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INTEGRATION BY PARTS FORMULA FOR McSHANE
AND KURZWEIL-HENSTOCK INTEGRALS
VIA DOUBLE LUSIN CONDITION

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Abstract. We prove an integration by parts formula for McShane and Kurzweil-Henstock integrals utilizing the double Lusin condition.

Keywords: McShane integral; Kurzweil-Henstock integral; differentiating integral; integration by parts; absolute continuity

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1. INTRODUCTION

It is known that for Lebesgue or Kurzweil-Henstock integration on a closed and bounded interval $[a, b]$, given an integrable function f with primitive F and a function G of bounded variation, fG is integrable and

$$(1.1) \quad \int_a^b fG = F(b)G(b) - \int_a^b F \, dG,$$

where the integral on the right-hand side of the equation is Riemann-Stieltjes integral. One may see, for example, [1] or [6] for further discussions. In the case of Lebesgue integration, or equivalently McShane, considering that a Lebesgue primitive is absolutely continuous, hence of bounded variation, the condition that f and g are integrable with primitives F and G , respectively, is enough to conclude that the product fG is integrable and equation (1.1) holds.

For Kurzweil-Henstock integration, since a Kurzweil-Henstock primitive is not necessarily of bounded variation, then we cannot immediately have an analogous

extension. However, in [10], an integral equivalent to Kurzweil-Henstock integral called *SL*-integral was introduced. It was shown that if f and g are *SL*-integrable with primitives F and G , respectively, then $fG + Fg$ is integrable and

$$(1.2) \quad \int_a^b (fG + Fg) = F(b)G(b) - F(a)G(a).$$

If we further add conditions assuring the existence of $\int_a^b Fg$, then we can have a formula similar to (1.1).

The goal of this paper is to provide a new proof for the integration by parts formulas for McShane and Kurzweil-Henstock integrals in the form of (1.2). More precisely, it is aimed to utilize the double Lusin condition characterization for McShane and Kurzweil-Henstock integrals to prove that given integrable functions f and g with primitives F and G , respectively, $Fg + fG$ is integrable with primitive FG and equation (1.2) holds.

The double Lusin condition, the inequalities in (1.3) below, which characterizes Kurzweil-Henstock integral, was introduced in [3] and its name double Lusin condition was given in [9]. To be precise, a function $f: [a, b] \rightarrow \mathbb{R}$ is Kurzweil-Henstock integrable if there exists a function F such that for every $\varepsilon > 0$ there exists a gauge δ on $[a, b]$ for which for any δ -fine partial division $D = \{(\xi, [u, v])\}$ of $[a, b]$ in $\Gamma_\varepsilon(F, f)$ we have

$$(1.3) \quad (D) \sum |F(v) - F(u)| < \varepsilon \quad \text{and} \quad (D) \sum |f(\xi)|(v - u) < \varepsilon,$$

where

$$\Gamma_\varepsilon(F, f) = \{(\xi, [u, v]): |F(v) - F(u) - f(\xi)(v - u)| \geq \varepsilon(v - u)\}.$$

A proof of a version of this result for McShane integral will be included in this paper.

Note that the double Lusin condition is a fundamental theorem of calculus that utilizes the idea of inner variation. The $\Gamma_\varepsilon(F, f)$ is an inner cover for the singularity of F relative to f . In the context of differentiating integrals, it was noted in [4] that inner variation is necessary on the real line when Vitali's covering theorem is inapplicable. This approach of differentiating integrals, particularly using Γ_ε , was initially discussed in [7] and independently in [11].

So far, in the literature, the available development after utilizing Γ_ε for a characterization via differentiating integrals is to prove a convergence theorem and discuss uniform differentiability and equiintegrability. See, for example, the results in [2], [3], [5], [12] for further details. In this paper, we move forward by proving integration by parts formulas via double Lusin condition characterization of McShane

and Kurzweil-Henstock integrals. While Vitali's covering theorem is applicable to Kurzweil-Henstock integration in the real-line, we did not limit our result to McShane integration to emphasize the development of integration by parts formula via double Lusin condition to a wider class of integrals and to introduce a proof of integration by parts formula in the division spaces in the real-line.

