0377 \\ -0.0377 & 7.1061 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 12.6557 & -0.0667 \\ -0.0667 & 12.5245 \end{bmatrix}, Q_3 = \begin{bmatrix} 432.3567 & 35.8151 \\ 35.8151 & 154.1902 \end{bmatrix}, Q_4 = \begin{bmatrix} 655.5405 & 51.1455 \\ 51.1455 & 257.6060 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 0.2454 & 0.0069 \\ 0.0069 & 0.02521 \end{bmatrix}, T_2 = \begin{bmatrix} 0.4118 & 0.0088 \\ 0.0088 & 0.4212 \end{bmatrix}, T_3 = \begin{bmatrix} 0.2455 & 0.0039 \\ 0.0039 & 0.2494 \end{bmatrix},$$

$$T_4 = \begin{bmatrix} 0.1305 & 0.0009 \\ 0.0009 & 0.1314 \end{bmatrix}, X_1 = \begin{bmatrix} 18.1162 & -0.1125 \\ -0.1125 & 17.9124 \end{bmatrix}, X_2 = \begin{bmatrix} 27.0286 & -0.1404 \\ -0.1404 & 26.7514 \end{bmatrix},$$

$$L = 10^3 \times \begin{bmatrix} 6.1240 & 0 \\ 0 & 5.1268 \end{bmatrix}, S = \begin{bmatrix} 870.8465 & 0 \\ 0 & 287.0032 \end{bmatrix}, M_1 = \begin{bmatrix} -0.2052 & 0.0148 \\ 0.0148 & -0.1210 \end{bmatrix},$$

$$M_2 = 10^3 \times \begin{bmatrix} 0.0658 & -0.6740 \\ -0.6740 & -0.2752 \end{bmatrix}, M_3 = \begin{bmatrix} 0.0930 & 0.0038 \\ 0.0038 & 0.0981 \end{bmatrix},$$

$$M_5 = \begin{bmatrix} 0.2946 & 0.2981 \\ 0.2981 & -0.3589 \end{bmatrix}, M_6 = \begin{bmatrix} -0.2283 & 0.0041 \\ 0.0041 & 0.0092 \end{bmatrix}, M_7 = \begin{bmatrix} -0.0116 & -0.0066 \\ -0.0066 & 0.0051 \end{bmatrix},$$

$$M_8 = \begin{bmatrix} 0.0026 & 0.0018 \\ 0.0018 & -0.0013 \end{bmatrix}, N_1 = \begin{bmatrix} -0.0720 & 0.0054 \\ 0.0054 & -0.0113 \end{bmatrix}, N_2 = \begin{bmatrix} 0.2607 & 0.0065 \\ 0.0065 & 0.2681 \end{bmatrix},$$

$$N_3 = \begin{bmatrix} -0.2602 & -0.0076 \\ -0.0076 & -0.2680 \end{bmatrix}, N_4 = \begin{bmatrix} 0.6438 & -0.4825 \\ -0.4825 & 0.3821 \end{bmatrix}, N_5 = \begin{bmatrix} 0.2124 & 0.1873 \\ 0.1873 & -0.0754 \end{bmatrix},$$

$$N_6 = \begin{bmatrix} -0.1449 & -0.0053 \\ -0.0053 & -0.0131 \end{bmatrix}, N_7 = \begin{bmatrix} -0.0108 & -0.0033 \\ -0.0033 & 0.0006 \end{bmatrix}, N_8 = \begin{bmatrix} 0.0017 & 0.0010 \\ 0.0010 & -0.0004 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} -0.0187 & 0.0132 \\ 0.0132 & -0.0102 \end{bmatrix}, U_2 = \begin{bmatrix} -0.2606 & -0.0070 \\ -0.0070 & -0.2684 \end{bmatrix}, U_3 = \begin{bmatrix} -0.1075 & 0.0972 \\ 0.0972 & 0.4607 \end{bmatrix},$$

