Kybernetika

S. Sandhiya; Anuradha Dhanapal

Solving neutrosophic bi-objective transportation problem using neutrosophic TOPSIS approach

Kybernetika, Vol. 62 (2026), No. 4, 584–617

Persistent URL: http://dml.cz/dmlcz/153776

Terms of use:

© Institute of Information Theory and Automation AS CR, 2026

Institute of Mathematics of the Czech Academy of Sciences provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This document has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* http://dml.cz

KYBERNETIKA — VOLUME *62* (2026), NUMBER 4, PAGES 584–617

SOLVING NEUTROSOPHIC BI-OBJECTIVE TRANSPORTATION PROBLEM USING NEUTROSOPHIC TOPSIS APPROACH

S. Sandhiya and Anuradha Dhanapal

The Transportation Problem is an optimization problem which aims to optimize the objective function of the products from various sources to destinations. In real-world situations, decision makers face a variety of conflicting objectives such as transportation costs, time, deterioration rates and so on which yields them with comprehensive profits. In these situations, the transportation problem becomes Multi-objective Transportation Problem. In real life, the parameters are unpredictable due to factors such as incomplete input data, market fluctuations, environmental influences and so on. To address these uncertainties, the parameters are expressed as single-valued trapezoidal neutrosophic number. This article explores a bi-objective transportation problem in a neutrosophic environment, aiming to determine the Pareto optimal solution. First, the neutrosophic problem is transformed into its deterministic problem using the Weighted Possibility Mean. In road safety, delineator design and placement often involve conflicting objectives like visibility, cost, installation time and safety. Balancing these is challenging due to complex road conditions, traffic, geometry and weather. In this point of view, we have proposed the approach namely, Neutrosophic TOPSIS approach for single valued trapezoidal neutrosophic numbers to transform the bi-objective problem into its single objective problem. Then the problem is solved by LINGO software to determine the Pareto optimal solution. Our approach provides a systematized method to assess alternatives and identify the optimal delineator design by balancing conflicting objectives, aiding decision makers in selecting the most suitable road configuration. A Sensitivity Analysis is performed to determine the sensitivity ranges for all the parameters of the problem. To illustrate the applicability, effectiveness and usefulness of the approach, a numerical example is provided and the comparison was made with the existing approaches. It is observed that the obtained Pareto optimal solution extracted from the proposed approach provides better result than the existing approaches. Finally, the conclusions are drawn along with the future work.

Keywords: bi-objective transportation problem, single-valued trapezoidal neutrosophic numbers, weighted possibility mean, neutrosophic TOPSIS approach, sensitivity analysis

Classification: 90B06, 90C29, 90C70

DOI: 10.14736/kyb-2026-4-0584

Abbreviations

BOTP Bi-Objective Transportation Problem
DM Decision Maker
FGPA Fuzzy Goal Programming Approach
FS Fuzzy Sets
IFS Intuitionistic Fuzzy Sets
MF Membership Function
MOFCSTP Multi-Objective Fixed Charge Solid Transportation Problem
MOOP Multi-Objective Optimization Problem
MOTP Multi-Objective Transportation Problem
NBOTP Neutrosophic Bi-Objective Transportation Problem
NFPA Neutrosophic Fuzzy Programming Approach
NIS Negative Ideal Solution
NMF Non-Membership Function
NS Neutrosophic Set
PIS Positive Ideal Solution
SA Sensitivity Analysis
SOOP Single Objective Optimization Problem
SVNS Single-Valued Neutrosophic Sets
SVTrNNs Single-Valued Trapezoidal Neutrosophic Numbers
TOPSIS Technique for Order Preference by Similarity to Ideal Solution
TP Transportation Problem
WPM Weighted Possibility Mean

1. INTRODUCTION

Transportation Problem (TP) is a crucial application in the decision making problem. The goal of TP is to optimize the objective function, considering availability and requirements, by determining the products that transported from sources to destinations. Hitchcock [19] was the first to introduce the TP in 1941 and then Koopmans [30] in 1947 developed the solution procedure to obtain the optimal solution. Many real-life applications can be modelled as TPs with multiple, conflicting, and incompatible objective functions. To handle these situations, single objective TP sets off into Multi-objective TP (MOTP). Some unpredictable factors arises in transporting the products such as lack of accurate information in the system, inadequate transportation policy, fluctuation and fabricated disasters in the financial market and so on. Fuzzy Set (FS) was introduced by Zadeh [68], provides a mathematical framework for modelling and analysing systems characterized by vagueness, imprecision and uncertainty. It is defined by Membership Function (MF) that assign a degree of belonging between 0 and 1. Although classical FS effectively represent uncertainty using only a single degree of MF, which may be insufficient in situations involving incomplete or conflicting information. To overcome this, Intuitionistic Fuzzy Set (IFS) was introduced by Atanassov [4] which is an extension of FS by incorporating an additional degree of uncertainty. While FS represent uncertainty using MF that defines the degree of truth (or belongingness) of an element, IFS which include both the MF and non-membership function (NMF) for each element.

While IFS explicitly models hesitation, it still assumes that the Decision Maker (DM) can assign precise membership and non-membership values. In many real-world scenarios, however DMs may hesitate among several possible membership degrees rather than a single value. So Hesitant fuzzy set was introduced by Torra [65] when defining the membership of an element, the difficulty of establishing the membership degree is not because we have a margin of error or some possibility distribution on the possible values, but we have a set of possible values. Another important extension is the Bipolar fuzzy set introduced by Lee [33] which addresses situations where both positive and negative information coexist. Unlike FS that model only the degree of satisfaction, bipolar fuzzy sets allow membership values in the interval $[-1, 1]$, where positive values represent favorable information and negative values represent unfavorable information. In short, Bipolar fuzzy sets extend the FS by using a membership range of $[-1]$ instead of $[0]$, which allows the representation of both positive and negative aspects of an element simultaneously. The picture fuzzy set was introduced by Cuong [11] which is an extension of the IFS that incorporates an additional degree to represent neutrality or abstention, along with membership and non-membership degrees. Despite these advancements, the above frameworks still impose certain constraints on the relationships among uncertainty components. To address this issue, Neutrosophic Set (NS) is an extension of IFS introduced by Smarandache [59] which extends the framework by incorporating three independent components namely, truth (T), indeterminacy (I) and falsity (F) each ranging independently within the interval $[0, 1]$. The independent treatment of indeterminacy is a major motivation for adopting neutrosophic sets in the present problem. In many real-world decision-making situations, uncertainty does not arise from partial truth and falsity, but also involves an additional component of indeterminacy caused by ambiguity, inconsistency, hesitation, or incomplete information. Fuzzy sets address uncertainty through membership degree, while intuitionistic fuzzy sets improve this representation by considering both membership and non-membership degrees. However, these frameworks may not explicitly capture situations where available information is neutral, conflicting, or unresolved. So Neutrosophic sets overcome this limitation by introducing an independent indeterminacy membership, which is particularly significant in practical optimization and decision-making problems with uncertain parameters. For instance, transportation costs may fluctuate due to change in fuel prices or market instability, while supply and demand values may be affected by incomplete data or forecasting errors. In such cases, uncertainty is not simply a matter of assigning degrees of truth or falsity, but also involves indeterminate information that cannot be neglected. Such indeterminate situations cannot be adequately represented using fuzzy or intuitionistic fuzzy frameworks, whereas neutrosophic sets can explicitly model them. As a result, fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, bipolar fuzzy sets, and picture fuzzy sets can all be viewed as particular cases of neutrosophic sets under suitable constraints.

Fuzzy set theory and neutrosophic set theory are closely connected through their common objective of modelling uncertainty in decision-making optimization problems. Fuzzy sets represent uncertainty through a membership degree indicating the extent to which an element belongs to a set. This framework was later extended by intuitionistic fuzzy sets through the inclusion of both membership and non-membership degrees, thereby

providing a richer description of uncertainty. Neutrosophic sets are a further generalization of fuzzy-based frameworks. They extend fuzzy and intuitionistic fuzzy sets by introducing three independent functions: truth-membership, indeterminacy-membership, and falsity-membership. This additional indeterminacy component establishes the fundamental connection between fuzzy and neutrosophic sets. In this sense, fuzzy sets can be viewed as a special case or foundational precursor of neutrosophic sets. In fuzzy optimization, uncertainty is commonly handled by expressing the parameters using fuzzy numbers such as triangular, trapezoidal fuzzy numbers and so on which offer simplicity and computational efficiency. Small changes in the input of triangular fuzzy numbers may significantly influence the outcome because uncertainty is focused on a single peak value. Trapezoidal fuzzy numbers generalize the triangular fuzzy numbers which consider a range of values to be equally acceptable and provide additional flexibility to represent DMs opinions and imprecise data. Building upon this framework, neutrosophic optimization provides a more generalized and flexible representation of uncertainty by allowing the parameters to be modelled using neutrosophic numbers such as triangular, trapezoidal neutrosophic numbers and so on. Trapezoidal neutrosophic numbers represent uncertainty over an interval and explicitly account for indeterminacy which reduces sensitivity to minor variations. As a result, the obtained solutions are more stable and reliable, particularly in decision-making problems involving uncertain data. The parameters are represented using single-valued trapezoidal neutrosophic numbers (SVTrNNs) whose truth, indeterminacy and falsity membership values are finite and bounded within the interval [0,1]. Accordingly, all neutrosophic concepts employed in this study are finite and well-defined, ensuring mathematical and computational feasibility. When the parameters of bi-objective TP (BOTP) such as transportation cost, deterioration cost, supply and demand are represented in neutrosophic data then the classical BOTP is transformed into neutrosophic bi-objective TP (NBOTP). As uncertain data cannot be solved explicitly, sometimes it provides the negative value for the decision variable which violates the non-negativity constraints of the decision-making problem. To avoid such violations and ensure the feasibility of solutions, it is necessary to convert neutrosophic data into deterministic representations before model implementation. Accordingly, various de-neutrosophication techniques have been proposed in the literature such as score function [56], ranking function [66], weighted possibility mean [51] and so on to transform the SVTrNNs into its deterministic numbers. Here in this paper, we have applied the Weighted Possibility Mean (WPM) for SVTrNNs to transform the neutrosophic numbers into its deterministic numbers.

In some situations, the decision-making problem involves multiple and conflicting objectives, such as cost, time and efficiency which cannot be optimized simultaneously using classical optimization techniques. In general, there does not exist a single solution that optimizes all objective functions at the same time instead, a set of Pareto-optimal solutions is obtained. Solving multi-objective problem directly is computationally challenging and does not yield a unique optimal solution which is difficult for decision makers to select the most appropriate alternative. Therefore, it is necessary to transform the multi-objective problem into single-objective problem which helps the DMs by reducing computational complexity in decision-making problems and decreasing the cognitive load required to interpret trade-offs. This leads to faster convergence and improve overall

outcomes. Accordingly, Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a method of multi-criteria decision-making employed to assess and rank a set of alternatives. The core idea behind TOPSIS is that the optimal alternative should be closest to the Positive Ideal Solution (PIS) and farthest from the Negative Ideal Solution (NIS). The TOPSIS method was initially proposed by Hwang and Yoon [21] which has become a widely adopted tool for multi-attribute decision-making problems. Abo-Sinna et al. [1] utilized TOPSIS method for solving the multi-objective non-linear programming problem. The proposed TOPSIS approach is solved using the LINGO software which significantly reduces run time (calculated as 0.01s), particularly for large-scale and complex datasets. The classical method such as MODI method are suitable only for small-sized data and highly time-consuming, whereas their applicability becomes limited for large and complex multi-objective problems due to increased computational burden and solution complexity. In contrast, our proposed approach improves efficiency, reduces computational time, and enables the handling of large-dimensional data more effectively. Moreover, solving a multi-objective optimization problem in its original form often leads to significant complexity due to the simultaneous consideration of conflicting objectives, making direct solution procedures computationally challenging and difficult for decision makers to interpret. Therefore, transforming the multi-objective problem into an equivalent single-objective model through the TOPSIS approach provides a practical and efficient alternative. This transformation reduces computational complexity, simplifies the decision-making process, and facilitates faster convergence toward an optimal compromise solution. In addition, the use of TOPSIS approach implemented through LINGO software minimizes run time, enhances computational efficiency and makes the proposed approach suitable for solving large-scale complex problems. In this paper, we have proposed the neutrosophic TOPSIS approach with different weights that corresponds based on priority of objective functions, DMs can systematically explore a set of Pareto-optimal solutions under different priority scenarios. This provides valuable insights into the trade-offs between conflicting objectives and allows DMs to make decisions. Sensitivity Analysis (SA) analyse the variations in the objective functions and constraints with respect to their optimal values and validity ranges. A sensitivity analysis identifies the sensitivity ranges without changing the current optimal solution, which minimizes error risks.

1.1. Literature review

This section consists of the literature survey of MOTP, fuzzy MOTP, intuitionistic MOTP, neutrosophic MOTP and TOPSIS approach. Pandian and Natarajan [42] introduced the zero point method to obtain the optimal solution for TP. Lee and Moore [32] modified goal programming approach to solve the MOTP. Aneja et al. [3] proposed the approach to determine the extreme points for bi-criteria TP. Isermann [22] proposed an algorithm for solving MOTP to obtain the efficient solutions. Numerous researchers such as Gupta and Gupta [17], Ringuest and Rinks [46], Kasana and Kumar [26], Bai and Yao [6], Pandian and Anuradha [43], Nomani et al. [41] and Kaur et al. [27] employed various approaches for obtaining the optimal compromise solution to solve the MOTP. Niluminda and Ekanayake [40] utilized the geometric mean and penalty technique for solving the MOTP. Rajbhar and Das [44] discussed the advancements techniques for

solving the MOTP. Zhu et al. [69] proposed approximation algorithms for solving the fixed charge TP. Mondal et al. [38] utilized the artificial bee colony and multi-objective optimization techniques for solving the fixed charge MOTP. Various authors like Gupta et al. [16], Dhanasekar et al. [13], Singh et al. [57], Bagheri et al. [5], Niksirat [39], Sobana and Anuradha [58] and Kacher et al. [23] have developed the various methods for obtaining the fuzzy optimal compromise solution to solve the fuzzy MOTP. Revathi et al. [45] utilized the neutrosophic compromise approach to obtain the Pareto optimal solution for solving the chance constraint model for uncertain MOTP. Roy et al. [48] extended TOPSIS, weighted goal programming and fuzzy programming approaches for obtaining the Pareto optimal solution to solve the multi-objective multi-item fixed charge solid TP. Kuiri and Das [31] utilized generalized reduced gradient technique and applied TOPSIS method for solving the fuzzy multi-objective multi-item solid TP. Kartli et al. [25] proposed a new heuristic algorithm for solving the fuzzy TP to obtain fuzzy optimal solution. The weighted average operator and average value approach was developed by Bera and Mondal [8] to solve the triangular intuitionistic MOTP. Wang et al. [67] analyses the relations and operations over single-valued neutrosophic sets (SVNS). Deli and Subas [12] addressed the specific forms of Single-valued trapezoidal neutrosophic numbers (SVTrNNs) and applied weighted aggregation operator to solve the multi-criteria decision making problem. Hosseinzadeh and Tayyebi [20] utilized the neutrosophic FPA to solve the neutrosophic bi-objective linear programming problem. The weighted possibility mean (WPM) was proposed by Sandhiya and Dhanapal [51] to convert the neutrosophic data into its deterministic data and then solved using the dripping method for the neutrosophic bi-objective TP. Sandhiya and Dhanapal [52] proposed the min zero min cost approach for solving the neutrosophic four dimensional fixed charge TP. Salama et al. [53] discussed the neutrosophic logic with fuzzy inference in decision making under uncertainty. Smarandache and Mohana [64] discussed the fuzzy, intuitionistic and neutrosophic set theories and its applications in decision analysis.