2. THE DOUBLE LUSIN CONDITION FOR MCSHANE AND KURZWEIL-HENSTOCK INTEGRALS

Let $[a, b]$ be a compact interval in \mathbb{R} . A partial division $D = \{(\xi, [u, v])\}$ is a finite collection of point-interval pairs $(\xi, [u, v])$ with $\xi \in [u, v]$, $[u, v] \subset [a, b]$, and the subintervals $[u, v]$ are nonoverlapping. In case the union of the subintervals $[u, v]$ in D is $[a, b]$, then we simply say that D is a division of $[a, b]$. A gauge on a set X is a function from X to the set of positive real numbers. Given a gauge δ on $[a, b]$, a point-interval pair $(\xi, [u, v])$ is said to be δ -fine if $[u, v] \subset (\xi - \delta(\xi), \xi + \delta(\xi))$. A partial division $D = \{(\xi, [u, v])\}$ is said to be δ -fine if every point-interval pair $(\xi, [u, v]) \in D$ is δ -fine. Note that for two gauges δ_1 and δ_2 , with $\delta_1(x) \leq \delta_2(x)$ for all $x \in [a, b]$, every δ_1 -fine partial division of $[a, b]$ is also δ_2 -fine. Recall that a function f on $[a, b]$ is said to be Henstock integrable if there exists a real number A for which given $\epsilon > 0$ there exists a gauge δ on $[a, b]$ such that for any δ -fine division $D = \{(\xi, [u, v])\}$ we have

$$\left| (D) \sum f(\xi)(v - u) - A \right| < \epsilon.$$

See for example [1], [6], [8] or [10]. Here A is the integral of f on $[a, b]$, it is unique, and we write

$$(H) \int_a^b f = A.$$

If for a point-interval pair $(\xi, [u, v])$, in the above discussions, we allow ξ to be anywhere within $[a, b]$ instead of being just confined in $[u, v]$, we obtain McShane point-interval pair, McShane partial division, McShane division and finally the definition of the McShane integral and we write

$$(M) \int_a^b f = A.$$

If a function f is McShane (or Kurzweil-Henstock) integrable on $[a, b]$, then it is McShane (or Kurzweil-Henstock) integrable on any subinterval $[u, v]$ of $[a, b]$. It

follows that given a McShane (or Kurzweil-Henstock) integrable function f on $[a, b]$, we can define a function F by $F(a) = 0$ and

$$F(x) = (M) \int_a^x f \quad \text{for } x \in (a, b]$$

$$\left(\text{or } F(x) = (H) \int_a^x f \quad \text{for } x \in (a, b] \right).$$

The function F is called the primitive of f .

A McShane primitive is $AC_\delta([a, b])$, in fact $AC[a, b]$, and a Kurzweil-Henstock primitive is $AC_\delta(X_i)$ for each i , for a collection of sets $\{X_i\}$ whose union is $[a, b]$. To be precise, F is $AC_\delta(X_i)$ if given $\varepsilon > 0$, there exists a positive number η and a gauge δ on $[a, b]$ such that for any McShane partial division $D = \{(\xi, [u, v])\}$ of $[a, b]$ whose tags $\xi \in X_i$ with $(D) \sum (v - u) < \eta$, we have $(D) \sum |F(v) - F(u)| < \varepsilon$.

As pointed out earlier in the introduction, a function $f: [a, b] \rightarrow \mathbb{R}$ is Kurzweil-Henstock integrable if there exists a function F such that for every $\varepsilon > 0$ there exists a gauge δ on $[a, b]$ for which for any δ -fine partial division $D = \{(\xi, [u, v])\}$ of $[a, b]$ in $\Gamma_\varepsilon(F, f)$ we have

$$(D) \sum |F(v) - F(u)| < \varepsilon \quad \text{and} \quad (D) \sum |f(\xi)|(v - u) < \varepsilon.$$

We will now prove the same result for the McShane integral.

Theorem 2.1. *Let f and F be functions on $[a, b]$. Then f is McShane integrable and F is its primitive if and only if given $\varepsilon > 0$ there exists a gauge δ on $[a, b]$ such that for any δ -fine McShane-partial partition $D = \{([u, v], \xi)\}$ of $[a, b]$ in $\Gamma_\varepsilon(F, f)$, we have*

$$(D) \sum |F(v) - F(u)| < \varepsilon \quad \text{and} \quad (D) \sum |f(\xi)(v - u)| < \varepsilon.$$

