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DATA TRANSFORMATION TECHNIQUE IN THE DATA INFORMATIVITY APPROACH VIA ALGEBRAIC SEQUENCES

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The data-informativity approach in data-driven control focuses on data and their matching model sets for system design and analysis. The approach offers a new mathematical formulation different from model-based control and is expected to progress. In model-based control, the introduction of equivalent transformations has made system analysis and design easier and facilitated theoretical development. In this study, we focus on data transformations and their transformation of matching model sets. We first introduce an algebraic sequence representing the relationship between the data and model set, and using this algebraic approach, we utilize propositions from homology theory, such as kernel universality, to analyze data and model transformations. This technique is significant not only mathematically but also in engineering. Further, we demonstrate how this technique can be applied to derive controllability judgments for data informativity-based analysis. Finally, we prove that design problems can be reduced to analysis problems involving controller inclusion.

Keywords: data-driven control, data informativity-based analysis, analysis and design problems, algebraic sequence, homology theory

Classification: 93A99, 15A06

1. INTRODUCTION

In systems and control theory, given a set of data, there are two major methods for performing system analysis and controller design. One is constructing a mathematical model through system identification [6], followed by analyzing and designing the system based on model-based control. The model-based control is a reasonable method once the model is determined, and it is equipped with a wide variety of mathematical tools. However, it is often difficult to conduct experiments for system identification. In addition, the actual dynamics of the system cannot be represented because the system information is simplified as a model. The other method involves using the data directly for the control design and analysis, which is referred to as data-driven control [3, 4, 9]. This method is more suited to actual systems than model-based control.

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In model-based control, the mathematical model of a dynamical system using differential equations is used to view the behavior of a system as a solution for the differential equations. Model-based control has been studied and utilized historically for a long time, and it includes various basic mathematical techniques such as equivalence transformation. The equivalence transformation technique is useful in various scenarios, such as for proving controllability and observability (left figure in Fig. 1).

A key challenge in data-driven control is obtaining important theoretical results that have already been clarified in the model-based control framework using only the data with theoretical guarantees. For this issue, the result called Willems's fundamental lemma [19] has been developed and applied to various problems such as stabilization and optimal control [7]. In addition, the notion of data informativity has been proposed in recent years for formulating control problems and investigating system properties in the data-driven framework to show that the system exhibited by a model consistent with the data has desirable properties [13]. Since its proposal, this framework has been applied to various aspects of system control theory [2, 14, 15, 16, 17, 18].

The study based on the data informativity framework considers the entire set of models that can satisfy the given data. Thus, it can present a new mathematical view of the relationship between models and data different from that of model-based control. In model-based control, the theory and application areas have been broadened because of basic mathematical techniques such as equivalent transformations. The data-informativity approach can be improved in terms of the richness of mathematical tools. One of them involves a transformation method that simplifies system analysis and design while preserving system properties such as controllability and observability, which play the role of the equivalent transformation of model-based control (right figure in Fig. 1).

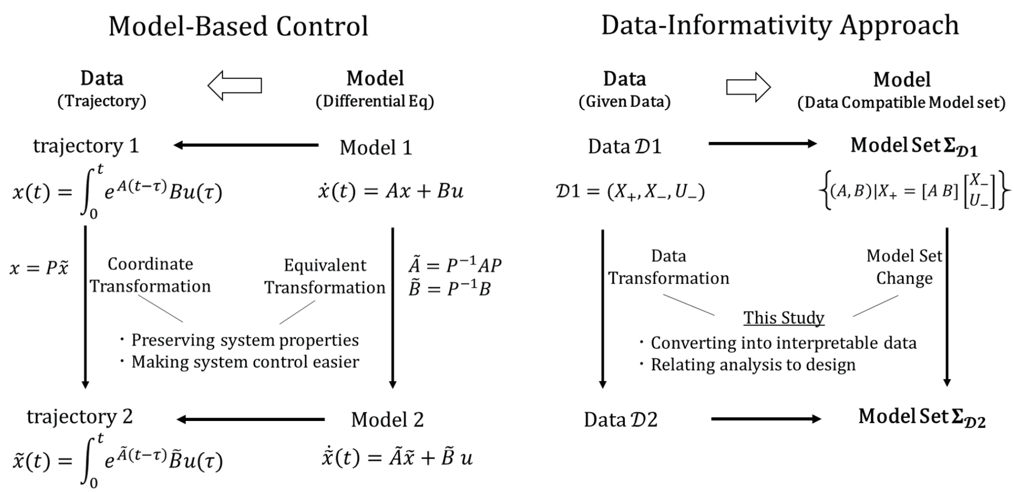

Fig. 1. Comparison of model-based control and data informativity approach.

In terms of the data informativity framework, there are few studies on the relationship between the transformation of data and the transformation of the set of models that satisfy the corresponding data except the conference paper published by the authors [12]. In [12], the authors formulated a data transformation, although in a limited form, and showed that controllability and observability are preserved. However, the limited formulation of the data transformation restricts the applicability of the transformation. In this paper, we provide a new relevant result on the relationship between the data transformation and model set change. We present the relationship between data and model sets in the data informativity-based approach using an algebraic sequence. This algebraic representation allows us to use fundamental propositions from homology algebra such as kernel universality. This data transformation technique is important considering both theoretical and practical points of view. In the theoretical sense, we show that the results of the controllability analysis problem in the data-informativity framework described in [13] can be derived using our data transformation technique. Further, we illustrate that design problems, which were discussed in [13], can be attributed to analytical problems of the closed-loop system involving controllers.