Various researchers such as Kavitha and Anuradha [28],[29] have analysed the SA. Hansen et al. [18] performed the SA for multi-objective programming. The SA was investigated to the parameters for the multi-objective fixed charge TP by Midya and Roy [36]. The SA was investigated for the intuitionistic multi-objective fixed charge solid TP (MOFCSTP) by Ghosh et al. [15]. Ghosh et al. [14] investigated the SA to the right hand side parameters for MOFCSTP. Midya and Roy [37] examined the SA to the right hand side constraints for the intuitionistic multi-stage multi-objective FCSTP. Samanta et al. [54] and Bind et al. [9] performed the SA for the objective function parameters. Table 1 represents the list of literature survey for bi-objective and multi-objective TP using various transformation approaches.

1.2. Research gap, motivation and contribution

The research gap, motivation and novelty of the presented work are summarised as follows:

- Based on the above literature survey, it is clear that the work primarily focuses on bi-objective transportation problem (BOTP) for triangular fuzzy numbers [55] and triangular intuitionistic fuzzy numbers [49] using TOPSIS approach by as-

Authors, years	Parameters			Objective		Transformation approaches	Solutions obtained
	F	IF	NF	Two	More than 2		
Zimmermann [70], 1978	✓			✓		Fuzzy programming approach	Optimal compromise solution
Ahmad and Adhami [2], 2019	✓			✓		Neutrosophic compromise programming approach	
Bodkhe [10], 2023	✓			✓		Fuzzy programming approach	
Miah et al. [35], 2023	✓				✓	Fuzzy programming approach	
Roy and Midya [49], 2019		✓			✓	Intuitionistic fuzzy programming and TOPSIS approaches	Pareto-optimal solution
Roy et al. [50], 2018		✓			✓	Intuitionistic fuzzy programming and goal programming approaches	Optimal compromise solution
Mahajan and Gupta [34], 2021		✓		✓		Various membership functions	
Shivani and Rani [55], 2024		✓		✓		Weighted Tchebycheff metrics programming, Intuitionistic fuzzy programming approach and Fuzzy TOPSIS approach	Pareto-optimal solution
Risk Allah et al. [47], 2018			✓	✓		Neutrosophic fuzzy programming approach	Optimal compromise solution
Present paper			✓	✓		Proposed Neutrosophic TOPSIS approach	Pareto-optimal solution
Note: F- Fuzzy; IF- Intuitionistic fuzzy; NF- Neutrosophic fuzzy							

Tab. 1. Literature survey for Bi-objective and Multi-objective TP using various transformation approaches.

signing equal weights which transforms the multi-objective optimization problem (MOOP) into the single objective optimization problem (SOOP). To the best of our knowledge, BOTP under neutrosophic environment using TOPSIS approach by assigning different weights have not proposed for single valued trapezoidal neutrosophic numbers (SVTrNNs).

- As the problem has multiple objective functions, each aiming for optimal values. Obtaining an optimal solution that simultaneously optimizes all objective functions is highly challenging and time-consuming. In this situation, the multi-objective is reduced to single objective problem in order to reduce the computational efforts for the DM. This motivates us to propose the neutrosophic TOPSIS approach for SVTrNNs which is based on intuitionistic TOPSIS approach [49] for triangular intuitionistic fuzzy numbers. By employing the proposed approach with different weights that corresponds based on priority of objective functions, DMs can systematically explore a set of Pareto-optimal solutions under different priority scenarios is the novelty of this study. This provides valuable insights into the trade-offs between conflicting objectives and allows DMs to make decisions.

- In this paper, we have considered the neutrosophic bi-objective transportation problem where the first objective and second objective represents the SVTrN trans-

portation cost and deterioration cost respectively. First, the weighted possibility mean [51] is applied to convert the neutrosophic data into its equivalent deterministic data. The proposed neutrosophic TOPSIS approach is utilized to transform the bi-objective TP into the single objective TP. Then the problem is solved by LINGO software to determine the Pareto optimal solution for various weights and distance functions for $1 \leq q < \infty$. The sensitivity analysis is performed to determine the sensitivity ranges for all the parameters of the problem. To illustrate the applicability, effectiveness and usefulness of the proposed approach, example is illustrated and the comparison was made with the existing approaches. The proposed approach demonstrates strong computational efficiency and the capability to assess the relative performance of each alternative.

The paper is classified into following categories: Section 2 follows with basic concepts and preliminaries. The mathematical formulation of BOTP under neutrosophic environment are represented in Section 3. Section 4 presents the solution approach using the proposed neutrosophic TOPSIS approach to determine the Pareto optimal solution. Section 5 illustrates the numerical example while Section 6 presents the sensitivity analysis. Section 7 provides the comparative analysis while Section 8 presents the concluding remarks and explores future prospects.

2. PRELIMINARIES AND DEFINITIONS OF NEUTROSOPHIC SETS

In this section, some basic definitions of NS [59], SVN sets [67], SVTrNNs and their arithmetic operations [12] are presented. Table 2 represents the comparison between classical, fuzzy and generalized sets.

Set Type	**Membership Representation**	**Key Components**	**Constraints**	**Generalization**
Classical (Crisp) Set	0 or 1	Belonging or not belonging	Strictly binary	Fuzzy sets
Fuzzy Set [68]	Membership value in $[0, 1]$	Truth (T)	$0 \leq T \leq 1$	Intuitionistic fuzzy sets
Intuitionistic Fuzzy Set [4]	Membership (T) and non-membership (F)	Truth (T), Falsity (F)	$T + F \leq 1$	Picture fuzzy sets, Neutrosophic sets
Hesitant Fuzzy Set [65]	Set of possible membership	$\{T_1, T_2, \ldots, T_k\}$	Each $T_i \in [0, 1]$	Neutrosophic sets
Bipolar Fuzzy Set [33]	Positive and negative membership	Positive (T^+) and Negative (T^-)	$T^+, T^- \in [0, 1]$	Neutrosophic sets
Picture Fuzzy Set [11]	Positive, neutral and negative	Positive (T), Neutral (I) and Falsity (F)	$T + I + F \leq 1$	Neutrosophic sets
Neutrosophic Set [59]	Truth, indeterminacy and falsity	T (truth), I (indeterminacy), F (falsity)	$T, I, F \in [0, 1^+]$	General framework

Tab. 2. Comparison between classical, fuzzy and generalized sets.

Neutrosophic set serves as a general framework that encompasses and extends the capabilities of other sets like fuzzy, intuitionistic fuzzy, hesitant fuzzy, bipolar fuzzy and picture fuzzy sets. This makes it a powerful and unifying tool for handling various types of imperfect information. In this paper, we have considered the single valued trapezoidal neutrosophic numbers to deal the uncertainties.

Definition 2.1. Neutrosophic Set [59]: Let X be a universe discourse. A NS L in X is characterized by a truth MF $A_{\bar{L}^N}(x)$, indeterminacy MF $B_{\bar{L}^N}(x)$ and a falsity MF $C_{\bar{L}^N}(x)$. $A_{\bar{L}^N}(x), B_{\bar{L}^N}(x)$ and $C_{\bar{L}^N}(x)$ are real standard elements of [0,1]. It can be written as

$$\bar{L}^N = \left\{ \langle x, A_{\bar{L}^N}(x), B_{\bar{L}^N}(x), C_{\bar{L}^N}(x) \rangle : x \in X, A_{\bar{L}^N}(x), B_{\bar{L}^N}(x), C_{\bar{L}^N}(x) \in \left]0^-, 1^+\right[\right\}$$

There is no restriction on the sum of $A_{\bar{L}^N}(x), B_{\bar{L}^N}(x)$ and $C_{\bar{L}^N}(x)$, so $0^- \leq A_{\bar{L}^N}(x) + B_{\bar{L}^N}(x) + C_{\bar{L}^N}(x) \leq 3^+$. Here we have defined the infinite neutrosophic sets.

Definition 2.2. Single-valued neutrosophic sets [67]: A SVNS $\bar{L}^{SVN}$ of a non-empty set X is defined as follows:

$$\bar{L}^{SVN} = \{ \langle x, A_{\bar{L}^N}(x), B_{\bar{L}^N}(x), C_{\bar{L}^N}(x) \rangle : x \in X \}$$

where $A_{\bar{L}^N}(x), B_{\bar{L}^N}(x), C_{\bar{L}^N}(x) \in [0,1]$ for each $x \in X$ and $0 \leq A_{\bar{L}^N}(x) + B_{\bar{L}^N}(x) + C_{\bar{L}^N}(x) \leq 3$.

Definition 2.3. Single-valued trapezoidal neutrosophic number [12]:
Let $\sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \in [0,1]$ and $p, q, r, s \in \Re$ such that $p \leq q \leq r \leq s$. Then a SVTrNN, $\tilde{d}^N = \left\langle (p,q,r,s); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \right\rangle$ is a special NS on $\Re$, whose truth membership, indeterminacy membership and falsity membership functions are given below:

$$\mu_{\tilde{d}^N} = \begin{cases} \sigma_{\tilde{d}^N} \left(\dfrac{x-p}{q-p} \right), & p \leq x < q \\ \sigma_{\tilde{d}^N}, & q \leq x \leq r \\ \sigma_{\tilde{d}^N} \left(\dfrac{s-x}{s-r} \right), & r \leq x \leq s \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_{\tilde{d}^N} = \begin{cases} \dfrac{q - x + \lambda_{\tilde{d}^N}(x-p)}{l-k}, & p \leq x < q \\ \lambda_{\tilde{d}^N}, & q \leq x \leq r \\ \dfrac{x - r + \lambda_{\tilde{d}^N}(s-x)}{s-r}, & r \leq x \leq s \\ 0, & \text{otherwise} \end{cases}$$

$$\rho_{\tilde{d}^N} = \begin{cases} \dfrac{q - x + \tau_{\tilde{d}^N}(x-p)}{q-p}, & p \leq x < q \\ \tau_{\tilde{d}^N}, & q \leq x \leq r \\ \dfrac{x - r + \tau_{\tilde{d}^N}(s-x)}{s-r}, & r \leq x \leq s \\ 1, & \text{otherwise} \end{cases}$$

where $\sigma_{\tilde{d}^N}$, $\lambda_{\tilde{d}^N}$ and $\tau_{\tilde{d}^N}$ denote the maximum truth, minimum indeterminacy, and minimum falsity membership degrees respectively. A SVTrNN $\tilde{d}^N = \left\langle (p,q,r,s); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \right\rangle$ may be expressed as an ill-defined quantity of p, which is approximately equal to [q,r].

Definition 2.4. Arithmetic Operations on SVTrNNs [12]:
Let $\tilde{d}^N = \langle (p,q,r,s); \sigma_{\tilde{d}^N}, \lambda_{\tilde{d}^N}, \tau_{\tilde{d}^N} \rangle$ and $\tilde{g}^N = \left\langle (p^{'},q^{'},r^{'},s^{'}); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \right\rangle$ be two SVTrNNs. The arithmetic operations on $\tilde{d}^N$ and $\tilde{g}^N$ are:

1. $\tilde{d}^N + \tilde{g}^N = \left\langle (p+p^{'},q+q^{'},r+r^{'},s+s^{'}); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle$
2. $\tilde{d}^N - \tilde{g}^N = \left\langle (p-p^{'},q-q^{'},r-r^{'},s-s^{'}); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle$
3. $\tilde{d}^N * \tilde{g}^N = \begin{cases} \left\langle (pp^{'},qq^{'},rr^{'},ss^{'}); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s>0, s^{'}>0 \\ \left\langle (ps^{'},qr^{'},qr^{'},ps^{'}); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s<0, s^{'}>0 \\ \left\langle (ss^{'},qq^{'},rr^{'},pp^{'}); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s<0, s^{'}<0 \end{cases}$
4. $\tilde{d}^N / \tilde{g}^N = \begin{cases} \left\langle (p/s^{'},q/r^{'},r/q^{'},s/p^{'}); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s>0, s^{'}>0 \\ \left\langle (s/s^{'},r/r^{'},q/q^{'},p/p^{'}); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s<0, s^{'}>0 \\ \left\langle (s/p^{'},r/q^{'},q/r^{'},p/s^{'}); \sigma_{\tilde{d}^N} \wedge \sigma_{\tilde{g}^N}, \lambda_{\tilde{d}^N} \vee \lambda_{\tilde{g}^N}, \tau_{\tilde{d}^N} \vee \tau_{\tilde{g}^N} \right\rangle, s<0, s^{'}<0 \end{cases}$
5. $d\tilde{g}^N = h(x) = \begin{cases} \langle (dp,dq,dr,ds); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \rangle, d>0 \\ \langle (ds,dr,dq,dp); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \rangle, d<0 \end{cases}$
6. $\tilde{g}^{N^{-1}} = \left\langle (1/s^{'},1/r^{'},1/q^{'},1/p^{'}); \sigma_{\tilde{g}^N}, \lambda_{\tilde{g}^N}, \tau_{\tilde{g}^N} \right\rangle, \tilde{g}^N \neq 0.$