$$U_4 = \begin{bmatrix} 0.2589 & 0.0067 \\ 0.0067 & 0.2669 \end{bmatrix}, U_5 = \begin{bmatrix} 0.0138 & 0.0581 \\ 0.0581 & -0.2713 \end{bmatrix}, U_6 = \begin{bmatrix} -0.0379 & 0.0139 \\ 0.0139 & 0.0248 \end{bmatrix},$$

$$U_7 = \begin{bmatrix} -0.0026 & -0.0004 \\ -0.0004 & 0.0039 \end{bmatrix}, U_8 = \begin{bmatrix} -0.0080 & -0.0018 \\ -0.0018 & -0.0088 \end{bmatrix}, Z_1 = \begin{bmatrix} -1.6385 & 0.6544 \\ 0.6544 & -0.3471 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 2.5340 & 0.0210 \\ 0.0210 & 2.4667 \end{bmatrix}, M_4 = 10^3 \times \begin{bmatrix} 0.8794 & -0.7704 \\ -0.7704 & 0.8027 \end{bmatrix}, \lambda_1 = 215.1348,$$

$\lambda_2 = 247.3246, \lambda_3 = 198.5241, \gamma = 405.2402$. For given initial state $x(t) = [0.5 \quad -0.5]^T$, Figure 1 gives the state trajectory of the neural network (1) under zero input, which shows that the neural network is stable.

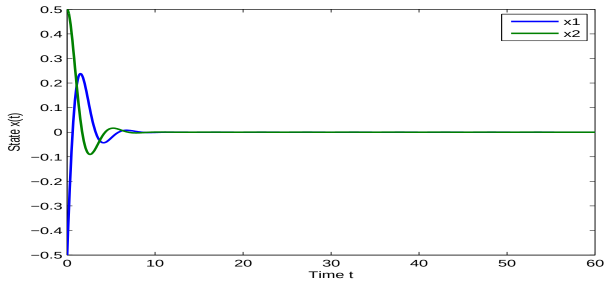

Fig. 1. State trajectory of the NNs in Example 1.

Example 2. Consider the uncertain stochastic neural network with time varying delays and impulses (52) with the following parameters

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 & -3.5 \\ 0.1 & 0.3 \end{bmatrix}, \quad C = \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.7 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Sigma_1 = \begin{bmatrix} 0 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} -0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \quad \Sigma_3 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.2 \end{bmatrix}, \quad G_2 = \begin{bmatrix} -0.1 & -0.2 \\ 0.2 & 0.1 \end{bmatrix}, \quad G_3 = \begin{bmatrix} 0.1 & -0.1 \\ 0.05 & 0.1 \end{bmatrix}, \quad G_4 = \begin{bmatrix} 0.1 & 0.2 \\ -0.1 & -0.1 \end{bmatrix}$$

$$I_k = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$

The activation functions are taken as follows:

$$g_1(x) = \frac{1}{20}(\mid x+1 \mid + \mid x-1 \mid), \; g_2(x) = \frac{1}{10}(\mid x+1 \mid + \mid x-1 \mid).$$

It can be verified that Assumption (H1) is satisfied with $F_1^- = -0.1$, $F_1^+ = 0.1$, $F_2^- = -0.2$, $F_2^+ = 0.2$. Thus

$$F_1 = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.04 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