1.1. Notation

We adopt the following notations in this study. Let $\mathbb{R}$, $\mathbb{R}^q$, and $\mathbb{R}^{p\times q}$ represent the set of real numbers, set of real vectors of size q, and set of real matrices of size $p \times q$, respectively. The set consisting of all $n \times m$ matrices is denoted by $M_{n,m}$. When $n = m$, $M_{n,m}$ is denoted by M_n. We denote the transpose of a matrix A by A^T. Let I represent the identity matrix. Let $A = (a_{ij}) \in M_{n,m}$. Then, $A[p : q, r : s]$ denotes the submatrix of A consisting of the p th, $p+1, \dots, q$ th rows and the r th, $r+1, \dots, s$ columns. For a vector space V, id_V denotes the identity map, $id_V : V \to V$. For a vector v, its Hermitian transpose is denoted by v^*. Given a general matrix M, we denote the vector of the ith column by M_i.

2. PRELIMINARIES

2.1. Fundamentals of the homological algebraic approach

The notation utilized in homological algebra and the basic results used in this study, following [1, 5, 8, 10], are explained. The application of the idea of homological algebra to the data informativity framework was considered in [11] and [12]. Let k be a field. In this study, k is assumed to be $\mathbb{R}$ or $\mathbb{C}$. For k-linear spaces V, W and a k-linear map $f : V \to W$, the kernel of f is

$$\mathrm{Ker}(f) = \{v \in V : f(v) = 0\}.$$

We say f is injective when $\mathrm{Ker}(f) = \{0\}$. To emphasize that f is injective, we write

$$f : 0 \to V \to W.$$

$\mathrm{Ker}(f)$ represents a subspace of V, and the inclusion map

$$\psi : 0 \to \mathrm{Ker}(f) \to V$$

is defined. The image of f is the set $\mathrm{Im}(f) := f(V)$, which is a subspace of W. The Cokernel of f is

$$\mathrm{Cok}(f) := W/\mathrm{Im}(f).$$

It is surjective when $\mathrm{Cok}(f) = 0$. To emphasize that f is surjective, we write

$$f : V \to W \to 0.$$

If f is both injective and surjective, then f is called an isomorphism. When we want to emphasize that f is an isomorphism, we write

$$f : V \xrightarrow{\sim} W.$$

A general morphism, which is not necessarily linear, is called bijective if it is both surjective and injective.

An algebraic sequence of k-linear spaces and k-linear maps

$$\ldots \to V_{i-1} \xrightarrow{f_i} V_i \xrightarrow{f_{i+1}} V_{i+1} \to \ldots \tag{1}$$

is called an exact sequence if each f_i satisfies $\mathrm{Ker}(f_{i+1}) = \mathrm{Im}(f_i)$. In this paper, we focus on the exact sequence

$$0 \to \mathrm{Ker}(f) \xrightarrow{\psi} V \xrightarrow{f} W$$

obtained from a linear map $f : V \to W$ and an inclusion map $\psi : 0 \to \mathrm{Ker}(f) \to V$.

For two k-linear spaces V and W, the set $\{k\text{-linear map } V \to W\}$ forms a k-linear space and is denoted by $\mathrm{Hom}(V, W)$. For $f, g \in \mathrm{Hom}(V, W)$, let $f + g$ be defined by $(f + g)(x) = f(x) + g(x)$, where $x \in V$. For $a \in k$ and $f \in \mathrm{Hom}(V, W)$ we define af by $(af)(x) = a \cdot f(x)$, where $x \in V$. Because $f + g$ and af are linear maps (this indicates that $f + g, af \in \mathrm{Hom}(V, W)$), we can see that $\mathrm{Hom}(V, W)$ denotes a vector space. This set is called the homomorphism space. Next, we discuss the morphism between homomorphism spaces induced naturally from a linear map, which plays an important role in this paper. If a linear space W is fixed, then the linear map $f : V \to V'$ induces another linear map,

$$f_* : \begin{array}{ccc} \mathrm{Hom}(V', W) & \to & \mathrm{Hom}(V, W) \\ g & \mapsto & g \circ f \end{array}.$$

We present an important lemma for f_* induced from f.

Lemma 2.1. For a linear map $f : V \to W$, the following conditions are equivalent:

1. f is an isomorphism.

2. For any linear space W', $f_* : \mathrm{Hom}(W, W') \to \mathrm{Hom}(V, W')$ is an isomorphism.

Proof. We first prove that (A) $\Rightarrow$ (B). Let f be the inverse map of g. For any $h \in \mathrm{Hom}(W, W')$, we have $g_* \circ f_*(h) = h \circ f \circ g = h$, and for any $h' \in \mathrm{Hom}(V, W')$, we have $f_* \circ g_*(h') = h' \circ g \circ f = h'$. Thus, g_* is the inverse map of f_*, and (B) is satisfied.

Subsequently, we prove (B) $\Rightarrow$ (A). Let $W' = V$. Then, there exists a map $g : W \to V$ such that $f_*(g) = g \circ f = id_V$ (where id_V denotes the identity map on V). Let $W' = W$. Consequently, we obtain $f_*(f \circ g) = f \circ g \circ f = f = f_*(id_W)$. Therefore, $f \circ g = id_W$, which implies that g is the inverse map of f. Thus, (A) is satisfied. $\square$

Finally, we present the following lemma on the morphism based on the universality of the kernel [5].

Lemma 2.2. The kernel (Ker, i) of the linear map $f : V \to W$ satisfies the following universality (where $i : \mathrm{Ker}(f) \to V$ is an inclusion map).