Definition 2.5. Weighted Possibility Mean of single valued trapezoidal neutrosophic numbers [51]: Let $\tilde{r} = \left(s_{\tilde{r}}^1, s_{\tilde{r}}^2, s_{\tilde{r}}^3, s_{\tilde{r}}^4; k_{\tilde{r}}, l_{\tilde{r}}, m_{\tilde{r}}\right)$ be SVTrNN and the possibilistic mean of truth, indeterminacy and falsity MF denoted by $g(\tilde{r}(\alpha))$, $g(\tilde{r}(\beta))$ and $g(\tilde{r}(\gamma))$ respectively. The weighted possibility mean is defined as $B(\tilde{r},\mu)$ in which the decision makers (DMs) always aim to maximize the degree of truth membership, minimize the degree of indeterminacy and falsity membership, as follows:

$$B(\tilde{r},\mu) = \mu g(\tilde{r}(\alpha)) + (1-\mu) g(\tilde{r}(\beta)) + (1-\mu) g(\tilde{r}(\gamma)) \tag{1}$$

where $\mu \in [0,1]$ represents the different preferences for the possibility means of truth, indeterminacy and falsity degree.

$$g(\tilde{r}(\alpha)) = \left[\frac{s_{\tilde{r}}^1 + 2s_{\tilde{r}}^2 + 2s_{\tilde{r}}^3 + s_{\tilde{r}}^4}{6} \right] k_{\tilde{r}}^2 \tag{2}$$

$$g(\tilde{r}(\beta)) = \left[\frac{(2s_{\tilde{r}}^1 + s_{\tilde{r}}^2 + s_{\tilde{r}}^3 + 2s_{\tilde{r}}^4) - (s_{\tilde{r}}^1 - s_{\tilde{r}}^2 - s_{\tilde{r}}^3 + s_{\tilde{r}}^4) l_{\tilde{r}} - (s_{\tilde{r}}^1 + 2s_{\tilde{r}}^2 + 2s_{\tilde{r}}^3 + s_{\tilde{r}}^4) l_{\tilde{r}}^2}{6} \right] \tag{3}$$

$$g(\tilde{r}(\gamma)) = \left[\frac{(2s_{\tilde{r}}^1 + s_{\tilde{r}}^2 + s_{\tilde{r}}^3 + 2s_{\tilde{r}}^4) - (s_{\tilde{r}}^1 - s_{\tilde{r}}^2 - s_{\tilde{r}}^3 + s_{\tilde{r}}^4) m_{\tilde{r}} - (s_{\tilde{r}}^1 + 2s_{\tilde{r}}^2 + 2s_{\tilde{r}}^3 + s_{\tilde{r}}^4) m_{\tilde{r}}^2}{6} \right]. \tag{4}$$

Example 2.6. Let us consider $\tilde{r} = (1, 3, 5, 8; 0.8, 0.5, 0.2)$ be a single-valued trapezoidal neutrosophic number. The PM of truth MF $\left[g(\tilde{c}_{11}^{N}(\alpha))\right]$ is as follows:

$$g(\tilde{c}_{11}^{N}(\alpha)) = \left[\frac{s_{\tilde{r}}^{1} + 2s_{\tilde{r}}^{2} + 2s_{\tilde{r}}^{3} + s_{\tilde{r}}^{4}}{6}\right] k_{\tilde{r}}^{2} = \left[\frac{(1) + 2(3) + 2(5) + 8}{6}\right] (0.8)^{2} = 2.66 \approx 3.$$

The PM of indeterminacy MF $\left[g(\tilde{c}_{11}^{N}(\beta))\right]$ is as follows:

$$g(\tilde{c}_{11}^{N}(\beta)) = \left[\frac{(2s_{\tilde{r}}^{1} + s_{\tilde{r}}^{2} + s_{\tilde{r}}^{3} + 2s_{\tilde{r}}^{4}) - (s_{\tilde{r}}^{1} - s_{\tilde{r}}^{2} - s_{\tilde{r}}^{3} + s_{\tilde{r}}^{4})l_{\tilde{r}} - (s_{\tilde{r}}^{1} + 2s_{\tilde{r}}^{2} + 2s_{\tilde{r}}^{3} + s_{\tilde{r}}^{4})l_{\tilde{r}}^{2}}{6}\right]$$

$$= \left[\frac{(2(1) + 3 + 5 + 2(8)) - (1 - 3 - 5 + 8)(0.5) - (1 + 2(3) + 2(5) + (8))(0.5)^{2}}{6}\right]$$

$$= 3.20 \approx 3.$$

The PM of falsity MF $\left[g(\tilde{c}_{11}^{N}(\gamma))\right]$ is as follows:

$$g(\tilde{c}_{11}^{N}(\gamma)) = \left[\frac{(2s_{\tilde{r}}^{1} + s_{\tilde{r}}^{2} + s_{\tilde{r}}^{3} + 2s_{\tilde{r}}^{4}) - (s_{\tilde{r}}^{1} - s_{\tilde{r}}^{2} - s_{\tilde{r}}^{3} + s_{\tilde{r}}^{4})m_{\tilde{r}} - (s_{\tilde{r}}^{1} + 2s_{\tilde{r}}^{2} + 2s_{\tilde{r}}^{3} + s_{\tilde{r}}^{4})m_{\tilde{r}}^{2}}{6}\right]$$

$$= \left[\frac{(2(1) + 3 + 5 + 2(8)) - (1 - 3 - 5 + 8)(0.4) - (1 + 2(3) + 2(5) + (8))(0.4)^{2}}{6}\right]$$

$$= 3.6 \approx 4.$$

By substituting the PM values of truth, indeterminacy and falsity MF in equation 1, we obtain $B(\tilde{c}_{11}^{N}, \mu) = \mu(3) + (1 - \mu)(3 + 4)$ where μ be any value between 0 and 1. Here $\mu = 0$ is chosen to reduce the WPM of neutrosophic number into its deterministic number, then we obtain $B(\tilde{c}_{11}^{N}, \mu) = 7$. Figure 1 represents the geometric interpretation of WPM for SVTrNNs.

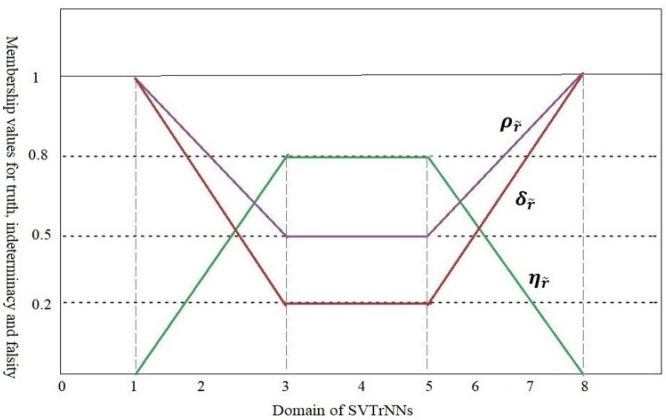


Fig. 1. Representation of WPM for SVTrNNs.

Definition 2.7. Pareto optimal solution [49]: A feasible solution $x^* = (\ x_{ij}^{l*} : l = 1, 2, \ldots, s; i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ is said to be POS of the problem if there exists no other feasible solution $x = (\ x_{ij}^{l} : l = 1, 2, \ldots, s; i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ such that

$$\tilde{Z}^{(m)}(x) \leq \tilde{Z}^{(m)}(x^*) \quad \text{for } m = 1, 2, \ldots \tag{5}$$

$$\tilde{Z}^{(m)}(x) < \tilde{Z}^{(m)}(x^*) \quad \text{for at least one m.} \tag{6}$$

Definition 2.8. Neutrosophic Pareto optimal solution: A feasible solution $x^* = (\ x_{ij}^{l*} : l = 1, 2, \ldots, s; i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ is said to be neutrosophic POS of the problem if there exists no other feasible solution $x = (\ x_{ij}^{l} : l = 1, 2, \ldots, s; i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ such that

$$\tilde{Z}^{N(m)}(x) \leq \tilde{Z}^{N(m)}(x^*) \quad \text{for } m = 1, 2, \ldots \tag{7}$$

$$\tilde{Z}^{N(m)}(x) < \tilde{Z}^{N(m)}(x^*) \quad \text{for at least one m.} \tag{8}$$

That is, A feasible solution is a Pareto optimal (neutrosophic Pareto optimal) if no other feasible solution improves all objectives simultaneously, with at least one objective strictly better.

A Pareto optimal solution is defined under precise objective values, whereas a neutrosophic Pareto optimal solution extends this concept by incorporating uncertainty under neutrosophic representations.

Example 2.9. Consider bi-objective neutrosophic problem with two objective functions: $\tilde{Z}^{N(1)} = (6, 10, 13, 15; 0.7, 0.3, 0.4)$ and $\tilde{Z}^{N(2)} = (14, 7, 21, 28; 0.8, 0.2, 0.6)$
Let two feasible solutions be x and x^* with objective values:
$\tilde{Z}^{N(1)}(x) = (6, 10, 13, 15; 0.7, 0.3, 0.4)$ and $\tilde{Z}^{N(2)}(x) = (14, 7, 21, 28; 0.8, 0.2, 0.6)$
$\tilde{Z}^{N(1)}(x^*) = (8, 12, 16, 18; 0.6, 0.4, 0.5)$ and $\tilde{Z}^{N(2)}(x^*) = (16, 9, 24, 30; 0.7, 0.3, 0.7)$
Since $\tilde{Z}^{N(m)}(x) \leq \tilde{Z}^{N(m)}(x^*)$ for m=1,2 and $\tilde{Z}^{N(m)}(x) < \tilde{Z}^{N(m)}(x^*)$ the solution x^* is not neutrosophic Pareto optimal, whereas x dominates x^* and hence it is a neutrosophic Pareto optimal solution.

In this article, the TP with two conflicting objective functions under neutrosophic environment is formulated. The neutrosophic BOTP is transformed into its deterministic BOTP using WPM. The bi-objective TP is converted into its single objective TP using the proposed neutrosophic TOPSIS approach and then it is solved by LINGO software to determine the Pareto optimal solution. The mathematical formulation of BOTP under neutrosophic environment is elaborated in the next section.

3. FORMULATION OF NEUTROSOPHIC BI-OBJECTIVE TRANSPORTATION PROBLEM (NBOTP)

This work addresses the bi-objective transportation problem under neutrosophic environment, a framework designed to handle the real-world complexities of uncertainty, imprecision and indeterminacy. The problem is contextualized using the application of road safety delineator logistics of Axnoy Industries, a leading manufacturer of road safety products like stainless steel delineators and traffic safety equipment. In this system, Axnoy operates three sources in Delhi, Jaipur, Pune supplies the delineators to

three destinations in Chennai, Kolkata, Bangalore. Each source has a specific yearly supply capacity $\tilde{a}_i^N$ and each destination has a known yearly demand $\tilde{b}_j^N$. The core challenge is to determine the optimal shipping quantities $\tilde{x}_{ij}^N$ that efficiently ship the delineators from suppliers to customers to minimize the total transportation cost $\tilde{c}_{ij}^N$ and deterioration cost $\tilde{d}_{ij}^N$ of the delineators. A major complication in real-world logistics is that factors such as supply, demand, transportation and deterioration costs are often vague, incomplete or indeterminate. Since a neutrosophic set extends fuzzy and intuitionistic fuzzy set by independently capturing the degrees of truth, falsity and indeterminacy membership for a parameter, making it exceptionally powerful for handling ambiguous and inconsistent information. Therefore, all parameters in this problem are considered as SVTrNNs. The problem is mathematically formulated as:

$$(G_1)\ \text{Minimize } \tilde{Z}^{(1)N}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^N \tilde{x}_{ij}^N \tag{9}$$

$$\text{Minimize } \tilde{Z}^{(2)N}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{d}_{ij}^N \tilde{x}_{ij}^N \tag{10}$$

Subject to:

$$\sum_{j=1}^{n} \tilde{x}_{ij}^N \leq \tilde{a}_i^N, i = 1, 2, \ldots, m \tag{11}$$

$$\sum_{i=1}^{m} \tilde{x}_{ij}^N \geq \tilde{b}_j^N, j = 1, 2, \ldots, n \tag{12}$$

$$\tilde{x}_{ij}^N \geq 0,\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n, \tag{13}$$

where $\tilde{a}_i^N = (a_i^1, a_i^2, a_i^3, a_i^4; a_i^{'1}, a_i^{'2}, a_i^{'3})$ for $i = 1, 2, \ldots, m$ refers to SVTrN supply at i^{th} origin and $\tilde{b}_j^N = (b_j^1, b_j^2, b_j^3, b_j^4; b_j^{'1}, b_j^{'2}, b_j^{'3})$ for $j = 1, 2, \ldots, n$ refers to SVTrN demand at j^{th} destination; $\tilde{c}_{ij}^N = (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4; c_{ij}^{'1}, c_{ij}^{'2}, c_{ij}^{'3})$ and $\tilde{d}_{ij}^N = (d_{ij}^1, d_{ij}^2, d_{ij}^3, d_{ij}^4; d_{ij}^{'1}, d_{ij}^{'2}, d_{ij}^{'3})$ denote the first and second objective of SVTrN transportation cost and deterioration cost transported from ith source to jth destination respectively. Let $\tilde{x}_{ij}^N = (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4; x_{ij}^{'1}, x_{ij}^{'2}, x_{ij}^{'3})$ represents the quantity for transporting the products from ith source to jth destination.