Proof. In case the conditions above hold, for a δ -fine McShane division $D = \{(\xi, [u, v])\}$ we have

$$\begin{aligned} \left| F(b) - F(a) - (D) \sum f(\xi)(v - u) \right| &\leq (D) \sum |F(v) - F(u) - f(\xi)(v - u)| \\ &= (D \cap \Gamma_\varepsilon) \sum |F(v) - F(u) - f(\xi)(v - u)| \\ &\quad + (D \setminus \Gamma_\varepsilon) \sum |F(v) - F(u) - f(\xi)(v - u)| \\ &< \varepsilon + \varepsilon + \varepsilon(b - a) \\ &= \varepsilon(2 + b - a). \end{aligned}$$

For the other direction, let $E_k = \{x \in [a, b]: k - 1 \leq |f(x)| < k\}$. Since f is McShane integrable with primitive F , then for all ε there exists a gauge $\delta_k: [a, b] \rightarrow (0, 1)$ such that for δ_k -fine McShane partial division D_k , we have

$$(D_k) \sum |F(v) - F(u) - f(\xi)(v - u)| < \frac{\varepsilon^2}{k2^{k+1}}.$$

Define a gauge δ such that if $\xi \in E_k$, $\delta(x) \leq \delta_k(\xi)$. Hence, if $D = \{(\xi, [u, v])\}$ is a δ -fine McShane partial division of $[a, b]$ contained in $\Gamma_\varepsilon(F, f)$,

$$\begin{aligned} (D) \sum |f(\xi)|(v - u) &< \sum_{k=1}^{\infty} k(D) \sum_{\xi \in E_k} |v - u| \\ &\leq \sum_{k=1}^{\infty} \frac{k}{\varepsilon} (D) \sum_{\xi \in E_k} |F(v) - F(u) - f(\xi)(v - u)| \\ &< \sum_{k=1}^{\infty} \frac{k}{\varepsilon} \cdot \frac{\varepsilon^2}{k2^{k+1}} \\ &= \frac{\varepsilon}{2}. \end{aligned}$$

Furthermore, δ may be modified, so that $(D) \sum |F(v) - F(u) - f(\xi)(v - u)| < \varepsilon/2$, and in which case

$$\begin{aligned} (D) \sum |F(v) - F(u)| &\leq (D) \sum |F(v) - F(v) - f(\xi)(v - u)| \\ &\quad + (D) \sum |f(\xi)|(v - u) < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon. \end{aligned}$$

□

3. INTEGRATION BY PARTS

We begin this section by presenting a result that describes how the singularity point-interval pairs Γ_ε behave when we are multiplying primitives.

Lemma 3.1. *Let f and g be McShane integrable functions on $[a, b]$ with primitives F and G , respectively. Then given $\varepsilon > 0$ there exists a gauge δ such that for any δ -fine partial division $D = \{(\xi, [u, v])\}$ we have*

$$D \cap \Gamma_\varepsilon(FG, Fg + fG) \subset (D \cap \Gamma_{\varepsilon_0}(F, f)) \cup (D \cap \Gamma_{\varepsilon_0}(G, g)),$$

where $\varepsilon_0 = \varepsilon/(2M + 2)$ and M is the higher value between the maximum values of F and G .

PROOF. Let $\varepsilon > 0$. Then by continuity of F and G , there exists a gauge δ on $[a, b]$ such that for any $\xi \in [a, b]$ and any t satisfying $|t - \xi| < \delta(\xi)$ we have

$$(3.1) \quad |f(\xi)[G(t) - G(\xi)]| < \varepsilon \quad \text{and} \quad |g(\xi)[F(t) - F(\xi)]| < \varepsilon_0.$$

Let $(\xi, [u, v])$ be a δ -fine McShane point-interval pair in $[a, b]$ such that $(\xi, [u, v]) \notin \Gamma_\varepsilon(F, f, \delta) \cup \Gamma_\varepsilon(G, g, \delta)$, that is,

$$(3.2) \quad |F(v) - F(u) - f(\xi)(v - u)| < \varepsilon_0(v - u)$$

and

$$(3.3) \quad |G(v) - G(u) - g(\xi)(v - u)| < \varepsilon_0(v - u).$$

It follows from (3.3), (3.2) and (3.1), applied successively to the terms of the right-hand side of the inequality below, that

$$\begin{aligned} & |F(v)G(v) - F(u)G(u) - (F(\xi)g(\xi) + f(\xi)G(\xi))(v - u)| \\ & \leq M|G(v) - G(u) - g(\xi)(v - u)| + M|F(v) - F(u) - f(\xi)(v - u)| \\ & \quad + |g(\xi)[F(v) - F(\xi)]|(v - u) + |f(\xi)[G(u) - G(\xi)]|(v - u) \\ & < M\varepsilon_0(v - u) + M\varepsilon_0(v - u) + 2\varepsilon_0(v - u) \\ & = \varepsilon(v - u). \end{aligned}$$

□

We now present our integration by parts formulas for McShane and Kurzweil-Henstock integrals.