1. $f \circ i = 0$.
2. For any vector space L and $g : L \to V$ satisfying $f \circ g = 0$, there exists a unique map such that $h : L \to \mathrm{Ker}(f)$ and $g = i \circ h$

$$\begin{tikzcd} L \arrow[d, dotted, "\exists! h"'] \arrow[dr, "g"] \arrow[drr, bend left, "0"] & & \\ \mathrm{Ker}(f) \arrow[r, "i"] & V \arrow[r, "f"] & W. \end{tikzcd}$$

Proof. When $g : L \to V$ satisfies $f \circ g = 0$, $g(x) \in \mathrm{Ker}(f)$ for all $x \in L$. Hence, there exists a unique $y \in \mathrm{Ker}(f)$ such that $g(x) = i(y)$. Let $h(x) = y$. Then, $h : L \to \mathrm{Ker}(f)$ is a homomorphism, and satisfies $g = i \circ h$. Conversely, $h : L \to \mathrm{Ker}(f)$, which satisfies $i \circ h = g$ as the only one similar to this. $\square$

In model-based control frameworks, an equivalent transformation focuses on the correspondence of each model. However, in the data-informativity framework, the correspondence of each model set must be considered. To clarify this point, the morphism induced by the universality of the kernel plays a crucial role in this study.

2.2. Data informativity

Following [13], we formulate and discuss the analysis and design problems related to the data informativity-based approach to data-driven control. Let Σ denote the class of models, i. e., a set of predetermined systems that includes the true system. Given a dataset $\mathcal{D}$, we define $\Sigma_{\mathcal{D}} \subseteq \Sigma$ as the set of all systems within Σ that can generate these data. In this paper, Σ is considered the set of all discrete-time input/state systems of the following format:

$$x(t+1) = Ax(t) + Bu(t), \tag{2a}$$
$$y(t) = Cx(t) + Du(t), \tag{2b}$$

where x, u, and y denote the state variable, input variable, and output variable, respectively. We represent the matrices composed of time-series data corresponding to input, state, and output data as follows.

$$U_- := [u(0)\ u(1)\ \cdots\ u(T-1)] \in \mathbb{R}^{\ell \times T}, \tag{3a}$$

$$X := [x(0)\ x(1)\ \cdots\ x(T)] \in \mathbb{R}^{n \times (T+1)}, \tag{3b}$$

$$X_- := [x(0)\ x(1)\ \cdots\ x(T-1)] \in \mathbb{R}^{n \times T}, \tag{3c}$$

$$X_+ := [x(1)\ x(2)\ \cdots\ x(T)] \in \mathbb{R}^{n \times T}, \tag{3d}$$

$$Y_- := [y(0)\ y(1)\ \cdots\ y(T-1)] \in \mathbb{R}^{m \times T}. \tag{3e}$$

Here, n represents the dimension of the state variables, and ℓ and m represent the dimensions of the input and output variables, respectively. For a given data set $\mathcal{D}$ $= (X_+, X_-, U_-)$, the model set $\Sigma_{\mathcal{D}}$ can be described by

$$\Sigma_{\mathcal{D}} = \left\{ (A, B) : X_+ = [A, B] \begin{bmatrix} X_- \\ U_- \end{bmatrix} \right\}. \tag{4}$$

In the case of $\mathcal{D} = (X_+, X_-, U_-)$, we define the mapping $f_{\mathcal{D}} : \mathbb{R}^T \to \mathbb{R}^{2n+\ell}$ as

$$\begin{array}{cccc} f_{\mathcal{D}} : & \mathbb{R}^T & \to & \mathbb{R}^{2n+\ell} \\ & r & \mapsto & \begin{bmatrix} X_+ \\ X_- \\ U_- \end{bmatrix} r. \end{array}$$

As explained in Section 2.1, $f_{\mathcal{D}}$ induces

$$f_{\mathcal{D},*} : \mathrm{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n) \to \mathrm{Hom}(\mathbb{R}^T, \mathbb{R}^n). \tag{5}$$

2.2.1. Analysis problems for data-informativity framework

Let $\mathcal{P}$ be a system-theoretical property and let $\Sigma_{\mathcal{P}}$ be the set of all systems in the system set Σ that satisfy the system property $\mathcal{P}$. Examples of system property $\mathcal{P}$ are stability, controllability, observability and dissipativity, etc. We then define the data informativity of data $\mathcal{D}$ concerning the system property $\mathcal{P}$ in terms of $\Sigma_{\mathcal{P}}$ as follows.

Definition 2.3. (Data informativity) Data $\mathcal{D}$ is said to be informative with respect to property $\mathcal{P}$ if $\Sigma_{\mathcal{D}} \subseteq \Sigma_{\mathcal{P}}$ holds.

Using this definition, the general analysis problem of data-driven control can be formulated as follows in terms of data informativity.

Problem 2.4. (Data informativity analysis problem): Provide a necessary and sufficient condition for data $\mathcal{D}$ to be informative for the property $\mathcal{P}$.

For this data informativity analysis problem, we formulate the data informativity for controllability and discuss the results. We define the system set Σ_{cont} as follows.

$$\Sigma_{\text{cont}} := \{(A, B) : (A, B) \text{ is controllable}\}. \tag{6}$$

Using this definition, data informativity for controllability is defined as follows:

Definition 2.5. Data $\mathcal{D} = (X_+, X_-, U_-)$ are said to be informative for controllability if $\Sigma_{\mathcal{D}} \subseteq \Sigma_{\text{cont}}$.

Using this formulation, the following results are obtained [13].

Proposition 2.6. Data $\mathcal{D} = (X_+, X_-, U_-)$ are informative for controllability if and only if

$$\operatorname{rank}(X_+ - \lambda X_-) = n \qquad {}^{\forall}\lambda \in \mathbb{C}. \tag{7}$$

2.2.2. Design problem for data-informativity framework

We formulate problems related to the design of data-driven control. The goal of the design problem is to formulate a data-driven controller such that the closed-loop system obtained by interconnecting the "true" system S and the controller exhibits certain properties [13]. Let $\mathcal{P}(K)$ be a system-theoretical property parametrized by the controller K to be connected. Here, we define the data informativity for the control design of $\mathcal{P}$ as follows.