The problem (G_1) can be represented in the form of SVTrNNs as:

$$(G_2)\ \text{Minimize } \tilde{Z}^{(1)N}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} \left(c_{ij}^{1(1)}, c_{ij}^{2(1)}, c_{ij}^{3(1)}, c_{ij}^{4(1)}; c_{ij}^{'1(1)}, c_{ij}^{'2(1)}, c_{ij}^{'3(1)}\right) \tilde{x}_{ij}^N$$

$$\text{Minimize } \tilde{Z}^{(2)N}(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} \left(d_{ij}^{1(2)}, d_{ij}^{2(2)}, d_{ij}^{3(2)}, d_{ij}^{4(2)}; d_{ij}^{'1(2)}, d_{ij}^{'2(2)}, d_{ij}^{'3(2)}\right) \tilde{x}_{ij}^N$$

Subject to:

$$\sum_{j=1}^{n} \tilde{x}_{ij}^N \leq (a_i^1, a_i^2, a_i^3, a_i^4; a_i^{'1}, a_i^{'2}, a_i^{'3}), i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij}^{N} \geq (b_j^1, b_j^2, b_j^3, b_j^4; b_j^{'1}, b_j^{'2}, b_j^{'3}), j = 1, 2, \ldots, n$$
$$\tilde{x}_{ij}^{N} \geq 0,\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.$$

To convert the neutrosophic BOTP into its deterministic BOTP, a ranking function based on the weighted possibility mean [51] is employed. This ranking function is denoted by $\Re$ which maps SVTrNN to a real number by aggregating its truth, indeterminacy and falsity components in a weighted manner. An accurate representation is achieved by the weighted possibility mean, which enables the quantification of the certainty associated with each parameter. Applying $\Re$ to the problem (G_2) gives

$$(G_3)\ \text{Minimize } \Re\left(\tilde{Z}^{(1)N}(x)\right) = \Re\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij}^{1(1)}, c_{ij}^{2(1)}, c_{ij}^{3(1)}, c_{ij}^{4(1)}; c_{ij}^{'1(1)}, c_{ij}^{'2(1)}, c_{ij}^{'3(1)}\right)\tilde{x}_{ij}^{N}\right)$$
$$\text{Minimize } \Re\left(\tilde{Z}^{(2)N}(x)\right) = \Re\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\left(d_{ij}^{1(2)}, d_{ij}^{2(2)}, d_{ij}^{3(2)}, d_{ij}^{4(2)}; d_{ij}^{'1(2)}, d_{ij}^{'2(2)}, d_{ij}^{'3(2)}\right)\tilde{x}_{ij}^{N}\right)$$
Subject to:
$$\sum_{j=1}^{n} \tilde{x}_{ij}^{N} \leq \Re\left(a_i^1, a_i^2, a_i^3, a_i^4; a_i^{'1}, a_i^{'2}, a_i^{'3}\right), i = 1, 2, \ldots, m$$
$$\sum_{i=1}^{m} \tilde{x}_{ij}^{N} \geq \Re\left(b_j^1, b_j^2, b_j^3, b_j^4; b_j^{'1}, b_j^{'2}, b_j^{'3}\right), j = 1, 2, \ldots, n$$
$$\tilde{x}_{ij}^{N} \geq 0,\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.$$

Since $\Re$ produces deterministic values, $\Re\left(\tilde{Z}^{(1)N}(x)\right) = Z^{(1)}(x)$, $\Re\left(\tilde{Z}^{(2)N}(x)\right) = Z^{(2)}(x)$.

Remark 3.1. Initially, the problem G_3 is solved to obtain its corresponding deterministic solution. This deterministic solution is then substituted into the problem G_1, and the required arithmetic operations are performed. As a result, problem G_3 is transformed back into problem G_1. The same procedure is subsequently repeated in a similar manner to ensure consistency and completeness of the solution process.

As the above deterministic problem is extremely difficult and consumes more time to obtain an optimal solution that optimizes each objective function. In this situation, the multi-objective problem is reduced to single objective problem in order to reduce the computational efforts for the DM. To handle this, we have proposed the neutrosophic TOPSIS approach for SVTrNNs to transform the multi-objective optimization problem (MOOP) into the single objective optimization problem (SOOP). The next section describes the solution approach using the proposed neutrosophic TOPSIS approach.

4. SOLUTION APPROACH

The solution approach using the proposed neutrosophic TOPSIS approach to obtain the Pareto optimal solution is discussed in this section. The procedure of the solution approach using the proposed approach is as follows:

Step 1: Input: Consider the bi-objective TP under neutrosophic environment (NBOTP) (G_1).

Step 2: Convert the neutrosophic BOTP (G_1) into its deterministic BOTP (G_2) using the definition 2.5 [51].

Step 3: Output: Solve $Z^1(x)$ and $Z^2(x)$ of deterministic BOTP to obtain the optimal solution using any existing methods available in the literature. Then using the obtained optimal solution, the pay-off table is formed in Table 3.

	$Z^{(1)}$	$Z^{(2)}$
X_1	$Z^{(1)}(X_1)$	$Z^{(2)}(X_1)$
X_2	$Z^{(1)}(X_2)$	$Z^{(2)}(X_2)$

Tab. 3. Pay-off matrix.

Step 4: Reduce the bi-objective TP into single objective TP using the proposed neutrosophic TOPSIS approach. The technique of the proposed approach for SVTrNNs is as follows:

Proposed Neutrosophic TOPSIS approach : The Neutrosophic Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision-making technique proposed to identify the most suitable solution among a set of available solutions. The essential principle of TOPSIS is that the solution should be closest to Positive Ideal Solution (PIS) and farthest from the Negative Ideal Solution (NIS). The method is formulated to construct two objective functions from m number of objectives. The first one is to minimize the distance from PIS, while the second one is to maximize distance from NIS. In the final stage, the Pareto optimal solution is obtained using the max–min operator proposed by Bellman and Zadeh [7]. This approach provides a systematic and consistent framework to estimate and compare complex alternatives based on multiple criteria, which have different weights. The flexibility of TOPSIS is valid in handling both qualitative and quantitative data which enhances its applicability across various domains. Overall, TOPSIS enhances objectivity, transparency, accuracy and applied in the multi-criteria decision-making process. The additional indeterminacy information improves interpretability and supports the DMs.

Step 4a: From Table 3, calculate the max and min values for each objective function using $Z^{(m)+} = min(Z^{(m)})$, $Z^{(m)-} = max(Z^{(m)})$. Then calculate the positive ideal solution (PIS) (Z^+) and negative ideal solution (NIS) (Z^-) using $Z^+ = (Z^{(1)+}, Z^{(2)+})$ and $Z^- = (Z^{(1)-}, Z^{(2)-})$ respectively.

Step 4b: Determine the distance function from PIS ($d_q^{PIS}(x)$) and NIS ($d_q^{NIS}(x)$) as follows:

$$d_q^{PIS}(x) = \left[\sum_{m=1}^{2}\left[w_m\left(\frac{Z^{(m)}(x) - Z^{(m)-}}{Z^{(m)+} - Z^{(m)-}}\right)\right]^q\right]^{\frac{1}{q}} \tag{14}$$

$$d_q^{NIS}(x) = \left[\sum_{m=1}^{2}\left[w_m\left(\frac{Z^{(m)-} - Z^{(m)}(x)}{Z^{(m)+} - Z^{(m)-}}\right)\right]^q\right]^{\frac{1}{q}}. \tag{15}$$

Here $w_m, (m = 1, 2)$ signifies the weights assigned to objective functions. The priority of objective function is assigned by both equal and different weights. In TOPSIS approach, the POS is determined by $q = 1, 2, \ldots, \infty$ to address the multi-objective problems. Here $q = 1$, $q = 2$ and $q = 3$ represent the Manhattan distance (farthest), Euclidean distance (least) and Tchebycheff distance (shortest) respectively. Due to their lack of practical value, other distances are less frequently used.

Step 4c: Input: Assign the distance $q = 2$ subject to the constraints Eqn. 11 to Eqn. 13. The problem can be reformulated into the following bi-objective problem is as follows:

$$(G_4)\text{Minimize } d_2^{PIS}(x) = \left[\sum_{m=1}^{2}\left[w_m\left(\frac{Z^{(m)}(x) - Z^{(m)-}}{Z^{(m)+} - Z^{(m)-}}\right)\right]^2\right]^{\frac{1}{2}}$$

$$\text{Maximize } d_2^{NIS}(x) = \left[\sum_{m=1}^{2}\left[w_m\left(\frac{Z^{(m)-} - Z^{(m)}(x)}{Z^{(m)+} - Z^{(m)-}}\right)\right]^2\right]^{\frac{1}{2}}$$

Subject to the constraints Eqn. 11 to Eqn. 13.

Step 4d: Output: Determine minimize $d_2^{PIS}(x)$ and maximize $d_2^{NIS}(x)$) values subject to the constraints Eqn.11 to Eqn.13. Now we form the pay-off table of ideal solution which is shown in Table 4.

Tab. 4. Pay-off matrix of ideal solution

	$d_2^{PIS}(x)$	$d_2^{NIS}(x)$
x^{PIS}	$(d_2^{PIS})^*$	$(d_2^{NIS})'$
x_{NIS}	$(d_2^{PIS})'$	$(d_2^{NIS})^*$

Step 4e: The truth MF $\alpha_1(x)$ and $\alpha_2(x)$, indeterminacy MF $\beta_1(x)$ and $\beta_2(x)$ and falsity MF $\gamma_1(x)$ and $\gamma_2(x)$ for two objective functions are formulated as follows:

$$\alpha_1(x) = \begin{cases} 1 & , \ d_2^{PIS}(x) \le (d_2^{PIS})^* \\ \dfrac{(d_2^{PIS})' - d_2^{PIS}(x)}{(d_2^{PIS})' - (d_2^{PIS})^*} & , \ (d_2^{PIS})^* \le d_2^{PIS}(x) \le (d_2^{PIS})' \\ 0 & , \ (d_2^{PIS})' \le d_2^{PIS}(x) \end{cases}$$

$$\alpha_2(x) = \begin{cases} 1 & , \ (d_2^{NIS})^* \leq d_2^{NIS}(x) \\ \dfrac{d_2^{NIS}(x) - (d_2^{NIS})'}{(d_2^{NIS})^* - (d_2^{NIS})'} & , \ (d_2^{NIS})' \leq d_2^{NIS}(x) \leq (d_2^{NIS})^* \\ 0 & , \ d_2^{NIS}(x) \leq (d_2^{NIS})' - \eta_2 \end{cases}$$

$$\beta_1(x) = \begin{cases} 0 & , \ d_2^{PIS}(x) \leq (d_2^{PIS})^* \\ \dfrac{d_2^{PIS}(x) - (d_2^{PIS})^*}{(d_2^{PIS})' + \eta_1 - (d_2^{PIS})^*} & , \ (d_2^{PIS})^* \leq d_2^{PIS}(x) \leq (d_2^{PIS})' + \eta_1 \\ 1 & , \ (d_2^{PIS})' + \eta_1 \leq d_2^{PIS}(x) \end{cases}$$

$$\beta_2(x) = \begin{cases} 0 & , \ (d_2^{NIS})^* \leq d_2^{NIS}(x) \\ \dfrac{(d_2^{NIS})^* - d_2^{NIS}(x)}{(d_2^{NIS})^* - ((d_2^{NIS})' - \eta_2)} & , \ (d_2^{NIS})' - \eta_2 \leq d_2^{NIS}(x) \leq (d_2^{NIS})^* \\ 1 & , \ d_2^{NIS}(x) \leq (d_2^{NIS})' - \eta_2 \end{cases}$$

$$\gamma_1(x) = \begin{cases} 0 & , \ d_2^{PIS}(x) \leq (d_2^{PIS})^* \\ \dfrac{d_2^{PIS}(x) - (d_2^{PIS})^*}{(d_2^{PIS})' + \rho_1 - (d_2^{PIS})^*} & , \ (d_2^{PIS})^* \leq d_2^{PIS}(x) \leq (d_2^{PIS})' + \rho_1 \\ 1 & , \ (d_2^{PIS})' + \rho_1 \leq d_2^{PIS}(x) \end{cases}$$

$$\gamma_2(x) = \begin{cases} 0 & , \ (d_2^{NIS})^* \leq d_2^{NIS}(x) \\ \dfrac{(d_2^{NIS})^* - d_2^{NIS}(x)}{(d_2^{NIS})^* - ((d_2^{NIS})' - \rho_2)} & , \ (d_2^{NIS})' - \rho_2 \leq d_2^{NIS}(x) \leq (d_2^{NIS})^* \\ 1 & , \ d_2^{NIS}(x) \leq (d_2^{NIS})' - \rho_2. \end{cases}$$

Step 4f: Using the max-min operator, the truth, indeterminacy and falsity MF for two objective functions can be written as:

(G_5)Maximize $(\mu_2 - \delta_2 - \theta_2)$

Subject to the constraints Eqn.11 to Eqn.13

$\alpha_1(x) \geq \mu_2, \alpha_2(x) \geq \mu_2,$

$\beta_1(x) \leq \delta_2, \beta_2(x) \leq \delta_2,$

$\gamma_1(x) \leq \theta_2, \gamma_2(x) \leq \theta_2,$

$\mu_2 + \delta_2 + \theta_2 \leq 3.$

Here $\mu_2 = \min\{\alpha_1(x), \alpha_2(x)\}, \delta_2 = \max\{\beta_1(x), \beta_2(x)\}, \theta_2 = \max\{\gamma_1(x), \gamma_2(x)\}$

Step 5: Output: Solve the reduced problem to obtain the POS using LINGO software. Finally, to compute the neutrosophic POS substitute the optimal allocations of (G_3) in the problem (G_1).

Step 6: Assign different weights for the objective function in Step 4b and then repeat from Step 4c to Step 5 to obtain the set of POS. Figure 2 represents the flowchart for the solution approach.

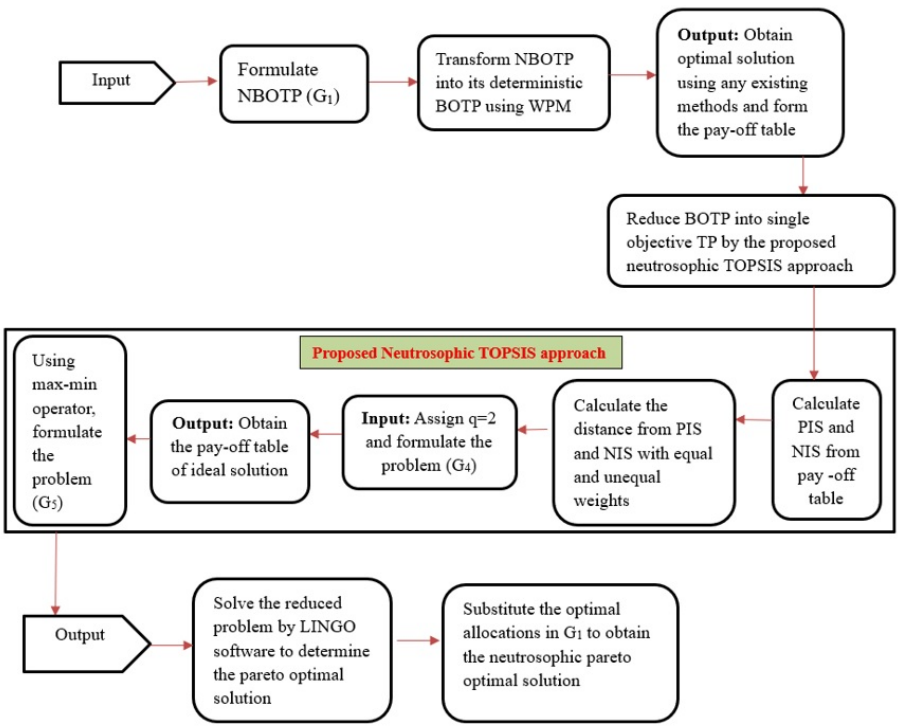


Fig. 2. Flowchart for the solution approach.

The solution approach for solving NBOTP is illustrated by a numerical illustration which is given in the following section. The data is taken from [51].