Theorem 3.1. *Let f and g be McShane integrable functions on $[a, b]$ with primitives F and G , respectively. Then $Fg + fG$ is McShane integrable with primitive FG .*

PROOF. First of all, since F and G are McShane primitives, then F and G are $AC_\delta([a, b])$. Let $\varepsilon > 0$. Then by taking minimum, there exist a common real-number $\eta < 1$ with

$$(3.4) \quad 0 < \eta < \varepsilon_0$$

and a common gauge λ such that for any λ -fine partial McShane partial division $D = \{(\xi, [u, v])\}$ of $[a, b]$, satisfying $\sum(v - u) < 2\eta$, we have

$$\sum |F(v) - F(u)| < \varepsilon_0 \quad \text{and} \quad \sum |G(v) - G(u)| < \varepsilon_0,$$

where

$$(3.5) \quad \varepsilon_0 = \min \left\{ \frac{\varepsilon}{2M+2}, \frac{\varepsilon}{4M}, \frac{\varepsilon}{2(b-a+2M)} \right\}$$

and with M being as defined in Lemma 3.1. By McShane-integrability of f and g , considering Theorem 2.1, there exist gauges δ_f and δ_g such that for any δ_f -fine McShane partial division D of $[a, b]$, we have

$$(3.6) \quad (D \cap \Gamma_{\eta^2}(F, f)) \sum |F(v) - F(u)| < \eta^2 \quad \text{and} \quad (D \cap \Gamma_{\eta^2}) \sum |f(\xi)(v-u)| < \eta^2,$$

and for any δ_g -fine partial McShane division D of $[a, b]$,

$$(3.7) \quad (D \cap \Gamma_{\eta^2}(G, g)) \sum |G(v) - G(u)| < \eta^2 \quad \text{and} \quad (D \cap \Gamma_{\eta^2}) \sum |g(\xi)(v-u)| < \eta^2.$$

Further, by Lemma 3.1, there exists a gauge γ on $[a, b]$ such that if $D = \{(\xi, [u, v])\}$ is γ -fine partial McShane division,

$$(3.8) \quad D \cap \Gamma_{\varepsilon}(FG, Fg + fG) \subset (D \cap \Gamma_{\varepsilon_0}(F, f)) \cup (D \cap \Gamma_{\varepsilon_0}(G, g)).$$

Define a gauge δ such that $\delta(x) = \min\{\lambda(x), \delta_f(x), \delta_g(x), \gamma(x)\}$ so that every δ -fine partial McShane division will also be λ -fine, δ_f -fine, δ_g -fine and γ -fine. Let $D = \{(\xi, [u, v])\}$ be a δ -fine McShane partial division of $[a, b]$ in $\Gamma_{\varepsilon}(FG, Fg + fG)$. Considering equation (3.5) and the containment in (3.8), we have

$$D \setminus \Gamma_{\varepsilon_0}(F, f) \subset D \cap \Gamma_{\varepsilon_0}(G, g) \quad \text{and} \quad D \setminus \Gamma_{\varepsilon_0}(G, g) \subset D \cap \Gamma_{\varepsilon_0}(F, f).$$

Note also that considering (3.4), $\eta^2 < \eta < \varepsilon_0$ and hence,

$$\Gamma_{\varepsilon_0}(F, f) \subset \Gamma_{\eta}(F, f) \subset \Gamma_{\eta^2}(F, f)$$

and

$$\Gamma_{\varepsilon_0}(G, g) \subset \Gamma_{\eta}(G, g) \subset \Gamma_{\eta^2}(G, g).$$