Definition 2.7. For the system property $\mathcal{P}$ and controller K, we denote $\Sigma_{\mathcal{P}(K)} = \{(A, B) : A + BK \text{ has system property } \mathcal{P}\}$. If there exists a controller K such that $\Sigma_{\mathcal{D}} \subseteq \Sigma_{\mathcal{P}(K)}$, we say that the data $\mathcal{D}$ is informative for the property $\mathcal{P}$.

The first step in the design problem of data-driven control is to determine whether an appropriate controller that satisfies the desired system characteristics for a given set of data $\mathcal{D}$ can be obtained. In other words, we consider the following problem.

Problem 2.8. For the data $\mathcal{D}$, provide the necessary and sufficient conditions for the existence of a controller K such that the data are informative for property $\mathcal{P}$.

The second step in the design problem of data-driven control involves the design of an appropriate controller. Here, we consider the following problem.

Problem 2.9. Assuming that the data $\mathcal{D}$ is informative for the property $\mathcal{P}$, find a controller K such that $\Sigma_{\mathcal{D}} \subseteq \Sigma_{P(K)}$.

3. MAIN RESULT

Data-driven control based on data informativity not only exhibits the advantages of data-driven control but also suggests a new mathematical view of the relationship between models and data, which differs from model-based control. In this paper, we provide isomorphic transformations of time series data in data informative content. By doing so, the conventional framework of data-driven control based on the data informativity framework, which considers the relationship between the data and the model set, is expanded to consider the relationship between the transformation of data and the transformation of the model set. In addition, we demonstrate an example of the application of this data transformation technique.

3.1. Purpose and motivation of this study

In this subsection, we explain the objectives of this paper with specific numerical examples. This study is motivated by three points.

1 (Providing techniques for converting measured data into easily interpretable data): In model-based control, the state matrix can be transformed into a more analyzable form (such as a diagonal or companion matrix) via an equivalence transformation. Similarly, in the data informativity approach, it is expected that data can be transformed into a form that is easier to interpret by the equivalence transformation in the context of the data informativity approach.

For example, suppose the following data series $\mathcal{D}_1$ is given.

$$\mathcal{D}_1 = \begin{bmatrix} 2 & 5 & 13 \\ 0 & 1 & 5 \end{bmatrix}.$$

We now consider making the data $\mathcal{D}_1$ more comprehensible by data transformation.

$$\begin{tikzcd} \text{Data } \mathcal{D}_1 \arrow[r, leftrightarrow] \arrow[d, "T\text{: Data Transformation}"'] & \Sigma_{\mathcal{D}_1} \arrow[d, "\text{Model Set Change}"] \\ \text{Data } \mathcal{D}_2 \arrow[r, leftrightarrow] & \Sigma_{\mathcal{D}_2} \end{tikzcd}$$

In this case, the data $\mathcal{D}_1$ is transformed by the data transformation

$$T = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix},$$

and we can get the data

$$\mathcal{D}_2 = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 2 & 5 & 13 \\ 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 9 \\ 1 & 2 & 4 \end{bmatrix}.$$

We can easily obtain

$$\Sigma_{\mathcal{D}_2} = \left\{ A : A \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 2 & 4 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \right\}.$$

In this way, the data can be transformed into an easily understandable form.

2 (Mathematical extensions in the data informativity approach): The data informativity approach considers data and the data-compatible model sets. This implies a new mathematical viewpoint that is different from the model-based control. Based on the new mathematical viewpoint, we formulate the relationship between the transformation of data and the transformation of model sets by providing a technique of data transformation(Figure 1 in section 1).

3 (Development of fundamental tools in the data informativity approach): In model-based control, equivalent transformations are useful in various situations such as proving controllability and observability and so on. It is expected that the same kind of technique as the equivalent transformation in model-based control can be given to be useful in the context of the data informativity approach. In fact, in [12], data informativity for observability is given as a special case usage of data transformation in this paper. We show that it is also possible to give proof of data informativity for controllability.

Another application of the data transformation technique is to solve the controller design problem of state feedback gains. In the model-based control approach, when designing K, the condition under which the state matrix $A+BK$ after implementing state feedback has desired characteristics (e. g., stability) is analyzed. In other words, we design K by applying the analysis method of the system (A,B) to the system $(A+BK,B)$. This design framework can be applied to the data informativity approach as well by the data transformation. In other words, we show that K can be designed by giving a data transformation representing the data after state feedback by

$$\mathcal{D} = \begin{bmatrix} X_+ \\ X_- \\ U_- \end{bmatrix} \mapsto \mathcal{D}(K) := \begin{bmatrix} X'_+ \\ X'_- \\ U'_- \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & -K & I \end{bmatrix} \begin{bmatrix} X_+ \\ X_- \\ U_- \end{bmatrix},$$

and considering an analysis problem on the data $\mathcal{D}(K)$.

3.2. Time-series data transformation in data informativity-based analysis

Proposition 3.1. Suppose that the data $\mathcal{D} = (X_+, X_-, U_-)$ and the data $\mathcal{D}' = (X'_+, X'_-, U'_-)$ exhibit the relation

$$\begin{bmatrix} X'_{+,i} \\ X'_{-,i} \\ U'_{-,i} \end{bmatrix} = \alpha^{-1}\left(\begin{bmatrix} X_{+,i} \\ X_{-,i} \\ U_{-,i} \end{bmatrix}\right), \qquad 1 \le i \le T \tag{8}$$

by the isomorphism

$$\begin{array}{cccc} \alpha : & \mathbb{R}^{2n+\ell} & \xrightarrow{\sim} & \mathbb{R}^{2n+\ell} \\ & \begin{bmatrix} x_1 \\ x_2 \\ u \end{bmatrix} & \mapsto & \alpha\left(\begin{bmatrix} x_1 \\ x_2 \\ u \end{bmatrix}\right), \end{array} \tag{9}$$

where for a matrix M, M_i denotes the vector consisting of the ith column. Then, there exists a unique isomorphism $h_\alpha : \mathrm{Ker}(f_{\mathcal{D},*}) \to \mathrm{Ker}(f_{\mathcal{D}',*})$ such that the following diagram

$$\begin{tikzcd} \mathrm{Ker}(f_{\mathcal{D},*}) \arrow[r, "\psi"] \arrow[d, dotted, "\exists! h_\alpha"'] \arrow[dr, phantom, "\circlearrowleft"] & \mathrm{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n) \arrow[d, "\alpha_*"] \\ \mathrm{Ker}(f_{\mathcal{D}',*}) \arrow[r, "\psi'"] & \mathrm{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n) \end{tikzcd}$$

is commutative, where ψ, ψ' is the inclusion map.