5. NUMERICAL EXAMPLE: APPLICATION TO ROAD SAFETY DELINEATOR LOGISTICS

Axnoy Industries is a well - established Delineators Manufacturers in Mumbai offering different types of road safety products. The range includes Stainless Steel Delineators, MS Delineators, Traffic Safety Batons, Bollard Barricades, Traffic Fence Barriers, etc. The offered Delineator is designed using modern technology with reflective strips that

are ideal for better night visibility. It is mainly used as road safety equipment for helping motorists in finding the right direction to drive. The famous cities of India such as Delhi, Jaipur and Pune are the three main places for manufacturing the delineators. There are three regions in India where the delineator can be imported namely, Chennai, Kolkata and Bangalore. These implementations involve neutrosophic uncertainty because the supply, demand, transportation cost and deterioration cost are influenced by fluctuating raw material availability, labor conditions, government road safety projects, traffic congestion and environmental factors. Therefore, the parameters are represented as single-valued trapezoidal neutrosophic numbers to achieve a realistic transportation decision framework. The yearly supplies of the delineator are $\tilde{a}_1^N$, $\tilde{a}_2^N$ and $\tilde{a}_3^N$ units and the yearly demands of the delineator are $\tilde{b}_1^N$, $\tilde{b}_2^N$, $\tilde{b}_3^N$ and $\tilde{b}_4^N$ units respectively. Let us assume that $\tilde{Z}^{(1)N}(x)$ and $\tilde{Z}^{(2)N}(x)$ are two objectives which aims to minimize the total transportation cost $\tilde{c}_{ij}^N$ and deterioration cost $\tilde{d}_{ij}^N$ of shipment for importing the delineator. The problem (G_1) is fully single-valued trapezoidal neutrosophic BOTP (SVTrNBOTP) where all the parameters are given in Table 5.

		P_1	P_2	P_3	P_4	Supply
M_1	$\tilde{Z}^{(1)N}$ $\tilde{Z}^{(2)N}$	$\tilde{c}_{11}^N$ $\tilde{d}_{11}^N$	$\tilde{c}_{12}^N$ $\tilde{d}_{12}^N$	$\tilde{c}_{13}^N$ $\tilde{d}_{13}^N$	$\tilde{c}_{14}^N$ $\tilde{d}_{14}^N$	$\tilde{a}_1^N$
M_2	$\tilde{Z}^{(1)N}$ $\tilde{Z}^{(2)N}$	$\tilde{c}_{21}^N$ $\tilde{d}_{21}^N$	$\tilde{c}_{22}^N$ $\tilde{d}_{22}^N$	$\tilde{c}_{23}^N$ $\tilde{d}_{23}^N$	$\tilde{c}_{24}^N$ $\tilde{d}_{24}^N$	$\tilde{a}_2^N$
M_3	$\tilde{Z}^{(1)N}$ $\tilde{Z}^{(2)N}$	$\tilde{c}_{31}^N$ $\tilde{d}_{31}^N$	$\tilde{c}_{32}^N$ $\tilde{d}_{32}^N$	$\tilde{c}_{33}^N$ $\tilde{d}_{33}^N$	$\tilde{c}_{34}^N$ $\tilde{d}_{34}^N$	$\tilde{a}_3^N$
Demand		$\tilde{b}_1^N$	$\tilde{b}_2^N$	$\tilde{b}_3^N$	$\tilde{b}_4^N$	

Tab. 5. SVTrNBOTP.

where
$\tilde{c}_{11}^N = (6, 10, 13, 15; 0.7, 0.3, 0.4)$; $\tilde{d}_{11}^N = (14, 7, 21, 28; 0.8, 0.2, 0.6)$;
$\tilde{c}_{12}^N = (18, 22, 30, 34; 0.5, 0.1, 0.4)$; $\tilde{d}_{12}^N = (4, 8, 11, 15; 0.6, 0.3, 0.2)$;
$\tilde{c}_{13}^N = (4, 8, 11, 15; 0.6, 0.3, 0.2)$; $\tilde{d}_{13}^N = (15, 17, 19, 22; 0.4, 0.8, 0.4)$;
$\tilde{c}_{14}^N = (26, 28, 30, 35; 0.9, 0.1, 0.1)$; $\tilde{d}_{14}^N = (12, 15, 19, 22; 0.6, 0.4, 0.5)$;
$\tilde{c}_{21}^N = (18, 24, 30, 35; 0.5, 0.1, 0.3)$; $\tilde{d}_{21}^N = (15, 18, 20, 22; 0.7, 0.5, 0.5)$;
$\tilde{c}_{22}^N = (26, 28, 30, 35; 0.9, 0.1, 0.1)$; $\tilde{d}_{22}^N = (3, 5, 6, 8; 0.6, 0.5, 0.4)$;
$\tilde{c}_{23}^N = (14, 7, 21, 28; 0.8, 0.2, 0.6)$; $\tilde{d}_{23}^N = (12, 15, 19, 22; 0.6, 0.4, 0.5)$;
$\tilde{c}_{24}^N = (18, 20, 22, 25; 0.7, 0.5, 0.5)$; $\tilde{d}_{24}^N = (15, 17, 19, 22; 0.4, 0.8, 0.4)$;
$\tilde{c}_{31}^N = (20, 26, 30, 31; 0.9, 0.1, 0.1)$; $\tilde{d}_{31}^N = (18, 20, 22, 25; 0.7, 0.5, 0.5)$;
$\tilde{c}_{32}^N = (9, 11, 14, 16; 0.5, 0.4, 0.7)$; $\tilde{d}_{32}^N = (5, 8, 10, 14; 0.3, 0.6, 0.6)$;
$\tilde{c}_{33}^N = (25, 35, 44, 50; 0.9, 0.2, 0.1)$; $\tilde{d}_{33}^N = (9, 11, 14, 16; 0.5, 0.4, 0.7)$
$\tilde{c}_{34}^N = (12, 15, 19, 22; 0.6, 0.4, 0.5)$; $\tilde{d}_{34}^N = (15, 18, 20, 22; 0.7, 0.5, 0.5)$
$\tilde{a}_1^N = \tilde{a}_2^N = (25, 35, 44, 50; 0.9, 0.2, 0.1)$, $\tilde{a}_3^N = (68, 75, 80, 85; 0.9, 0.1, 0.2)$,
$\tilde{b}_1^N = (19, 25, 30, 35; 0.5, 0.1, 0.2)$, $\tilde{b}_2^N = (26, 28, 30, 35; 0.9, 0.1, 0.1)$,
$\tilde{b}_3^N = (35, 45, 50, 55; 0.9, 0.1, 0.2)$, $\tilde{b}_4^N = (45, 55, 60, 64; 0.9, 0.1, 0.3)$.

Using **Step 2**, the WPM representation for SVTrNBOTP is shown in Table 6.

		P_1	P_2	P_3	P_4	Supply
M_1	$\tilde{Z}^{(1)N}$	$\mu\left[g(\tilde{c}_{11}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{11}^N(\beta))+g(\tilde{c}_{11}^N(\gamma))]$	$\mu\left[g(\tilde{c}_{12}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{12}^N(\beta))+g(\tilde{c}_{12}^N(\gamma))]$	$\mu\left[g(\tilde{c}_{13}^N(\alpha))\right](1-\mu)[g(\tilde{c}_{13}^N(\beta))+g(\tilde{c}_{13}^N(\gamma))]$	$\mu\left[g(\tilde{c}_{14}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{14}^N(\beta))+g(\tilde{c}_{14}^N(\gamma))]$	$\tilde{a}_1^N$
	$\tilde{Z}^{(2)N}$	$\mu\left[g(\tilde{d}_{11}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{11}^N(\beta))+g(\tilde{d}_{11}^N(\gamma))]$	$\mu\left[g(\tilde{d}_{12}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{12}^N(\beta))+g(\tilde{d}_{12}^N(\gamma))]$	$\mu\left[g(\tilde{d}_{13}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{13}^N(\beta))+g(\tilde{d}_{13}^N(\gamma))]$	$\mu\left[g(\tilde{d}_{14}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{14}^N(\beta))+g(\tilde{d}_{14}^N(\gamma))]$	
M_2	$\tilde{Z}^{(1)N}$	$\mu\left[g(\tilde{c}_{21}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{21}^N(\beta))+g(\tilde{c}_{21}^N(\gamma))]$	$\mu\left[g(\tilde{c}_{22}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{22}^N(\beta))+g(\tilde{c}_{22}^N(\gamma))]$	$\mu\left[g(\tilde{c}_{23}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{23}^N(\beta))+g(\tilde{c}_{23}^N(\gamma))]$	$\mu\left[g(\tilde{c}_{24}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{24}^N(\beta))+g(\tilde{c}_{24}^N(\gamma))]$	$\tilde{a}_2^N$
	$\tilde{Z}^{(2)N}$	$\mu\left[g(\tilde{d}_{21}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{21}^N(\beta))+g(\tilde{d}_{21}^N(\gamma))]$	$\mu\left[g(\tilde{d}_{22}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{22}^N(\beta))+g(\tilde{d}_{22}^N(\gamma))]$	$\mu\left[g(\tilde{d}_{23}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{23}^N(\beta))+g(\tilde{d}_{23}^N(\gamma))]$	$\mu\left[g(\tilde{d}_{24}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{24}^N(\beta))+g(\tilde{d}_{24}^N(\gamma))]$	
M_3	$\tilde{Z}^{(1)N}$	$\mu\left[g(\tilde{c}_{31}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{31}^N(\beta))+g(\tilde{c}_{31}^N(\gamma))]$	$\mu\left[g(\tilde{c}_{32}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{32}^N(\beta))+g(\tilde{c}_{32}^N(\gamma))]$	$\mu\left[g(\tilde{c}_{33}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{33}^N(\beta))+g(\tilde{c}_{33}^N(\gamma))]$	$\mu\left[g(\tilde{c}_{34}^N(\alpha))\right]+(1-\mu)[g(\tilde{c}_{34}^N(\beta))+g(\tilde{c}_{34}^N(\gamma))]$	$\tilde{a}_3^N$
	$\tilde{Z}^{(2)N}$	$\mu\left[g(\tilde{d}_{31}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{31}^N(\beta))+g(\tilde{t}_{31}^N(\gamma))]$	$\mu\left[g(\tilde{d}_{32}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{32}^N(\beta))+g(\tilde{d}_{32}^N(\gamma))]$	$\mu\left[g(\tilde{d}_{33}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{33}^N(\beta))+g(\tilde{d}_{33}^N(\gamma))]$	$\mu\left[g(\tilde{d}_{34}^N(\alpha))\right]+(1-\mu)[g(\tilde{d}_{34}^N(\beta))+g(\tilde{d}_{34}^N(\gamma))]$	
Demand		$\tilde{b}_1^N$	$\tilde{b}_2^N$	$\tilde{b}_3^N$	$\tilde{b}_4^N$	

Tab. 6. WPM representation of SVTrNBOTP.

where

$$\tilde{a}_1^N=\mu\left[g(\tilde{a}_1^N(\alpha))\right]+(1-\mu)\left[g(\tilde{a}_1^N(\beta))+g(\tilde{a}_1^N(\gamma))\right],$$
$$\tilde{a}_2^N=\mu\left[g(\tilde{a}_2^N(\alpha))\right]+(1-\mu)\left[g(\tilde{a}_2^N(\beta))+g(\tilde{a}_2^N(\gamma))\right],$$
$$\tilde{a}_3^N=\mu\left[g(\tilde{a}_3^N(\alpha))\right]+(1-\mu)\left[g(\tilde{a}_3^N(\beta))+g(\tilde{a}_3^N(\gamma))\right],$$
$$\tilde{b}_1^N=\mu\left[g(\tilde{b}_1^N(\alpha))\right]+(1-\mu)\left[g(\tilde{b}_1^N(\beta))+g(\tilde{b}_1^N(\gamma))\right],$$
$$\tilde{b}_2^N=\mu\left[g(\tilde{b}_2^N(\alpha))\right]+(1-\mu)\left[g(\tilde{b}_2^N(\beta))+g(\tilde{b}_2^N(\gamma))\right],$$
$$\tilde{b}_3^N=\mu\left[g(\tilde{b}_3^N(\alpha))\right]+(1-\mu)\left[g(\tilde{b}_3^N(\beta))+g(\tilde{b}_3^N(\gamma))\right],$$
$$\tilde{b}_4^N=\mu\left[g(\tilde{b}_4^N(\alpha))\right]+(1-\mu)\left[g(\tilde{b}_4^N(\beta))+g(\tilde{b}_4^N(\gamma))\right].$$

Now to compute the possibilistic mean values for truth $g(\tilde{r}(\alpha))$, indeterminacy $g(\tilde{r}(\beta))$ and falsity $g(\tilde{r}(\gamma))$ MF using the possibilistic mean of truth MF (Eqn. 2), possibilistic mean of indeterminacy MF (Eqn. 3) and possibilistic mean of falsity MF (Eqn. 4).

Choose the first cell value

$$B(\tilde{c}_{11}^N,\mu)=\mu\left[g(\tilde{c}_{11}^N(\alpha))\right]+(1-\mu)\left[g(\tilde{c}_{11}^N(\beta))+g(\tilde{c}_{11}^N(\gamma))\right]. \tag{16}$$

The possibilistic mean of truth MF $\left[g(\tilde{c}_{11}^N(\alpha))\right]$ using Eqn. 2 is

$$g(\tilde{c}_{11}^N(\alpha))=\left[\frac{s_{\tilde{r}}^1+2s_{\tilde{r}}^2+2s_{\tilde{r}}^3+s_{\tilde{r}}^4}{6}\right]k_{\tilde{r}}^2=\left[\frac{(6)+2(10)+2(13)+15}{6}\right](0.7)^2=5.47\approx 5.$$

The possibilistic mean of indeterminacy MF $\left[g(\tilde{c}_{11}^N(\beta))\right]$ using Eqn. 3 is

$$g(\tilde{c}_{11}^N(\beta)) = \left[\frac{(2s_{\tilde{r}}^1 + s_{\tilde{r}}^2 + s_{\tilde{r}}^3 + 2s_{\tilde{r}}^4) - (s_{\tilde{r}}^1 - s_{\tilde{r}}^2 - s_{\tilde{r}}^3 + s_{\tilde{r}}^4)l_{\tilde{r}} - (s_{\tilde{r}}^1 + 2s_{\tilde{r}}^2 + 2s_{\tilde{r}}^3 + s_{\tilde{r}}^4)l_{\tilde{r}}^2}{6}\right]$$

$$= \left[\frac{(2(6) + 10 + 13 + 2(15)) - (6 - 10 - 13 + 15)(0.3) - (6 + 2(10) + 2(13) + (15))(0.3)^2}{6}\right]$$

$$= 9.92 \approx 10.$$

The possibilistic mean of falsity MF $\left[g(\tilde{c}_{11}^N(\gamma))\right]$ using Eqn. 4 is

$$g(\tilde{c}_{11}^N(\gamma)) = \left[\frac{(2s_{\tilde{r}}^1 + s_{\tilde{r}}^2 + s_{\tilde{r}}^3 + 2s_{\tilde{r}}^4) - (s_{\tilde{r}}^1 - s_{\tilde{r}}^2 - s_{\tilde{r}}^3 + s_{\tilde{r}}^4)m_{\tilde{r}} - (s_{\tilde{r}}^1 + 2s_{\tilde{r}}^2 + 2s_{\tilde{r}}^3 + s_{\tilde{r}}^4)m_{\tilde{r}}^2}{6}\right]$$

$$= \left[\frac{(2(6) + 10 + 13 + 2(15)) - (6 - 10 - 13 + 15)(0.4) - (6 + 2(10) + 2(13) + (15))(0.4)^2}{6}\right]$$

$$= 9.18 \approx 9.$$

Now replacing the above values in $B(\tilde{c}_{11}^N, \mu) = \mu\left[g(\tilde{c}_{11}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{c}_{11}^N(\beta)) + g(\tilde{c}_{11}^N(\gamma))\right]$ we obtain $B(\tilde{c}_{11}^N, \mu) = \mu(5) + (1-\mu)(10+9)$. In the same manner, we compute the values of possibilistic mean of truth, indeterminacy and falsity MF of the other cells. Now the reduced WPM of SVTrNBOTP is shown in Table 7.