Now, since $D \cap \Gamma_{\eta}(G, g) \subset D \cap \Gamma_{\eta^2}(G, g)$, which implies that

$$\begin{aligned} \eta(D \cap \Gamma_{\eta}(G, g)) \sum |I| &\leq (D \cap \Gamma_{\eta}(G, g)) \sum |G(v) - G(u) - g(\xi)(v-u)| \\ &\leq (D \cap \Gamma_{\eta}(G, g)) \sum |G(v) - G(u)| \\ &\quad + (D \cap \Gamma_{\eta}(G, g)) \sum |g(\xi)(v-u)| \\ &\leq (D \cap \Gamma_{\eta^2}(G, g)) \sum |G(v) - G(u)| \\ &\quad + (D \cap \Gamma_{\eta^2}(G, g)) \sum |g(\xi)(v-u)|, \end{aligned}$$

then considering (3.7) we have

$$\eta(D \cap \Gamma_\eta(G, g)) \sum |I| \leq 2\eta^2.$$

Hence

$$(D \setminus \Gamma_{\varepsilon_0}(F, f)) \sum |I| \leq (D \cap \Gamma_{\varepsilon_0}(G, g)) \sum |I| \leq (D \cap \Gamma_\eta(G, g)) \sum |I| < 2\eta.$$

Analogously, it can also be shown that

$$(D \setminus \Gamma_{\varepsilon_0}(G, g)) \sum |I| < 2\eta.$$

It then follows that

$$(3.9) \quad (D \setminus \Gamma_{\varepsilon_0}(F, f)) \sum |F(v) - F(u)| < \varepsilon_0,$$

and

$$(3.10) \quad (D \setminus \Gamma_{\varepsilon_0}(G, g)) \sum |G(v) - G(u)| < \varepsilon_0.$$

Hence, considering inequalities (3.6), (3.7), (3.9) and (3.10),

$$\begin{aligned} (D) \sum |F(v)[G(v) - G(u)]| &= (D \setminus \Gamma_{\varepsilon_0}(G, g)) \sum |F(v)[G(v) - G(u)]| \\ &\quad + (D \cap \Gamma_{\varepsilon_0}(G, g)) \sum |F(v)[G(v) - G(u)]| \\ &< M\varepsilon_0 + M\varepsilon_0 \end{aligned}$$

and also

$$\begin{aligned} (D) \sum |G(v)[F(v) - F(u)]| &= (D \setminus \Gamma_{\varepsilon_0}(F, f)) \sum |G(v)[F(v) - F(u)]| \\ &\quad + (D \cap \Gamma_{\varepsilon_0}(F, f)) \sum |G(v)[F(v) - F(u)]| \\ &< M\varepsilon_0 + M\varepsilon_0. \end{aligned}$$

It follows from the recent two inequalities above and equation (3.5) that

$$\begin{aligned} (D) \sum |F(v)G(v) - F(u)G(u)| &\leq (D) \sum |F(v)[G(v) - G(u)]| \\ &\quad + (D) \sum |G(u)[F(v) - F(u)]| \\ &< 4M\varepsilon_0 \\ &\leq \varepsilon. \end{aligned}$$

Also, by inequalities (3.10), (3.7) and (3.4), and equation (3.5), we have

$$\begin{aligned}
 (D) \sum |F(\xi)g(\xi)(v-u)| &= M(D \setminus \Gamma_{\varepsilon_0}(G, f)) \sum |g(\xi)(v-u) - (G(v) - G(u))| \\
 &\quad + M(D \setminus \Gamma_{\varepsilon_0}(G, f)) \sum |G(v) - G(u)| \\
 &\quad + M(D \cap \Gamma_{\varepsilon_0}(G, f)) \sum |g(\xi)(v-u)| \\
 &< \varepsilon_0(b-a) + M\varepsilon_0 + M\varepsilon_0 \\
 &\leq \frac{\varepsilon}{2}.
 \end{aligned}$$

Similarly,

$$(D) \sum |G(\xi)f(\xi)(v-u)| < \varepsilon_0(b-a + 2M) \leq \frac{\varepsilon}{2}.$$

Hence

$$(D) \sum |(F(\xi)g(\xi) + f(\xi)G(\xi))|(v-u) < \varepsilon.$$

By Theorem 2.1, $Fg + fG$ is McShane integrable with primitive FG . □

The $AC_\delta([a, b])$ condition above may be replaced by only AC . The reason we appeal to the AC_δ condition is that a straightforward analogous proof will be immediately available for Kurzweil-Henstock integral.

Theorem 3.2. *Let f and g be Kurzweil-Henstock integrable functions on $[a, b]$ with primitives F and G , respectively. Then $Fg + fG$ is Henstock-integrable with primitive FG .*

Proof. Consider a countable collection of sets $\{X_i\}$ whose union is $[a, b]$ such that both F and G are $AC_\delta(X_i)$ for each i . Then the rest follows analogously to McShane version. □

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