P r o o f. For the data $\mathcal{D} = (X_+, X_-, U_-)$ and $\mathcal{D}' = (X'_+, X'_-, U'_-)$, $f_{\mathcal{D},*}$ and $f_{\mathcal{D}',*}$ described in Eq.(5) are obtained. Let us consider the algebraic exact sequences

$$0 \rightarrow \operatorname{Ker}(f_{\mathcal{D},*}) \stackrel{\psi}{\rightarrow} \operatorname{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n) \stackrel{f_{\mathcal{D},*}}{\longrightarrow} \operatorname{Hom}(\mathbb{R}^T, \mathbb{R}^n)$$

and

$$0 \rightarrow \operatorname{Ker}(f_{\mathcal{D}',*}) \stackrel{\psi'}{\rightarrow} \operatorname{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n) \stackrel{f_{\mathcal{D}',*}}{\longrightarrow} \operatorname{Hom}(\mathbb{R}^T, \mathbb{R}^n)$$

derived from $f_{\mathcal{D},*}$ and $f_{\mathcal{D}',*}$. Since $f_{\mathcal{D}',*} = (\alpha^{-1} \circ f_{\mathcal{D}})_* = f_{\mathcal{D},*} \circ (\alpha^{-1})_* = f_{\mathcal{D},*} \circ (\alpha_*)^{-1}$ and $f_{\mathcal{D}',*} \circ (\alpha_* \circ \psi) = 0$, there exists a unique morphism

$$h_\alpha : \operatorname{Ker}(f_{\mathcal{D},*}) \rightarrow \operatorname{Ker}(f_{\mathcal{D}',*})$$

that makes the diagram

$$\begin{tikzcd}
0 \arrow[r] & \operatorname{Ker}(f_{\mathcal{D},*}) \arrow[r, "\psi"] \arrow[d, dotted, "^{\exists!}h_\alpha"'] \arrow[dr, phantom, "\circlearrowleft"] & \operatorname{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n) \arrow[r, "f_{\mathcal{D},*}"] \arrow[d, "\alpha_*"'] \arrow[dr, phantom, "\circlearrowleft"] & \operatorname{Hom}(\mathbb{R}^T, \mathbb{R}^n) \arrow[d, "id"'] \\
0 \arrow[r] & \operatorname{Ker}(f_{\mathcal{D}',*}) \arrow[r, "\psi'"] & \operatorname{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n) \arrow[r, "f_{\mathcal{D}',*}"] & \operatorname{Hom}(\mathbb{R}^T, \mathbb{R}^n)
\end{tikzcd}$$

commutative by the universality of the kernel described in Lemma 2.2. By Lemma 2.1, α_* induced by the isomorphism α is also an isomorphism, which indicates that h_α is an isomorphism. □

Next, we consider the relationship among the isomorphism h_α, the set of models $\Sigma_{\mathcal{D}}$, and $\Sigma_{\mathcal{D}'}$. First, we discuss the equivalence relation, the quotient set induced by the equivalence relation, and the morphisms induced on the quotient set. We identify $\operatorname{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n)$ with $M_{n,2n+\ell}$. Let $\mathrm{GL}_n(\mathbb{R})$ be the general linear group defined as follows:

$$\mathrm{GL}_n(\mathbb{R}) = \{g \in M_n(\mathbb{R}) : \det(g) \neq 0\}.$$

We define the action of $\mathrm{GL}_n(\mathbb{R})$ on $M_{n,2n+\ell}(= \operatorname{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n))$ as

$$\begin{array}{ccc} \mathrm{GL}_n(\mathbb{R}) \times M_{n,2n+\ell} & \rightarrow & M_{n,2n+\ell} \\ (G \ \ , \ \ H) & \mapsto & GH \end{array} . \tag{10}$$

Similarly, we define the action of $\mathrm{GL}_n(\mathbb{R})$ on $M_{n,T}(= \operatorname{Hom}(\mathbb{R}^T, \mathbb{R}^n))$. We define the relation $\sim$ in $M_{n,2n+\ell}(= \operatorname{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n))$ as follows:

$$H \sim H' (\in M_{2n+\ell,n}) \iff {}^{\exists}G \in \mathrm{GL}_n(\mathbb{R}) : H = GH'. \tag{11}$$

Evidently, $\sim$ is an equivalence relation. Similarly, we introduce the equivalence relation $\sim'$ on $M_{n,T}(= \operatorname{Hom}(\mathbb{R}^T, \mathbb{R}^n))$. In the case of $H \sim H'$, as

$$f_{\mathcal{D},*}(H) \sim' f_{\mathcal{D},*}(H')$$

holds for $f_{\mathcal{D},*}$ in Eq.(5), the following equivalence classes

$$\overline{\operatorname{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n)} := \operatorname{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n)/ \sim \tag{12a}$$

$$\overline{\operatorname{Hom}(\mathbb{R}^T, \mathbb{R}^n)} := \operatorname{Hom}(\mathbb{R}^T, \mathbb{R}^n)/ \sim' \tag{12b}$$

naturally induce

$$\bar{f}_{\mathcal{D},*} : \overline{\mathrm{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n)} \quad \rightarrow \quad \overline{\mathrm{Hom}(\mathbb{R}^T, \mathbb{R}^n)}. \tag{13}$$

Moreover, $\Sigma_{\mathcal{D}}$ can be regarded as a subset of $\overline{\mathrm{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n)}$ defined by

$$\Sigma_{\mathcal{D}} \ni (A, B) \mapsto \begin{bmatrix} I & -A & -B \end{bmatrix} \in \overline{\mathrm{Hom}(\mathbb{R}^{2n+\ell}, \mathbb{R}^n)}. \tag{14}$$

From this identification, we obtain $\Sigma_{\mathcal{D}} \subseteq \mathrm{Ker}(\bar{f}_{\mathcal{D},*})$. In this context, we discuss time-series data transformation in data informativity-based analysis.