By solving the LMIs (54)−(58) in Theorem 4.1 using MATLAB LMI toolbox, one can obtain the feasible solution for any time delay satisfying $0 < d(t) \leq 1.6241$ and $0 \leq 1.0 \leq \tau(t) \leq 3.5783$ when $\mu = 0.2, \delta = 0.1$. Suppose, if we take $\mu = 0.2, \delta = 0.1, \tau_1 = 1.0, d = 0.3, \tau_2 = 2.3$, we can obtain the following feasible solutions

$$P_1 = \begin{bmatrix} 123.1307 & -2.6762 \\ -2.6762 & 126.2225 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 47.1324 & -0.9150 \\ -0.9150 & 56.3039 \end{bmatrix}, \quad P_3 = 10^4 \times \begin{bmatrix} 0.8462 & 0.0301 \\ 0.0301 & 2.8263 \end{bmatrix},$$

$$P_4=\begin{bmatrix}14.1132 & 0.7140\\ 0.7140 & 11.0384\end{bmatrix},\quad P_5=10^3\times\begin{bmatrix}1.0766 & 0.0020\\ 0.0020 & 0.5875\end{bmatrix},\quad L_1=\begin{bmatrix}43.0610 & -1.6700\\ -1.6700 & 43.8648\end{bmatrix},$$

$$L_2=\begin{bmatrix}60.7614 & 18.9541\\ 18.9541 & 60.9004\end{bmatrix},\quad L_3=\begin{bmatrix}906.9030 & 97.3614\\ 97.3614 & 484.7922\end{bmatrix},\quad R_1=\begin{bmatrix}20.5262 & -1.3446\\ -1.3446 & 25.1153\end{bmatrix},$$

$$R_2=\begin{bmatrix}38.7288 & 6.8815\\ 6.8815 & 32.1568\end{bmatrix},\quad R_3=\begin{bmatrix}269.8072 & 46.8835\\ 46.8835 & 156.1916\end{bmatrix},\quad S_1=\begin{bmatrix}15.3430 & 0.2558\\ 0.2558 & 13.4679\end{bmatrix},$$

$$S_2=\begin{bmatrix}34.5891 & 7.1417\\ 7.1417 & 30.1517\end{bmatrix},\quad S_3=\begin{bmatrix}386.7042 & 41.7548\\ 41.7548 & 221.4267\end{bmatrix},\quad Q_1=\begin{bmatrix}11.9561 & -0.4032\\ -0.4032 & 9.9872\end{bmatrix},$$

$$Q_2=\begin{bmatrix}7.1335 & -0.2417\\ -0.2417 & 5.953\end{bmatrix},\quad Q_3=\begin{bmatrix}363.5368 & 29.8191\\ 29.8191 & 171.1898\end{bmatrix},\quad Q_4=\begin{bmatrix}274.7815 & 25.0485\\ 25.0485 & 113.2372\end{bmatrix},$$

$$T_1=\begin{bmatrix}0.3657 & -0.0004\\ -0.0004 & 0.2838\end{bmatrix},\quad T_2=\begin{bmatrix}0.4195 & -0.0008\\ -0.0008 & 0.3240\end{bmatrix},\quad T_3=\begin{bmatrix}1.3479 & -0.0040\\ -0.0040 & 1.0312\end{bmatrix},$$

$$T_4=\begin{bmatrix}0.0933 & -0.0002\\ -0.0002 & 0.0760\end{bmatrix},\quad X_1=\begin{bmatrix}11.9820 & -0.4040\\ -0.4040 & 9.9964\end{bmatrix},\quad X_2=\begin{bmatrix}17.7797 & -0.5943\\ -0.5943 & 14.8716\end{bmatrix},$$

$$L=10^3\times\begin{bmatrix}2.3688 & 0\\ 0 & 2.3437\end{bmatrix},\quad S=\begin{bmatrix}509.7439 & 0\\ 0 & 216.5831\end{bmatrix},\quad M_1=\begin{bmatrix}-0.4444 & -0.0119\\ -0.0119 & -0.2591\end{bmatrix},$$

$$M_2=\begin{bmatrix}0.2137 & 0.4291\\ 0.4291 & 0.3288\end{bmatrix},\quad M_3=\begin{bmatrix}0.3304 & -0.0016\\ -0.0016 & 0.2543\end{bmatrix},\quad M_4=\begin{bmatrix}-0.5125 & -0.3566\\ -0.3566 & -0.6523\end{bmatrix},$$