		P_1	P_2	P_3	P_4	Supply
M_1	$\tilde{Z}^{(1)N}$	$\mu(5)+$ $(1-\mu)$ $(10+9)$	$\mu(6.5)+$ $(1-\mu)$ $(25.74+21.84)$	$\mu(3.42)+$ $(1-\mu)$ $(8.64+9.12)$	$\mu(23.89)+$ $(1-\mu)$ $(29.65+29.65)$	$\tilde{a}_1^N$
	$\tilde{Z}^{(2)N}$	$\mu(10.45)+$ $(1-\mu)$ $(17.54+11.38)$	$\mu(3.42)+$ $(1-\mu)$ $(8.64+9.12)$	$\mu(2.90)+$ $(1-\mu)$ $(6.57+15.36)$	$\mu(6.12)+$ $(1-\mu)$ $(14.28+12.75)$	
M_2	$\tilde{Z}^{(1)N}$	$\mu(6.70)+$ $(1-\mu)$ $(26.41+24.30)$	$\mu(23.89)+$ $(1-\mu)$ $(29.65+29.65)$	$\mu(10.45)+$ $(1-\mu)$ $(17.54+11.38)$	$\mu(10.37)+$ $(1-\mu)$ $(15.95+15.95)$	$\tilde{a}_2^N$
	$\tilde{Z}^{(2)N}$	$\mu(9.22)+$ $(1-\mu)$ $(14.04+14.04)$	$\mu(1.98)+$ $(1-\mu)$ $(4.12+4.62)$	$\mu(6.12)+$ $(1-\mu)$ $(14.28+12.75)$	$\mu(2.90)+$ $(1-\mu)$ $(6.57+15.36)$	
M_3	$\tilde{Z}^{(1)N}$	$\mu(22)+$ $(1-\mu)$ $(26.14+26.14)$	$\mu(3.12)+$ $(1-\mu)$ $(10.5+6.37)$	$\mu(31.45)+$ $(1-\mu)$ $(36.74+37.84)$	$\mu(6.12)+$ $(1-\mu)$ $(14.28+12.75)$	$\tilde{a}_3^N$
	$\tilde{Z}^{(2)N}$	$\mu(10.37)+$ $(1-\mu)$ $(15.95+15.95)$	$\mu(0.82)+$ $(1-\mu)$ $(5.93+5.93)$	$\mu(3.12)+$ $(1-\mu)$ $(10.5+6.37)$	$\mu(9.22)+$ $(1-\mu)$ $(14.04+14.04)$	
Demand		$\tilde{b}_1^N$	$\tilde{b}_2^N$	$\tilde{b}_3^N$	$\tilde{b}_4^N$	

Tab. 7. WPM of SVTrNBOTP.

where
$\tilde{a}_1^N = \tilde{a}_2^N = \mu(31.45) + (1-\mu)(36.74 + 37.84)$, $\tilde{a}_3^N = \mu(62.50) + (1-\mu)(76.09 + 73.81)$,
$\tilde{b}_1^N = \mu(6.83) + (1-\mu)(26.91 + 26.10)$, $\tilde{b}_2^N = \mu(23.89) + (1-\mu)(29.65 + 29.65)$,
$\tilde{b}_3^N = \mu(37.8) + (1-\mu)(45.45 + 44.13)$, $\tilde{b}_3^N = \mu(3.6) + (1-\mu)(55.03 + 50.71)$.

Transform the WPM of SVTrNBOTP into its equivalent deterministic BOTP by choosing the value of $\mu = 0$ where $0 \leq \mu \leq 1$ as shown in Table 8.

		P_1	P_2	P_3	P_4	Supply
M_1	$Z^{(1)}$	19	48	18	59	75
	$Z^{(2)}$	29	18	22	27	
M_2	$Z^{(1)}$	51	59	29	32	75
	$Z^{(2)}$	28	9	27	22	
M_3	$Z^{(1)}$	52	17	74	27	150
	$Z^{(2)}$	32	12	17	28	
Demand		53	59	90	106	

Tab. 8. Deterministic BOTP.

By **Step 3**, solve the deterministic BOTP using any existing methods available in the literature to obtain the optimal solution. The obtained optimal solutions of $Z^{(1)}(x)$ and $Z^{(2)}(x)$ are formed as pay-off matrix in Table 9.

	$Z^{(1)}$	$Z^{(2)}$	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}
X_1	7059	7267	53	0	22	0	0	0	68
X_2	12715	6031	45	0	0	30	0	0	0
	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	x_{41}	x_{42}	x_{43}	x_{44}
X_1	7	0	5	0	91	0	0	0	8
X_2	75	0	59	90	1	8	0	0	0

Tab. 9. Pay-off matrix.

As in **Step 4**, transform the bi-objective TP into its single objective TP using the proposed neutrosophic TOPSIS approach.

By **Step 4a**, from Table 9, the calculated max and min values are $Z^{(1)+} = 7059$, $Z^{(2)+} = 6031$, $Z^{(1)-} = 12715$ and $Z^{(2)-} = 7267$. The calculated PIS and NIS are $Z^{+} = (7059, 6031)$ and $Z^{-} = (12715, 7267)$.

As in **Step 4b**, assign the weights as $w_1 = 0.5$ and $w_2 = 0.5$ using Eqn. 14 and Eqn. 15, the obtained distance function from PIS $d_q^{PIS}(x)$ and NIS $d_q^{NIS}(x)$ are

$$d_q^{PIS}(x) = \left[\left(0.5 \left(\frac{Z^{(1)}(x) - 7059}{12715 - 7059} \right) \right)^q + \left(0.5 \left(\frac{Z^{(2)}(x) - 6031}{7267 - 6031} \right) \right)^q \right]^{\frac{1}{q}} \tag{17}$$

$$d_q^{NIS}(x) = \left[\left(0.5 \left(\frac{12715 - Z^{(1)}(x)}{12715 - 7059} \right) \right)^q + \left(0.5 \left(\frac{7267 - Z^{(2)}(x)}{7267 - 6031} \right) \right)^q \right]^{\frac{1}{q}}. \tag{18}$$

As in **Step 4c**, set $q = 2$ in the above equation then the problem is reformulated as

$$(G_6)\ \text{Minimize } d_2^{PIS}(x) = \left[\left(0.5\left(\frac{Z^{(1)}(x) - 7059}{12715 - 7059}\right)\right)^2 + \left(0.5\left(\frac{Z^{(2)}(x) - 6031}{7267 - 6031}\right)\right)^2\right]^{\frac{1}{2}}$$

$$\text{Maximize } d_2^{NIS}(x) = \left[\left(0.5\left(\frac{12715 - Z^{(1)}(x)}{12715 - 7059}\right)\right)^2 + \left(0.5\left(\frac{7267 - Z^{(2)}(x)}{7267 - 6031}\right)\right)^2\right]^{\frac{1}{2}}$$

Subject to the constraints Eqn. 11 to Eqn. 13. (19)

By **Step 4d**, solve the problem (G_6) using the LINGO software and form the pay-off table of ideal solution which is given in Table 10.

	$d_2^{PIS}(x)$	$d_2^{NIS}(x)$
x^{PIS}	1.000	0.203
x_{NIS}	1.381	0.853

Tab. 10. Pay-off matrix of ideal solution.

Here $(d_2^{PIS})^* = 1.000$, $(d_2^{PIS})' = 1.381$, $(d_2^{NIS})^* = 0.853$, $(d_2^{NIS})' = 0.203$.

By **Step 4e**, we have formulated the truth and falsity MF are as follows:

$$\alpha_1(x) = \frac{(d_2^{PIS})' - d_2^{PIS}(x)}{(d_2^{PIS})' - (d_2^{PIS})^*}$$

$$\alpha_1(x) = \frac{(1.381 - d_2^{PIS}(x)}{1.381 - 1.000}$$

$$\alpha_1(x) = \frac{(1.381 - d_2^{PIS}(x)}{0.381}$$

$$\gamma_1(x) = \frac{d_2^{PIS}(x) - (d_2^{PIS})^*}{(d_2^{PIS})' + \rho_1 - (d_2^{PIS})^*}$$

$$\gamma_1(x) = \frac{d_2^{PIS}(x) - 1.000}{1.381 + \rho_1 - 1.000}$$

$$\gamma_1(x) = \frac{d_2^{PIS}(x) - 1.000}{0.381 + \rho_1}.$$

In the same manner, we formulate the remaining truth, indeterminacy and falsity MF for both objective functions.

As in **Step 4f** using the max-min operator, the bi-objective problem is reduced into the single objective problem which are shown below.

(G_7) Maximize $(\mu_2 - \delta_2 - \theta_2)$

Subject to the constraints Eqn. 11 to Eqn. 13

$$Z^{(1)}(x) = 19x_{11} + 48x_{12} + 18x_{13} + 59x_{14} + 51x_{21} + 59x_{22} + 29x_{23} + 32x_{24} + 52x_{31} + 17x_{32} + 74x_{33} + 27x_{34} + 0x_{41} + 0x_{42} + 0x_{43} + 0x_{44} \quad (20)$$

$$Z^{(2)}(x) = 29x_{11} + 18x_{12} + 22x_{13} + 27x_{14} + 28x_{21} + 9x_{22} + 27x_{23} + 22x_{24} + 32x_{31} + 12x_{32} + 17x_{33} + 28x_{34} + 0x_{41} + 0x_{42} + 0x_{43} + 0x_{44} \quad (21)$$

$$\frac{1.381 - d_2^{PIS}(x)}{0.381} \geq \mu_2, \quad \frac{d_2^{NIS}(x) - 0.203}{0.650} \geq \mu_2, \quad (22)$$

$$\frac{d_2^{PIS}(x) - 1.000}{0.381 + \eta_1} \leq \delta_2, \quad \frac{0.853 - d_2^{NIS}(x)}{0.650 + \eta_2} \leq \delta_2, \quad (23)$$

$$\frac{d_2^{PIS}(x) - 1.000}{0.381 + \rho_1} \leq \theta_2, \quad \frac{0.853 - d_2^{NIS}(x)}{0.650 + \rho_2} \leq \theta_2, \quad (24)$$

$$\mu_2 + \delta_2 + \theta_2 \leq 3. \quad (25)$$

Here $\mu_2 = \min\{\alpha_1(x), \alpha_2(x)\}, \delta_2 = \max\{\beta_1(x), \beta_2(x)\}, \theta_2 = \max\{\gamma_1(x), \gamma_2(x)\}$

By **Step 5**, we obtain the Pareto optimal solution using the LINGO software of $Z^{(1)}(x)$ is 7059 and $Z^{(2)}(x)$ is 7267 at the allocations $x_{11} = 53$, $x_{13} = 22$, $x_{23} = 68$, $x_{24} = 7$, $x_{32} = 59$, $x_{34} = 91$, $x_{44} = 8$ and the rest are all zero for $q = 2$. Finally, the corresponding neutrosophic Pareto-optimal solution obtained for the proposed neutrosophic TOPSIS approach is $\{(3107, 3336, 5068, 6150; 0.5, 0.5, 0.7), (3653, 3994, 5366, 6446; 0.3, 0.8, 0.6)\}$.

Using Step 6, assign different weights based on priority of objective functions to obtain the set of Pareto-optimal solutions which are shown in Table 12.

6. SENSITIVITY ANALYSIS: IMPACT OF VARIATIONS IN PARAMETERS

The sensitivity analysis involves altering the parameters of objective function, supply and demand by increasing or decreasing a certain percentage of values, while holding the other parameters at their initial values. First we alter the parameters of objective function by decreasing the value as 5%. Then the problem (G_7) is formulated as

Maximize $(\mu_2 - \delta_2 - \theta_2)$

Subject to the constraints Eqn. 11 to Eqn. 13

$$Z^{(1)}(x) = 18.95x_{11} + 47.95x_{12} + 17.95x_{13} + 58.95x_{14} + 50.95x_{21} + 58.95x_{22} + 28.95x_{23} + 31.95x_{24} + 51.95x_{31} + 16.95x_{32} + 73.95x_{33} + 26.95x_{34} + 0x_{41} + 0x_{42} + 0x_{43} + 0x_{44}$$

$$Z^{(2)}(x) = 28.95x_{11} + 17.95x_{12} + 21.95x_{13} + 26.95x_{14} + 27.95x_{21} + 8.95x_{22} + 26.95x_{23} + 21.95x_{24} + 31.95x_{31} + 11.95x_{32} + 16.95x_{33} + 27.95x_{34} + 0x_{41} + 0x_{42} + 0x_{43} + 0x_{44}$$

$$\frac{1.381 - d_2^{PIS}(x)}{0.381} \geq \mu_2, \quad \frac{d_2^{NIS}(x) - 0.203}{0.650} \geq \mu_2,$$

$$\frac{d_2^{PIS}(x) - 1.000}{0.381 + \eta_1} \leq \delta_2, \quad \frac{0.853 - d_2^{NIS}(x)}{0.650 + \eta_2} \leq \delta_2,$$

$$\frac{d_2^{PIS}(x) - 1.000}{0.381 + \rho_1} \leq \theta_2, \quad \frac{0.853 - d_2^{NIS}(x)}{0.650 + \rho_2} \leq \theta_2,$$

$$\mu_2 + \delta_2 + \theta_2 \leq 3.$$

Here $\mu_2 = \min\{\alpha_1(x), \alpha_2(x)\}, \delta_2 = \max\{\beta_1(x), \beta_2(x)\}, \theta_2 = \max\{\gamma_1(x), \gamma_2(x)\}$.