Theorem 3.2. Suppose that data $\mathcal{D} = (X_+, X_-, U_-)$ and data $\mathcal{D}' = (X'_+, X'_-, U'_-)$ satisfy the conditions of Proposition 3.1. Further, let the isomorphism α in Eq.(9) be of the form

$$\begin{array}{cccc} \alpha : & \mathbb{R}^{2n+\ell} & \xrightarrow{\sim} & \mathbb{R}^{2n+\ell} \\ & \begin{bmatrix} x_1 \\ x_2 \\ u \end{bmatrix} & \mapsto & \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ 0 & T_{22} & T_{23} \\ 0 & T_{32} & T_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u \end{bmatrix}, \end{array}$$

where $T_{11}, T_{12}, T_{22} \in M_n, T_{13}, T_{32} \in M_{\ell,n}, T_{23} \in M_{n,\ell}$, and $T_{33} \in M_\ell$. The isomorphism h_α induced by the universality of kernel induces the natural bijective morphism

$$\bar{h}_\alpha : \overline{\mathrm{Ker}(f_{\mathcal{D},*})} = \mathrm{Ker}(\bar{f}_{\mathcal{D},*}) \xrightarrow{\sim} \mathrm{Ker}(\bar{f}_{\mathcal{D}',*}) = \overline{\mathrm{Ker}(f_{\mathcal{D}',*})} \tag{15}$$

on the quotient set determined by the aforementioned equivalence relation on the general linear group. Under the identity in Eq.(14), $\bar{h}_\alpha$ yields a one-to-one correspondence between $\Sigma_{\mathcal{D}}$ and $\Sigma_{\mathcal{D}'}$.

Proof. Let us consider the following commutative diagram.

$$\begin{tikzcd} \mathrm{Ker}(f_{\mathcal{D},*}) \arrow[r, "h_\alpha"] \arrow[d, "\pi_1"'] \arrow[dr, phantom, "\circlearrowleft"] & \mathrm{Ker}(f_{\mathcal{D}',*}) \arrow[d, "\pi_2"'] \\ \overline{\mathrm{Ker}(f_{\mathcal{D},*})} \arrow[r, "\bar{h}_\alpha"] & \overline{\mathrm{Ker}(f_{\mathcal{D}',*})} \end{tikzcd}$$

where $\pi_1 : \mathrm{Ker}(f_{\mathcal{D},*}) \rightarrow \overline{\mathrm{Ker}(f_{\mathcal{D},*})}$ and $\pi_2 : \mathrm{Ker}(f_{\mathcal{D}',*}) \rightarrow \overline{\mathrm{Ker}(f_{\mathcal{D}',*})}$ are natural maps to each quotient set. Since the map to the quotient set is surjective in general, π_1 and π_2 are surjective. Moreover, since h_α is an isomorphism we can see $\bar{h}_\alpha \circ \pi_1 = \pi_2 \circ h_\alpha$ is surjective. Therefore we see $\bar{h}_\alpha$ is surjective. Next, we show that $\bar{h}_\alpha$ is injective. From the surjectivity of π_1, there exist $v, v' \in \mathrm{Ker}(f_{\mathcal{D},*})$ satisfying $\pi_1(v) = x$ and $\pi_1(v') = x'$. From $\pi_2 \circ h_\alpha(v) = \pi_2 \circ h_\alpha(v')$ we have $h_\alpha(v) \sim h_\alpha(v')$. From the configuration of h_α we also know $v \sim v'$. Thus $x = x'$ holds and $\bar{h}_\alpha$ is injective. From the above, Eq.(15) can be shown. This isomorphism $\bar{h}_\alpha$ yields a one-to-one correspondence

$$\begin{array}{ccc} \Sigma_{\mathcal{D}} & \longrightarrow & \Sigma_{\mathcal{D}'} \\ [I \ -A \ -B] & \longrightarrow & [I \ -A \ -B] \circ \alpha \end{array} \tag{16}$$

for $\Sigma_{\mathcal{D}}$ and $\Sigma_{\mathcal{D}'}$ as a set. $\square$

Here, we explain a remark. In [12], they are considering equivalent transformations of data, and α in Theorem 3.2 has the special form $T_{12} = T_{13} = T_{23} = T_{32} = 0$ and $T_{33} = I$. Although equivalent transformations have some applications, such as proving data informativity for observability, we can achieve even more by generalizing α. In the next section, we will explore the richer range of applications that this generalization provides.

3.3. Application of the data transformation technique

The transformation of time-series data described in subsection 3.2 expands the data informative framework to include the relationship among transformations in the data space and those in the space of the functions acting on each data. In this section, we demonstrate that the Hautus test (method of investigating controllability, as described in Proposition 2.6.) in terms of the data informative framework can be proved via data transformation. This is an example of its application and proves that the design problem can be attributed to an analytical problem involving controllers.

