Preface to the original edition

This book comprises a theory of transformations of ordinary linear homogeneous differential equations of the second order; the concern of this theory is to apply results relating to transformation of variables to the solutions of such differential equations. It is a qualitative theory, in the real domain, and is of global character.

The theory of transformations of such linear second-order equations was founded by E. E. Kummer, whose work in this field is distinguished particularly by his discovery of that non-linear third-order differential equation which is fundamental to the theory. Further discoveries in this direction led to wide-ranging research on transformations of linear differential equations of the n-th order, in connection with the equivalence problem; notable studies in this field are those of E. Laguerre, F. Brioschi, G. H. Halphen, A. R. Forsyth, S. Lie and P. Appell. Within the framework of these studies, there occurs from time to time work on the transformation of linear differential equations of the second order in the complex domain. Linear differential equations of the second order have an exceptional position among those of order n ($n \geqslant 2$) since only in the case $n = 2$ are two differential equations always equivalent.

The transformation theory expounded in this book is a far-reaching development of certain new fundamental ideas, and consists essentially of two parts. The first part comprises the "theory of dispersions," named after its basic idea; it relates to oscillatory differential equations and proceeds from the concept of "central dispersion" to a constructive theory for integration of the Kummer differential equation. The other part consists of "general transformation theory", in which we study properties of the solutions of the Kummer differential equation under general conditions, in connection with the process of transforming linear differential equations of the second order. One section of this theory is devoted to questions regarding complete solutions of the Kummer equation; such complete solutions are distinguished by the fact that they provide functions which transform integrals of linear differential equations of the second order into each other over their entire domain.

The methods used in building up this transformation theory made it clear that much can be gained by a deepening and widening of certain concepts in the classical theory of linear second-order differential equations. This is particularly true of the concept of "phase" which one can recognize, in retrospect, as one of the most important ideas used in transformation theory. The significance of this concept is brought out in the chapter on "phase theory" which develops a body of results related to this notion in preparation for later use. Other theories, also of a preparatory character, are built up as well; these relate to the subjects of "conjugate numbers" and of centro-affine properties of plane curves.

My concern has been to give the book a form which shall be fresh and stimulating, and at the same time be a complete and unified whole. The material on which I have worked has not only called for the methods of classical analysis but has also given scope, in many directions, for use of the apparatus of modern algebra, particularly group theory; in this way has come the discovery of some deep results. The reader must judge how far I have succeeded in achieving the objectives I set myself; how far, indeed, potentialities have been turned into fact.

I take this opportunity of expressing warmest thanks to my colleagues, Dozents E. Barvínek and Fr. Neuman, for their careful reading of the manuscript and for their kindly advice. Dr. Neuman has, moreover, compiled the bibliography on work carried out by participants in my seminar and arising from the subject-matter of this book. I am grateful also to Mrs. H̆. Fendrychová for her careful preparation of the diagrams; finally, I am greatly indebted to the VEB Deutscher Verlag der Wissenschaften in Berlin for their generous and accurate collaboration.

Brno *O. Borůvka*