Therefore, the obtained POS of $Z^{(1)}(x)$ is 7044 and $Z^{(2)}(x)$ is 7252. Next, we alter the parameters of supply by decreasing the value as 5%. Then the problem (G_7) is formulated as

$$\text{Maximize } (\mu_2 - \delta_2 - \theta_2)$$

Subject to:

$$Z^{(1)}(x) = 19x_{11} + 48x_{12} + 18x_{13} + 59x_{14} + 51x_{21} + 59x_{22} + 29x_{23} + 32x_{24} + 52x_{31} + 17x_{32} + 74x_{33} + 27x_{34} + 0x_{41} + 0x_{42} + 0x_{43} + 0x_{44}$$

$$Z^{(2)}(x) = 29x_{11} + 18x_{12} + 22x_{13} + 27x_{14} + 28x_{21} + 9x_{22} + 27x_{23} + 22x_{24} + 32x_{31} + 12x_{32} + 17x_{33} + 28x_{34} + 0x_{41} + 0x_{42} + 0x_{43} + 0x_{44}$$

$$x_{11} + x_{12} + x_{13} + x_{14} = 74.95, x_{21} + x_{22} + x_{23} + x_{24} = 74.95$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 149.95, x_{41} + x_{42} + x_{43} + x_{44} = 7.95$$

$$x_{51} + x_{52} + x_{53} + x_{54} = 0.2, x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 53$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 59, x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 90$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 106$$

$$\frac{1.381 - d_2^{PIS}(x)}{0.381} \geq \mu_2, \quad \frac{d_2^{NIS}(x) - 0.203}{0.650} \geq \mu_2,$$

$$\frac{d_2^{PIS}(x) - 1.000}{0.381 + \eta_1} \leq \delta_2, \quad \frac{0.853 - d_2^{NIS}(x)}{0.650 + \eta_2} \leq \delta_2,$$

$$\frac{d_2^{PIS}(x) - 1.000}{0.381 + \rho_1} \leq \theta_2, \quad \frac{0.853 - d_2^{NIS}(x)}{0.650 + \rho_2} \leq \theta_2,$$

$$\mu_2 + \delta_2 + \theta_2 \leq 3.$$

Here $\mu_2 = \min\{\alpha_1(x), \alpha_2(x)\}, \delta_2 = \max\{\beta_1(x), \beta_2(x)\}, \theta_2 = \max\{\gamma_1(x), \gamma_2(x)\}$.

Therefore, the obtained POS of $Z^{(1)}(x)$ is 7071.30 and $Z^{(2)}(x)$ is 7165.85. In the same manner, we have altered the other parameters of objective functions, supply and demand by increasing or decreasing a certain percentage of values, while holding the other parameters at their initial values which are shown in Table 11.

Percentage values	Variations of objective function		Variations of Supply		Variations of Demand	
	Z_1	Z_2	Z_1	Z_2	Z_1	Z_2
-5%	7044	7252	7071.30	7165.85	7069.35	7166.55
-3%	7050	7258	7072.78	7167.91	7071.61	7168.33
-1%	7056	7264	7074.26	7169.97	7073.87	7170.11
0%	7059	7267	7059	7267	7059	7267
+1%	7062	7270	7074.54	7170.67	7074.93	7170.53
+3%	7068	7276	7073.62	7170.01	7074.79	7169.59
+5%	7074	7282	7072.70	7169.35	7074.65	7168.65

Tab. 11. Variations of objective functions, supply, and demand.

For better understanding, using Table 11 we have represented the graphical view of SA for variations of objective function, supply and demand parameters in Fig. 3, Fig. 4 and Fig. 5 respectively.

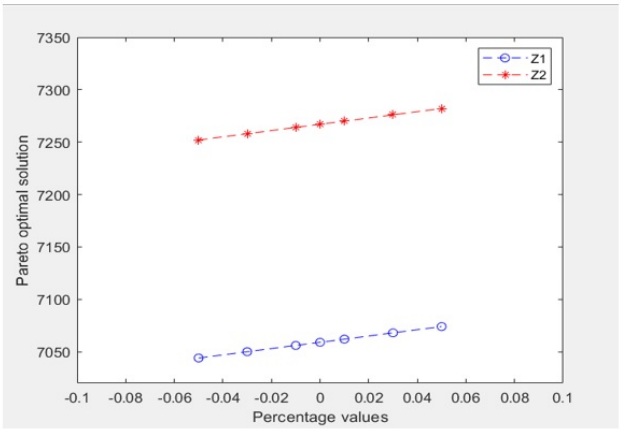


Fig. 3. Sensitivity Analysis for variations of objective function parameter.

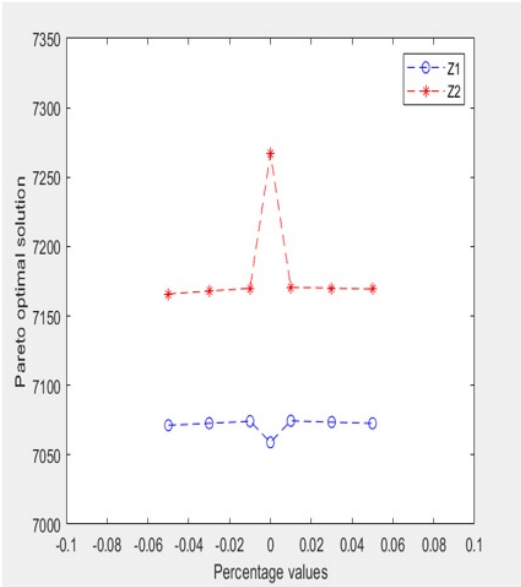


Fig. 4. Sensitivity Analysis for variations of supply parameter.

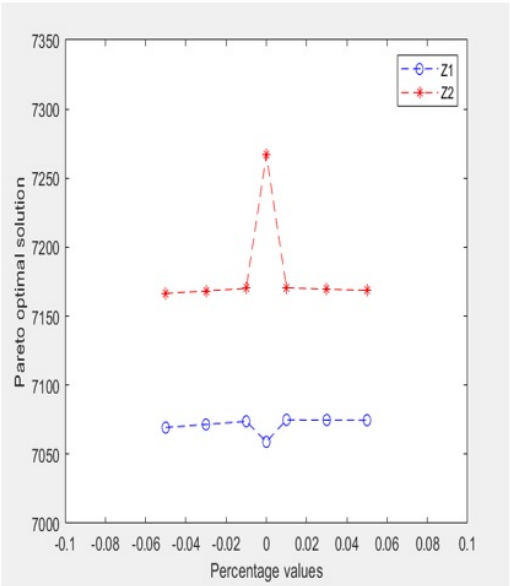


Fig. 5. Sensitivity Analysis for variations of demand parameter.

From Figure 3, we conclude that the Pareto optimal solution for both the objectives gradually increases as the percentage values rise which depends on the variations in the objective function parameter. Figures 4 and 5 indicate that the Pareto optimal solution for the first objective initially increases, reaches a peak and then subsequently declines with the percentage values whereas the Pareto optimal solution for the second objective initially increases, reaches a minimum value and then subsequently declines with the percentage values.

7. COMPARATIVE ANALYSIS: ASSES THE EFFECTIVENESS OF THE PROPOSED APPROACH

Table 12 displays the results of POS obtained using various optimization methods such as proposed neutrosophic TOPSIS approach, NFPA and fuzzy goal programming approach (FGPA). These methods were applied to the same numerical example to compare their outcomes and assess the effectiveness of each approach to achieve the optimal results. Consider the following scenarios by assigning different weights based on priority of objective functions are as follows:

1. The DMs should assign equal importance to both transportation cost and deterioration cost for pharmaceutical products such as vaccines, insulin and blood. During the long distance shipment, the transportation cost becomes a significant factor due to the need for specialized cold infrastructure. At the same time, these

products have limited shelf lives and strict temperature requirements which have an equal importance for the deterioration cost.

2. The DMs should assign greater importance to transportation cost than to deterioration cost for industrial materials. When transporting these materials over long distances, the use of heavy vehicles results in high transportation cost. Since industrial materials are highly durable and the cost associated with deterioration during transportation is minimal.

3. The DMs should assign greater importance to deterioration cost than to transportation cost for perishable products and baked foods. These products are highly perishable which makes the deterioration cost as dominant factor during transportation. While transportation costs are less significant than the potential losses from product spoilage.

Weights	**Proposed approach**	**NFPA**	**FGPA**
$w_1 = w_2$ $w_1 = w_2 = 0.5$ Values of DV	$x_{11} = 53$, $x_{13} = 22$, $x_{23} = 68$, $x_{24} = 7$, $x_{32} = 59$, $x_{34} = 91$, $x_{44} = 8$	$x_{11} = 42$, $x_{13} = 33$, $x_{21} = 3$, $x_{23} = 16$, $x_{24} = 56$, $x_{32} = 59$, $x_{33} = 41$, $x_{34} = 50$, $x_{41} = 8$	$x_{11} = 53$, $x_{12} = 7$, $x_{13} = 15$, $x_{23} = 75$, $x_{32} = 44$, $x_{34} = 106$, $x_{42} = 8$
POS	(7059, 7267)	(9188,6497)	(7398, 7514)
$w_1 > w_2$ $w_1 = 0.80, w_2 = 0.20$ Values of DV	$x_{11} = 47.52$, $x_{13} = 27.47$, $x_{23} = 62.52$, $x_{24} = 12.47$, $x_{32} = 59$, $x_{34} = 91$, $x_{41} = 5.47$, $x_{44} = 2.52$	-	-
POS	(7069.94, 7201.33)	-	-
$w_1 < w_2$ $w_1 = 0.20, w_2 = 0.80$ Values of DV	$x_{11} = 45$, $x_{13} = 20.78$, $x_{14} = 9.21$, $x_{23} = 26.42$, $x_{24} = 48.57$, $x_{32} = 59$, $x_{33} = 42.78$, $x_{34} = 48.21$, $x_{41} = 8$	-	-
POS	(9564.39, 6578.50)	-	-
Comparison for Proposed approach Vs Existing methods			
Aspect	**Proposed approach**	**NFPA**	**FGPA**
Handling of unequal weights	Handle unequal weights	Does not handle unequal weights	Does not handle unequal weights
Pareto optimal solutions	Set of POS	Only one POS	Only one POS
Adaptability in multi-objective environment	Effectively manage multiple conflicting objectives at the same time	Limited capability to handle multiple conflicting objectives at the same time	Limited capability to handle multiple conflicting objectives at the same time
Suitability for real-world applications	Highly suitable	Moderately suitable	Moderately suitable

Tab. 12. Results of Pareto optimal solution.

From Table 12, we analyse that our proposed approach with different weights that corresponds based on priority of objective functions, DMs can systematically explore a set of Pareto optimal solutions under different priority scenarios whereas the other exiting approaches provide only one Pareto optimal solution. In addition to this, we can identify the neutrosophic ideal solutions that is PIS has the best and NIS has the worst neutrosophic score on every criterion. Also, the DM can provide the actual

preference weights for the criteria, which can also be neutrosophic if their preferences are not perfectly clear. To enhance the clarity, the Pareto optimal solution obtained is compared with the existing approaches as depicted in Figure 6.

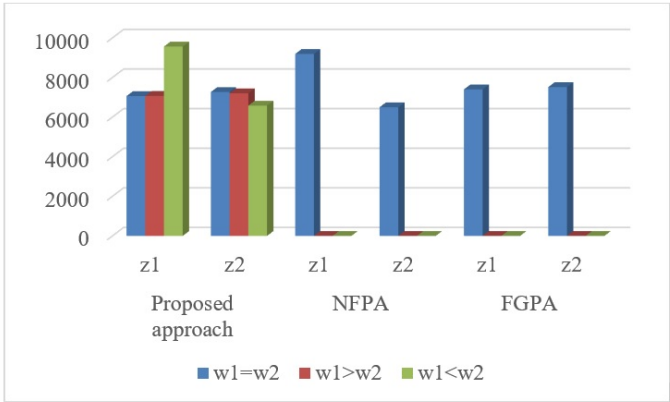


Fig. 6. Comparison of obtained Pareto-optimal solution.

From Table 12 and Figure 6, it can be seen that the obtained Pareto optimal solution of the solution approach using the proposed neutrosophic TOPSIS approach provides better result than the NFPA and FGPA by calculating the average of two objective functions.

8. CONCLUSIONS AND FUTURE SCOPE

In this study, the neutrosophic bi-objective transportation problem with the parameters of single-valued trapezoidal neutrosophic numbers are considered. The weighted possibility mean is employed to convert the neutrosophic problem into its deterministic problem. We have proposed the neutrosophic TOPSIS approach to convert the bi-objective problem into its single objective problem. To highlight the practical efficacy of the problem, we provide numerical examples and the comparison was made with the existing approaches. Our proposed approach provides a comprehensive and systematic framework for evaluating different delineator configurations within road environments. This approach not only aids decision-makers in selecting the most effective delineator design but also helps to identify the optimal configuration by weighing the trade-offs between competing objectives. For the future perspective, researchers could expand to neutrosophic environment such as multi-objective fixed charge transportation problem, multi-item multi-objective fixed charge transportation problem, multi-stage multi-objective fixed charge transportation problem and multi-level multi-item multi-objective fixed charge transportation problem. The framework can also be extended to stochastic environments where uncertainties in supply and demand are modelled using probability distributions such as normal, Pareto, Weibull distributions and so on. Moreover, other types of neutrosophic sets such as hyper neutrosophic sets [61], super hyper

neutrosophic sets [61], tree neutrosophic sets [63], double-valued neutrosophic sets [24], neutrosophic offsets [60] and plithogenic sets [62] can be formulated. This approach is not restricted to transportation problems and can also be applied to assignment problem and travelling salesman problem.

ACKNOWLEDGEMENT

The authors would like to thank the Editor and the anonymous referees for their valuable comments and suggestions.

(Received June 18, 2025)

REFERENCES

[1] M. A. Abo-Sinna, A. H. Amer, and A. S. Ibrahim: Extensions of TOPSIS for large scale multi-objective non-linear programming problems with block angular structure. Appl. Math. Model. *32* (2008), 3, 292–302. DOI:10.1111/j.1467-7717.2008.01040.x

[2] F. Ahmad and A. Y. Adhami: Neutrosophic programming approach to multi-objective nonlinear transportation problem with fuzzy parameters. Int. J. Manag. Sci. Eng. Manag. *14* (2019), 3, 218–229.