3.3.1. Proof of the Hautus test in the data informative framework utilizing the proposed data transformation technique

First, let us discuss the necessary condition. Suppose that the data $\mathcal{D} = (X_+, X_-, U_-)$ is data informative for controllability. For α in Eq.(9) in Proposition 3.1, we denote

$$\begin{array}{rccc} \alpha^\lambda : & \mathbb{R}^{2n+\ell} & \to & \mathbb{R}^{2n+\ell} \\ & v & \mapsto & \begin{bmatrix} I & -\lambda I & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} v \end{array}, \tag{17}$$

for any ${}^\forall\lambda \in \mathbb{C}$. In this case, $\mathcal{D}'$ in Proposition 3.1 and Theorem 3.2 yield $\mathcal{D}' = (X_+ + \lambda X_-, X_-, U_-)$ and a one-to-one correspondence,

$$\begin{array}{rccc} \bar{h}_{\alpha^\lambda} : & \mathrm{Ker}(\bar{f}_{\mathcal{D},*}) & \xrightarrow{\sim} & \mathrm{Ker}(\bar{f}_{\mathcal{D}',*}) \\ & \cup\!\mid & & \cup\!\mid \\ & \Sigma_{\mathcal{D}} & \longrightarrow & \Sigma_{\mathcal{D}'} \end{array}. \tag{18}$$

If Eq.(7) does not hold, then there exists some $v \neq 0$ satisfying $v^*[X_+ + \lambda X_-] = 0$, where v^* represents the Hermitian transpose of vector v. By swapping the coordinates appropriately, the first component of v may be assumed to be non-zero, and thus

$$G := \begin{bmatrix} v^* \\ I[2:n, 1:n] \end{bmatrix}$$

may be assumed to be regular. For any $(A, B) \in \Sigma_{\mathcal{D}}$, let $(\bar{A}, \bar{B})$ be defined as $[I \; -\bar{A} \; -\bar{B}] := \bar{h}_{\alpha^\lambda}([I \; -A \; -B])$. Using this $(\bar{A}, \bar{B})$ and v, consider

$$\tilde{g} := \begin{bmatrix} v^* & 0 & 0 \\ I[2:n, 1:n] & -\bar{A}[2:n, 1:n] & -\bar{B}[2:n, 1:n] \end{bmatrix},$$

We define $(\bar{A}', \bar{B}')$ as follows:

$$\begin{bmatrix} \bar{A}' & \bar{B}' \end{bmatrix} := G^{-1} \begin{bmatrix} 0 & 0 \\ \bar{A}[2:n, 1:n] & \bar{B}[2:n, 1:n] \end{bmatrix}.$$

We obtain

$$\tilde{g} = [I - \bar{A}' - \bar{B}'] \in \Sigma_{\mathcal{D}'} \subseteq \mathrm{Ker}(\bar{f}_{\mathcal{D}',*}).$$

Let $(\bar{\bar{A}}, \bar{\bar{B}})$ be defined by $[I - \bar{\bar{A}} - \bar{\bar{B}}] := \bar{h}_{\alpha^\lambda}^{-1}([I - \bar{A}' - \bar{B}'])$. Eq.(18) yields $(\bar{\bar{A}}, \bar{\bar{B}}) \in \Sigma_{\mathcal{D}}$. Conversely, $(\bar{A}', \bar{B}') = (\bar{\bar{A}} + \lambda I, \bar{\bar{B}})$, and by the definition of (A', B'),

$$\mathrm{rank}\left(\begin{bmatrix} \bar{\bar{A}} + \lambda I & \bar{\bar{B}} \end{bmatrix}\right) = \mathrm{rank}\left(\begin{bmatrix} \bar{A}' & \bar{B}' \end{bmatrix}\right) < n.$$

This contradicts the fact that the data $\mathcal{D}$ is data informative for controllability.

Next, we prove the converse. For any $(A, B) \in \Sigma_{\mathcal{D}}$, $g_{(A-\lambda I, B)}$ is defined by

$$\begin{array}{rccc} g_{(A-\lambda I,B)}: & \mathbb{R}^{n+\ell} & \to & \mathbb{R}^n \\ & \begin{bmatrix} x \\ u \end{bmatrix} & \mapsto & \begin{bmatrix} A - \lambda I & B \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \end{array}.$$

Let $V_- := \mathrm{Im}[X_-^T, U_-^T]^T$, $\widetilde{V_+^\lambda} := \mathrm{Im}[X_+ - \lambda X_-]$, where T represents a transpose (not Hermitian transpose), and consider the following exact sequence:

$$\begin{tikzcd} 0 \arrow[r] & V_- \arrow[r] \arrow[d, dotted, "\exists ! \phi"'] \arrow[dr, phantom, "\circlearrowleft"] & \mathbb{R}^{n+\ell} = V_- \oplus V_-^\perp \arrow[r] \arrow[d, "g_{(A-\lambda I,B)}"'] \arrow[dr, phantom, "\circlearrowleft"] & V_-^\perp \arrow[r] \arrow[d, "id"] & 0 \\ 0 \arrow[r] & \widetilde{V_+^\lambda} \arrow[r] & \mathbb{R}^n = \widetilde{V_+^\lambda} \oplus \widetilde{V_+^\lambda}^\perp \arrow[r] & \widetilde{V_+^\lambda}^\perp \arrow[r] & 0. \end{tikzcd}$$

By the assumption that $\mathrm{rank}[X_+ - \lambda X_-] = n$, we know that $\widetilde{V_+^\lambda}^\perp = 0$. Hence, $\widetilde{V_+^\lambda} = \mathbb{R}^n$. By the definition of $\widetilde{V_+^\lambda}$,

$$\widetilde{V_+^\lambda} \subseteq \mathrm{Im}\left(g_{(A-\lambda I,B)}|_{V_-}\right) \subseteq \mathbb{R}^n,$$

we can see $\mathbb{R}^n = \mathrm{Im}(g_{(A-\lambda I,B)}|_{V_-})$. From this, we can derive $\mathrm{rank}[A - \lambda I\ B] = n$, which implies that the data $\mathcal{D} = (X_+, X_-, U_-)$ is data informative for controllability.

