

# Life and work of Vojtěch Jarník

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[Photographs]

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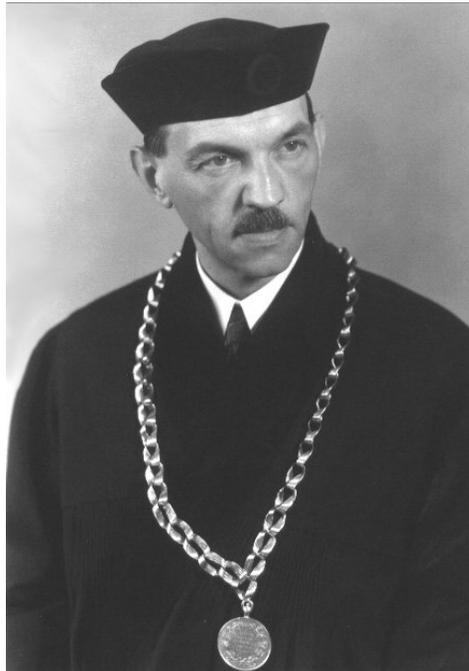
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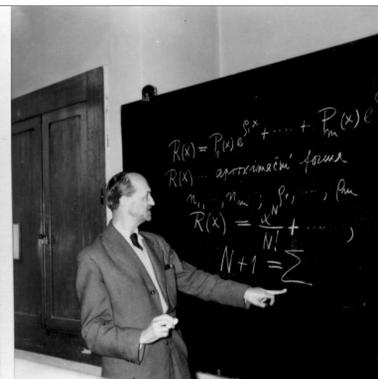
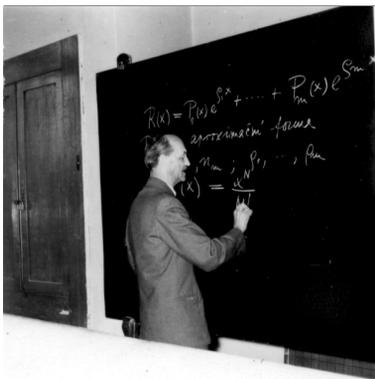
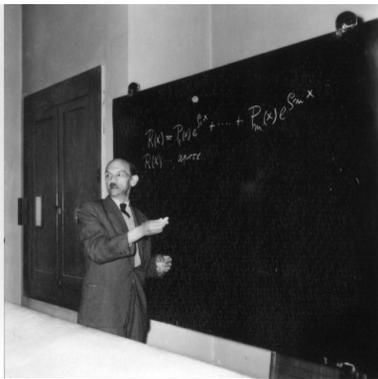
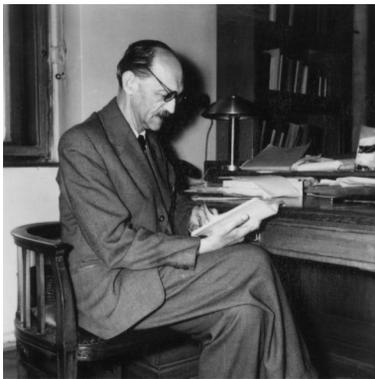
At eighteen, graduating from secondary school



As Dean of the Faculty of Mathematics and Physics,  
Charles University



Receiving an award from the President  
of the Czechoslovak Academy of Sciences



At his desk and at the blackboard



Meeting a foreign guest

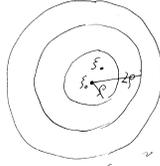


On Labour Day (with Prof. Vl. Kořinek)

Různé holomorfních funkcí.  
 Řada Taylorova a Laurentova.

Věta: Bude-li  $F_n$  holomorfní v oblasti  $\Omega \subset \mathbb{C}$ ,  
 $F_n \rightarrow F$  lokálně stejnoměrně v  $\Omega$ . Potom  
 $F$  je holomorfní v  $\Omega$ .  
 Platí  $F_n^{(p)} \rightarrow F^{(p)}$  lokálně stejno-  
 měrně v  $\Omega$  pro  $p=1,2,3,\dots$

Důkaz: 1)  $\forall z \in \Omega$  existuje  $\rho > 0$  tak, že  
 $U(z, \rho) \subset \Omega$  je holomorfní v  $U(z, \rho)$ .  
 $F_n \rightarrow F$  stejnoměrně v  $U(z, \rho)$ .  
 Volíme  $\xi$  (konvergenční bod)  $\xi \in U(z, \rho) \subset \Omega$