[3] Y. P. Aneja and K. P. Nair: Bicriteria transportation problem. Manag. Sci. *25* (1979), 1, 73–78. DOI:10.1287/mnsc.25.1.73

[4] K. T. Atanassov: Intuitionistic fuzzy sets. Fuzzy Sets Syst. *20* (1986), 1, 87–96. DOI:10.1016/S0165-0114(86)80034-3

[5] M. Bagheri, A. Ebrahimnejad, S. Razavyan, F. Hosseinzadeh Lotfi, and N. Malekmohammadi: Solving the fully fuzzy multi-objective transportation problem based on the common set of weights in DEA. J. Intell. Fuzzy Syst. *39* (2020), 3, 3099–3124. DOI:10.3233/JIFS-191560

[6] G. Bai and L. Yao: A simple algorithm for the multi-objective transportation model. In: 2011 International Conference on Business Management and Electronic Information, IEEE 2011, pp. 479–482.

[7] R. E. Bellman and L. A. Zadeh: Decision-making in a fuzzy environment. Management science. Manag. Sci. *17* (1970), 4, 8141–8164.

[8] R. K. Bera and S. K. Mondal: A multi-objective transportation problem under quantity dependent credit period and cost structure policies in triangular intuitionistic fuzzy environment. Eng. Appl. Artif. Intell. *123* (2023), 106396. DOI:10.1016/j.engappai.2023.106396

[9] A. K. Bind, D. Rani, A. Ebrahimnejad, and J. L. Verdegay: New strategy for solving multi-objective green four dimensional transportation problems under normal type-2 uncertain environment. Eng. Appl. Artif. Intell. *137* (2024), 109084.

[10] S. G. Bodkhe: Multi-objective transportation problem using fuzzy programming techniques based on exponential membership functions. Int. J. Stat. Appl. Math. *8* (2024), 5, 20–24.

[11] B. C. Cuong and V. Kreinovich: Picture fuzzy sets-a new concept for computational intelligence problems. In: 2013 third world congress on information and communication technologies (WICT 2013), IEEE 2013, pp. 1–6.

[12] I. Deli and Y. Subas: Single valued neutrosophic numbers and their applications multi-criteria decision making problem. Neutrosophic Sets Syst. *2* (2014), 1, 1–13.

[13] S. Dhanasekar, S. Hariharan, and P. Sekar: Fuzzy Hungarian MODI algorithm to solve fully fuzzy transportation problems. Int. J. Fuzzy Syst. *19* (2017), 1479–1491. DOI:10.1007/s40815-016-0251-4

[14] S. Ghosh, K. H. Küfer, S. K. Roy, and G. W. Weber: Type-2 zigzag uncertain multi-objective fixed-charge solid transportation problem: time window vs. preservation technology. Cent. Eur. J. Oper. Res.*31* (2023), 1, 337–362. DOI:10.1007/s10100-022-00811-7

[15] S. Ghosh, S. K. Roy, A. Ebrahimnejad, and J. L. Verdegay: Multi-objective fully intuitionistic fuzzy fixed-charge solid transportation problem. Complex Intell. Syst. *7* (2021), 1009–1023. DOI:10.1007/s40747-020-00251-3

[16] A. Gupta, A. Kumar, and A. Kaur: Mehar's method to find exact fuzzy optimal solution of unbalanced fully fuzzy multi-objective transportation problems. Optim. Lett. *6* (2012), 1737–1751. DOI:10.1007/s11590-011-0367-2

[17] B. I. N. A. Gupta and R. E. E. T. A. Gupta: Multi-criteria simplex method for a linear multiple objective transportation problem. Indian J. Pure Appl. Math. *14* (1983), 2, 222–232.

[18] P. Hansen, M. Labbé, and R. E. Wendell: Sensitivity analysis in multiple objective linear programming: The tolerance approach. Eur. J. Oper. Res. *38* (1989), 1, 63–69. DOI:10.1080/09578818908426501

[19] F. L. Hitchcock: The distribution of a product from several sources to numerous localities. J. Math. Phys. *20* (1941), 1–4, 224–230. DOI:10.1002/sapm1941201224

[20] E. Hosseinzadeh and J. Tayyebi: A compromise solution for the neutrosophic multi-objective linear programming problem and its application in transportation problem. J. Appl. Res. Ind. Eng. *10* (2023), 1, 1–10.

[21] C. L. Hwang and K. Yoon: Multiple Attribute Decision Making: Methods and Applications. Springer, New York 1981.

[22] H. Isermann: The enumeration of all efficient solutions for a linear multiple-objective transportation problem. Nav. Res. Logist. Q. *26* (1979), 1, 123–139. DOI:10.1002/nav.3800260112

[23] Y. Kacher and P. Singh: Fuzzy harmonic mean technique for solving fully fuzzy multi-objective transportation problem. J. Comput. Sci.*63* (2022), 101782. DOI:10.1016/j.jocs.2022.101782

[24] I. Kandasamy: Double-valued neutrosophic sets, their minimum spanning trees, and clustering algorithm. J. Intell. Syst. *27* (2018), 2, 163–182. DOI:10.1515/para-2018-0012

[25] N. Kartli, E. Bostanci, and M. S. Guzel: Heuristic algorithm for an optimal solution of fully fuzzy transportation problem. Computing *106* (2024), 10, 3195–3227. DOI:10.1007/s00607-024-01319-5

[26] H. S. Kasana and K. D. Kumar: An efficient algorithm for multi-objective transportation problems. Asia Pac. J. Oper. Res. *17* (2000), 1, 27.

[27] L. Kaur, M. Rakshit, and S. Singh: A new approach to solve multi-objective transportation problem. Appl. Appl. Math. *13* (2018), 1, 10.

[28] K. Kavitha and D. Anuradha: Heuristic algorithm for finding sensitivity analysis of a more for less solution to transportation problems. Global J. Pure Appl. Math. *11* (2015), 1, 479–485.

[29] K. Kavitha and D. Anuradha: Sensitivity analysis for spanning tree. In: Advances in Algebra and Analysis: International Conference on Advances in Mathematical Sciences, Vellore, Volume I, Springer International Publishing 2018, pp. 227–232.

[30] T. C. Koopmans: Optimum utilization of the transportation system. Econometrica (1949), 136–146.

[31] A. Kuiri and B. Das: An application of FISM and TOPSIS to a multi-objective multi-item solid transportation problem. Opsearch. *57* (2020), 4, 1299–1318. DOI:10.1007/s12597-020-00456-7

[32] S. M. Lee and L. J. Moore: Optimizing transportation problems with multiple objectives. AIIE Trans. *5* (1973), 4, 333–338. DOI:10.1016/0001-8716(73)80005-7

[33] K. M. Lee: Bipolar-valued fuzzy sets and their operations. In: Proc. Int. Conf. on Intelligent Technologies, Bangkok 2000, pp. 307–312.

[34] S. Mahajan and S. K. Gupta: On fully intuitionistic fuzzy multi-objective transportation problems using different membership functions. Ann. Oper. Res. *296* (2021), 211–241. DOI:10.1007/s10479-019-03318-8

[35] M. M. Miah, A. AlArjani, A. Rashid, A. R. Khan, M. S. Uddin, and E. A. Attia: Multi-objective optimization to the transportation problem considering non-linear fuzzy membership functions. AIMS Math. *8* (2023), 5, 10397–10419. DOI:10.3934/math.2023527

[36] S. Midya and S. K. Roy: Multi-objective fixed-charge transportation problem using rough programming. Int. J. Oper. Res.*37* (2020), 3, 377–395. DOI:10.1504/ijor.2020.105444

[37] S. Midya, S. K. Roy, and V. F. Yu: Intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain. Int. J. Mach. Learn. Cybern. *12* (2021), 699–717. DOI:10.1007/s13042-020-01197-1

[38] M. Mondal, G. Mandal, A. A. Shaikh, and A. K. Bhunia: Investigation of a multi-objective fixed charge transportation problem with quantity dependent transportation cost and discount policy via metaheuristics. Inf. Syst. Oper. Res. (2025), 1–30. DOI:10.1080/03155986.2025.2492739

[39] M. Niksirat: A new approach to solve fully fuzzy multi-objective transportation problem. Fuzzy Inf. Eng. (2022), 1–12. DOI:10.1080/16168658.2022.2152836

[40] K. P. O. Niluminda and E. M. U. S. B. Ekanayake: The multi-objective transportation problem solve with geometric mean and penalty methods. Indones. J. Innov. Appl. Sci. *3* (2023), 1, 74–85.

[41] M. A. Nomani, I. Ali, and A. Ahmed: A new approach for solving multi-objective transportation problems. Int. J. Manag. Sci. Eng. Manag. *12* (2017), 3, 165–173. DOI:10.1080/17509653.2016.1172994

[42] P. Pandian and G. Natarajan: A new method for finding an optimal solution for transportation problems. Int. J. Math. Sci. Eng. Appl. *4* (2010), 2, 59–65.

[43] P. Pandian and D. Anuradha: A new method for solving bi-objective transportation problems. Aust. J. Basic Appl. Sci. *5* (2011), 10, 67–74.

[44] R. Rajbhar and L. Das: Advancements in and multi-objective transportation problems: techniques and comparative analysis. Microsyst. Technol. (2025), 1–14.

[45] A. N. Revathi, S. Mohanaselvi, and B. Said: An efficient neutrosophic technique for uncertain multi objective transportation problem. Neutrosophic Sets Syst. *53* (2023), 1, 27.

[46] J. L. Ringuest and D. B. Rinks: Interactive solutions for the linear multi-objective transportation problem. Eur. J. Oper. Res. *32* (1987), 1, 96–106. DOI:10.1016/0377-2217(87)90274-8

[47] R. M. Rizk-Allah, A. E. Hassanien, and M. Elhoseny: A multi-objective transportation model under neutrosophic environment. Comput. Electr. Eng. *69* (2018), 705–719.

[48] S. K. Roy, S. Midya, and G. W. Weber: Multi-objective multi-item fixed-charge solid transportation problem under twofold uncertainty. Neural Comput. Appl. *31* (2019), 8593–8613. DOI:10.1007/s00521-019-04431-2

[49] S. K. Roy and S. Midya: Multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment. Appl. Intell. *49* (2019), 3524–3538. DOI:10.1007/s10489-019-01466-9

[50] S. K. Roy, A. Ebrahimnejad, J. L. Verdegay, and S. Das: New approach for solving intuitionistic fuzzy multi-objective transportation problem. Sadhana *43* (2019), 1, 1–12.

[51] S. Sandhiya and A. Dhanapal: Solving bi-objective transportation problem under neutrosophic environment. J. Appl. Math. Informatics *42* (2024), 4, 831–854.

[52] S. Sandhiya and A. Dhanapal: Solving neutrosophic multi-dimensional fixed charge transportation problem. Contemp. Math.*2024* (2024), 3601–3624. DOI:10.1093/imrn/rnad027

[53] A. A. Salama, H. E. Khalid, A. K. Essa, I. Yasser, and A. A. Abd El-Khalek: Integrating neutrosophic logic with fuzzy inference systems: A comprehensive survey on enhancing decision-making under uncertainty. Neutrosophic Sets Syst. *91* (2025), 851–868.

[54] S. Samanta, D. Chakraborty, and D. K. Jana: Neutrosophic multi-period two stage four-dimensional transportation problem for breakable items. Expert Syst. Appl. *246* (2024), 123266. DOI:10.1016/j.eswa.2024.123266

[55] Shivani and D. Rani: Multi-objective multi-item four dimensional green transportation problem in interval-valued intuitionistic fuzzy environment. Int. J. Syst. Assur. Eng. Manag. *15* (2024), 2, 727–744.

[56] K. P. Sikkannan and V. Shanmugavel: Unraveling neutrosophic transportation problem using costs mean and complete contingency cost table. Neutrosophic Sets Syst. *29* (2019), 165–173.

[57] P. Singh, S. Kumari, and P. Singh: Fuzzy efficient interactive goal programming approach for multi-objective transportation problems. Int. J. Appl. Comput. Math. *3* (2017), 505–525. DOI:10.1007/s40819-016-0155-x

[58] V.E. Sobana and D. Anuradha: On solving bi-objective fuzzy transportation problem. In: Advances in Algebra and Analysis: International Conference on Advances in Mathematical Sciences, Vellore, India 2017, Volume I, Springer International Publishing 2018, pp. 233–242.

[59] F. Smarandache: A unifying field in logics. Neutrosophy: Neutrosophic Probability, Set and Logic. American Research Press, Rehoboth 105, 1999.

[60] F. Smarandache: Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset. Similarly for Neutrosophic Over-/Under-/Off-Logic, Probability, and Statistics. Infinite Study, 2016.

[61] F. Smarandache: Hyperuncertain, Superuncertain, and Superhyperuncertain Sets/Logics/Probabilities/Statistics. Infinite Study, 2017.

[62] F. Smarandache: Plithogeny, Plithogenic Set, Logic, Probability, and Statistics. Infinite Study. Pons Editions, Brussels 2017.

[63] F. Smarandache: Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), pythagorean fuzzy set, spherical fuzzy set, and q-rung orthopair fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision. J. New Theory *29* (2019), 1–13. DOI:10.7748/phc.29.2.13.s11

[64] F. Smarandache and K. Mohana: Fuzzy, Intuitionistic and Neutrosophic Set Theories and their Applications in Decision Analysis. Infinite Study, 2025.

[65] V. Torra: Hesitant fuzzy sets. Int. J. Intell. Syst. *25* (2010), 6, 529–539.

[66] R. M. Umamageswari and G. Uthra: Generalized single valued neutrosophic trapezoidal numbers and their application to solve transportation problem. Studia Rosenthaliana *11* (2020), 164–170.

[67] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman: Single valued neutrosophic sets. Tech. Sci. Appl. Math. *16* (2010), 1, 10–14. DOI:10.1525/gfc.2010.10.2.14

[68] L. A. Zadeh: Fuzzy sets. Inf. Control *8* (1965), 3, 338–353. DOI:10.1016/S0019-9958(65)90241-X

[69] P. Zhu, G. Chen, Y. Chen, and A. Zhang: On the pure fixed charge transportation problem. Discret. Optim. *55* (2025), 100875. DOI:10.1016/S0255-0857(25)00088-X

[70] H. J. Zimmermann: Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Syst. *1* (1978), 1, 45–55. DOI:10.1016/0165-0114(78)90031-3

S. Sandhiya, Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu 632014. India.
e-mail: sandhiyanive98@gmail.com

Anuradha Dhanapal, Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu 632014. India.
e-mail: anuradhadhanapal1981@gmail.com