3.3.2. Reduction of design problems to analysis problems in the data informativity-based approach

As a second application of the proposed time-series data transformation technique, we demonstrate that the design problem can be reduced to an analysis problem involving controllers in a stepwise manner.

Theorem 3.3. For $K \in M_{n,\ell}$, assume that data $\mathcal{D} = (X_+, X_-, U_-)$ and $\mathcal{D}(K) = (X'_+, X'_-, U'_-)$ satisfy the relation

$$\begin{bmatrix} X'_+ \\ X'_- \\ U'_- \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & -K & I \end{bmatrix} \begin{bmatrix} X_+ \\ X_- \\ U_- \end{bmatrix}$$

under the isomorphism

$$\begin{array}{cccc} \alpha_K: & \mathbb{R}^{2n+\ell} & \to & \mathbb{R}^{2n+\ell} \\ & v & \mapsto & \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & K & I \end{bmatrix} v \end{array}. \tag{19}$$

Under this assumption, obtaining a solution to the analytical problem for the data $\mathcal{D}(K)$ is equivalent to obtaining a solution to the design problem for the data $\mathcal{D}$.

P r o o f. By Theorem 3.2, we obtain a one-to-one correspondence

$$\begin{array}{cccc} \bar{h}_{\alpha_K} & \mathrm{Ker}(\bar{f}_{\mathcal{D},*}) & \xrightarrow{\sim} & \mathrm{Ker}(\bar{f}_{\mathcal{D}(K),*}) \\ & \cup\!\mid & & \cup\!\mid \\ & \Sigma_{\mathcal{D}} & \longrightarrow & \Sigma_{\mathcal{D}(K)} \end{array} \tag{20}$$

between subsets $\Sigma_{\mathcal{D}}$ of $\mathrm{Ker}(\bar{f}_{\mathcal{D},*})$ and subsets $\Sigma_{\mathcal{D}(K)}$ of $\mathrm{Ker}(\bar{f}_{\mathcal{D}(K),*})$. As

$$\begin{array}{l} \Sigma_{\mathcal{D}} \subseteq \Sigma_{\mathcal{P}(K)} \\ \iff \text{for } {}^{\forall}(A,B) \in \Sigma_{\mathcal{D}}, \quad (A+BK,B):\mathcal{P} \\ \iff \text{for } {}^{\forall}(A,B) \in \Sigma_{\mathcal{D}}, \quad \bar{h}_{\alpha_K}(\{A,B\}):\mathcal{P} \end{array}$$

, where $(A,B):\mathcal{P}$ indicates that the system (A,B) satisfies the system property $\mathcal{P}$, and $\bar{h}_{\alpha_K}(\{A,B\})$ shows that, by Eq.(14), the system (A,B) is regarded as an element of a homomorphism space and then its projection onto the quotient set is mapped by $\bar{h}_{\alpha_K}$. Moreover, Eq.(20), for any $(A',B') \in \Sigma_{\mathcal{D}(K)}$,

$${}^{\exists!}(A,B) \in \Sigma_{\mathcal{D}} \text{ s.t } \bar{h}_{\alpha_K}(\{A,B\}) = (A',B').$$

Hence, we get

$$\Sigma_{\mathcal{D}} \subseteq \Sigma_{\mathcal{P}(K)} \Rightarrow \Sigma_{\mathcal{D}(K)} \subseteq \Sigma_{\mathcal{P}}.$$

To prove the converse, we note that by Eq.(20), for any $(A,B) \in \Sigma_{\mathcal{D}}$,

$${}^{\exists!}(A',B') \in \Sigma_{\mathcal{D}(K)} \text{ s.t } \bar{h}_{\alpha_K}^{-1}(\{A',B'\}) = (A,B).$$

This indicates

$$\Sigma_{\mathcal{D}(K)} \subseteq \Sigma_{\mathcal{P}} \Rightarrow \Sigma_{\mathcal{D}} \subseteq \Sigma_{\mathcal{P}(K)},$$

This proves the desired result. □

Remark 3.4. In model-based control, K is designed by analyzing the system $(A+BK,B)$ after feedback. Theorem 3.3 implies that the design problem can be solved by a solution of the analysis problem for the data $(X'_+, X'_-, U'_-) = (X_+, X_-, -KX_- + U_-)$ after feedback. Thus, in the context of the data informativity approach, a design method similar to the feedback design method of model-based control can be applied. As a concrete example, a study that applies this method to the design problem of dissipativity was proposed in our previous result [11]. The analytical problem of dissipativity was reported in [16], and based on the results of this analytical problem, Theorem 3.3 is applied to the design problem of dissipativity in [11] (in [11], the method is applied in a more concrete form than Theorem 3.3 presented in the discussion of the design problem of dissipativity).

4. CONCLUSIONS AND DIRECTIONS OF FUTURE RESEARCH

The study of data-driven control based on data informativity considers the data and the set of models that satisfy the data, given the data. It offers a new mathematical view different from that of model-based control and has been applied in various situations in system control theory. In the context of data-driven control based on data informativity, this study provides a correspondence between the transformation of data and the transformation of model sets by introducing a new algebraic method. This is an extension of the scope of the previous framework, which focused on the relationship between data and the model set that satisfies that data, to include the relationship between transformations of data and transformations of model sets. This data transformation technique is meaningful not only from a mathematical perspective but also from an engineering perspective. As an example, we demonstrate the derivation of controllability judgments for data informativity-based analysis via transformation theory, and that design problems and analysis problems, which have been discussed step-wise, can be reduced to analysis problems in the form of design problems involving controllers. Future directions of research include possible applications of the data transformation techniques to control systems other than those described in this paper.

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