Bude-li  $g$  konstanta  
 $g(t) = \xi_0 + 2\rho e^{it}$ ,  $0 \leq t \leq 2\pi$ .  
 Potom pro  $\xi \in U(z, \rho)$

$$F_n(\xi) = \frac{1}{2\pi i} \int_{\gamma} \frac{F_n(z)}{z-\xi} dz$$

konvergence je stejnoměrná  
 v kompaktní  $U(z, \rho)$ ,  
 tedy je možné prohodit, takže

$$\lim_{n \rightarrow \infty} \frac{1}{2\pi i} \int_{\gamma} \frac{F_n(z)}{z-\xi} dz = \frac{1}{2\pi i} \int_{\gamma} \frac{F(z)}{z-\xi} dz = G(\xi)$$

a je holomorfní v  $U(z, \rho)$  (věta 27)  
 Obě derivace, že  $G(\xi) = F(\xi)$  - tedy v  $U(z, \rho)$

musí dokázat holomorfní  $F$ . Je to  
 $F_n(z) \rightarrow F(z)$  stejnoměrně na  $[z, z]$ , a

$$\left| \frac{1}{z-\xi} \right| < \frac{1}{\rho} \text{ pro } z \in [z, z], \xi \in U(z, \rho)$$

$$(3) \text{ } \frac{1}{2\pi i} \int_{\gamma} \frac{F_n(z)}{z-\xi} dz \rightarrow \frac{1}{2\pi i} \int_{\gamma} \frac{F(z)}{z-\xi} dz$$

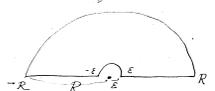
(4) (3) stejne  $F_n(\xi) \rightarrow G(\xi)$ ,  
 tedy  $G(\xi) = F(\xi)$  - tedy  $F$  je holomorfní,  
 a z (4) vyplývá i ostatní části stejne  $F_n^{(p)}(\xi) \rightarrow F^{(p)}(\xi)$

neboť  $\left| \frac{F_n(z) - F(z)}{(z-\xi)^{n+1}} \right| \leq \frac{\epsilon}{z-\xi} \rho^{n+1}$   
 pro  $n > n_0$  a všechna  
 $z \in [z, z], \xi \in U(z, \rho)$

Partikulární 1)  $\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)} dx = 2\pi i \sum_{\text{Im } z > 0} \text{Res} \frac{P(z)}{Q(z)}$

(zde  $P, Q$  jsou polynomy,  $\text{Im } z > 0$  znamená  $P+2$ ,  
 $Q(z)$  to má reálné kořeny)

$$2) \int_{-\infty}^{+\infty} \frac{\sin x}{x} dx = \lim_{R \rightarrow \infty} \int_{-R}^R \frac{\sin x}{x} dx = \frac{1}{2} \lim_{R \rightarrow \infty} \int_{-R}^R \frac{e^{ix} - e^{-ix}}{x} dx$$



$$\int_{-R}^{-\epsilon} \frac{e^{ix}}{x} dx + \int_{\epsilon}^R \frac{e^{ix}}{x} dx + \int_{\gamma_\epsilon} \frac{e^{iz}}{z} dz + \int_{\gamma_R} \frac{e^{iz}}{z} dz = 0$$

$$g_\epsilon(t) = \epsilon e^{i\epsilon t} \quad (0 \leq t \leq \pi)$$

$$\int_{\gamma_\epsilon} \frac{e^{iz}}{z} dz = i \int_0^\pi e^{-\epsilon t} dt = i \int_0^\pi e^{-\epsilon t} dt$$

$$\int_{\gamma_\epsilon} \rightarrow \pi i \text{ pro } \epsilon \rightarrow 0$$

$$\left| \int_{\gamma_R} \frac{e^{iz}}{z} dz \right| \leq 2 \int_0^\pi e^{-R \sin t} dt$$

$$\leq 2 \int_0^\pi e^{-R \frac{2}{\pi} t} dt < 2 \cdot \frac{\pi}{2R} \rightarrow 0 \text{ pro } R \rightarrow \infty$$

$$\lim_{R \rightarrow \infty} \int_{-R}^{-\epsilon} \frac{e^{ix}}{x} dx + \int_{\epsilon}^R \frac{e^{ix}}{x} dx = \lim_{R \rightarrow \infty} 2i \int_{\epsilon}^R \frac{\sin x}{x} dx = \int_{-\infty}^{\infty} \frac{\sin x}{x} dx$$

$$2i \int_{\epsilon}^R \frac{e^{ix}}{x} dx = \int_{\epsilon}^R \frac{e^{ix}}{x} dx \rightarrow \pi i; \int_{\epsilon}^R \frac{\sin x}{x} dx = \frac{\pi}{2}$$