$$M_5=\begin{bmatrix}0.1783 & 0.1182\\ 0.1182 & 0.0496\end{bmatrix},\quad M_6=\begin{bmatrix}-0.2253 & -0.0570\\ -0.0570 & -0.0205\end{bmatrix},\quad M_7=\begin{bmatrix}0.0125 & 0.0003\\ 0.0003 & -0.0008\end{bmatrix},$$

$$M_8=\begin{bmatrix}0.0022 & 0.0028\\ 0.0028 & 0.0002\end{bmatrix},\quad N_1=\begin{bmatrix}-0.0753 & -0.0085\\ -0.0085 & -0.0014\end{bmatrix},\quad N_2=\begin{bmatrix}0.2967 & -0.0009\\ -0.0009 & 0.2269\end{bmatrix},$$

$$N_3=\begin{bmatrix}-0.2987 & -0.0004\\ -0.0004 & -0.2275\end{bmatrix},\quad N_4=\begin{bmatrix}-0.1970 & -0.1884\\ -0.1884 & -0.3900\end{bmatrix},\quad N_5=\begin{bmatrix}0.1288 & 0.0692\\ 0.06920 & 0.0412\end{bmatrix},$$

$$N_6=\begin{bmatrix}-0.1526 & -0.0380\\ -0.0380 & -0.0211\end{bmatrix},\quad N_7=\begin{bmatrix}-0.0014 & 0.0036\\ 0.0036 & -0.0009\end{bmatrix},\quad N_8=\begin{bmatrix}0.0015 & 0.0019\\ 0.0019 & 0.0002\end{bmatrix},$$

$$U_1 = \begin{bmatrix} -0.0153 & 0.0066 \\ 0.0066 & 0.0009 \end{bmatrix}, \quad U_2 = \begin{bmatrix} -0.2964 & 0.0009 \\ 0.0009 & -0.2268 \end{bmatrix}, \quad U_3 = \begin{bmatrix} -0.0896 & -0.2758 \\ -0.2758 & 0.8246 \end{bmatrix},$$

$$U_4 = \begin{bmatrix} 0.2944 & -0.0009 \\ -0.0009 & 0.2252 \end{bmatrix}, \quad U_5 = \begin{bmatrix} -0.0060 & 0.0274 \\ 0.0274 & -0.0932 \end{bmatrix}, \quad U_6 = \begin{bmatrix} -0.0210 & 0.0037 \\ 0.0037 & 0.0290 \end{bmatrix},$$

$$U_7 = \begin{bmatrix} 0.0015 & -0.0003 \\ -0.0003 & -0.0008 \end{bmatrix}, \quad U_8 = \begin{bmatrix} -0.0147 & -0.0026 \\ -0.0026 & -0.0112 \end{bmatrix}, \quad Z_1 = \begin{bmatrix} -1.6241 & 0.1507 \\ 0.1507 & -0.3396 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 1.9428 & -0.0044 \\ -0.0044 & 1.4590 \end{bmatrix}, \quad \lambda_1 = 218.3413, \qquad \lambda_2 = 254.9511, \quad \lambda_3 = 218.1895,$$

$$\gamma = 451.8541.$$

6. CONCLUSION

In this paper we have studied the passivity issue for a new class of impulsive stochastic neural networks with time delays in the leakage terms and mixed time delays are studied under two cases: with known or unknown parameters. In order to prove the passivity for the suggested system, many techniques such as Lyapunov stability theory, stochastic analysis and linear matrix inequalities techniques have been successfully used in this paper. Finally, numerical examples have been provided to demonstrate the validity of the approach. By utilizing the proposed idea of this paper, future works will focus on stabilization for various dynamic systems with time-delays such as switched generalized neural networks and Memristor-Based Recurrent Neural Networks, Complex valued neural networks, Chaotic Lur'e Systems. The corresponding results will appear in the near future.

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