

Topological papers of Eduard Čech

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LIFE AND WORK OF EDUARD ČECH

Eduard Čech, professor of Charles University, Prague and member of the Czechoslovak Academy of Sciences, was the greatest Czechoslovak mathematician and one of the leading world specialists in the fields of differential geometry and topology. To these fields he contributed works of basic importance.

He was born on June 29, 1893 in Stračov in northwestern Bohemia. He attended the secondary school in Hradec Králové. In 1912 he began to study mathematics at Charles University in Prague. He learnt especially by reading the mathematical literature in the library of the Association of Czechoslovak Mathematicians and Physicists. Within a period of five semesters he studied thoroughly a considerable amount of mathematical literature of his own choice. In this manner he acquired knowledge in a number of mathematical disciplines without any guidance. In studying some treatises on elementary mathematics he discovered logical gaps in their theorems and proofs; he took a special liking to correcting and completing them. This was the origin of his interest in didactic questions of mathematics. As at that time two disciplines were necessary to become secondary school teacher he chose as the other discipline descriptive geometry and devoted himself to the study of different branches of geometry.

Čech spent only five semesters at Charles University. In 1915 he was forced to interrupt his studies and to leave for service in the army. After the war he terminated his studies with State examinations and for a short period he taught mathematics at a secondary school in Prague.

In 1920 he submitted his thesis in mathematics under the title "On curve and plane elements of the third order", thus gaining the degree of Doctor of Philosophy. Thereafter Čech became deeply interested in mathematical work. He started studying in a systematic manner the differential projective properties of geometric objects. He familiarized himself with the papers of the outstanding Italian geometer G. Fubini and, having obtained a scholarship, he spent the school year 1921—22 in Turin.

Professor Fubini saw the extraordinary capabilities of young Čech and offered to collaborate with him on a scientific book. As a result of this cooperation two volumes of “*Geometria proiettiva differenziale*” appeared in 1926 and 1927. The authors then wrote another book under the title “*Introduction à la géométrie projective différentielle des surfaces*” published in Paris in 1931.

In 1922 Čech became Docent at the Faculty of Science in Prague. His thesis was concerned with projective differential geometry. A year later, while not yet 30, he was appointed Professor Extraordinary at the Faculty of Science of the University in Brno where the chair previously held by Matyáš Lerch became vacant. As the chair of geometry was occupied at that time, he was given the task of lecturing on certain branches of mathematical analysis and algebra. He therefore started to study these disciplines intensively. In a short space of time he made a study of the appropriate literature and for twelve years he lectured on analysis and algebra at the University of Brno. This circumstance seems to be important for the interest of Professor Čech in topology.

In 1928 he was appointed Professor. At that time he manifested a deep interest in topology. The principal sources of his study were the papers published in “*Fundamenta Mathematica*”. He was also influenced by the papers of outstanding American and Soviet topologists. After 1931 he published no more papers on differential geometry and devoted himself entirely to research in the field of general and combinatorial topology. Let us mention in the first place two papers of pioneer character published in 1932. One of them is concerned with the general theory of homology in arbitrary spaces and the other with the general theory of manifolds and with the theorems of duality; with these papers Čech ranked among the best experts in the field of combinatorial topology. In September 1935 he was invited to a conference on combinatorial topology held in Moscow. This meeting was attended by a number of the best European and American topologists. Professor Čech reported there on the results of his research which met with such attention that he was invited to lecture at the Institute for Advanced Study in Princeton, centre of mathematical research in the U.S.A.

After his return from the U.S.A. in 1936, Čech organized a mathematical school in Brno. With a group of young people deeply interested in mathematics he founded a topological seminar where at the beginning the papers of P. S. Alexandrov and P. Urysohn were systematically discussed. The atmosphere of the seminar, as well as the personality of Čech who continued to give fresh impulses to work, had a favourable influence on all the members. Many problems raised by Čech were solved there and, within a period of three years, 26 scientific papers originated at the seminar. Čech’s paper on bcompact spaces was among them. In this paper Čech investigated topological spaces now known as the maximal of Stone-Čech compactification. The topological seminar existed till 1939 when after the German occupation of Bohemia and Moravia all Czech universities were closed. Nevertheless, even then Čech created with his nearest collaborators B. Pospíšil and J. Novák a working group which met

regularly in Pospíšil's flat till the arrest of the latter by the Gestapo in 1941. Čech's topological seminar occupies an important place in the history of Czechoslovak mathematics. Čech introduced there a new progressive form for mathematical research, i.e. systematic team-work.

After twenty-two years of teaching and scientific activity in Brno Professor Čech came to the Faculty of Science in Prague. He developed there a great effort to organize Czechoslovak mathematical activities. In 1947 he became director of the Mathematical Institute of the Czech Academy of Sciences and Arts and led it until 1950 when the Central Mathematical Institute was founded; Čech was appointed its director. When in 1952 the Czechoslovak Academy of Sciences was founded, Čech became one of its first members. He was then charged with the direction of the Mathematical Institute of the Czechoslovak Academy of Sciences. He set up the work and research program of this institute, aiming at the development of Czechoslovak mathematics not only in the theoretical field but also in applications, especially in technical. In 1954 he moved to the Faculty of Mathematics and Physics where he organized the Charles University Mathematical Institute. At that time and until his death he worked systematically in the field of differential geometry and published altogether 17 papers. Nevertheless, he was also interested in topology and wrote the book [IX].

The scientific, teaching and organizational activity of Professor Čech contributed substantially to the development of mathematics in Czechoslovakia. In addition to this rich activity, he was deeply interested in the problems of mathematical teaching. He was one of those mathematicians who understood that there should exist a close cooperation between university professors and secondary school teachers. Led by this conviction, he wrote mathematical textbooks for junior classes of secondary schools. In these textbooks he centred his attention on the formation of mathematical concepts in the mind of the pupils and on the development of abstract logical reasoning.

In a series of pedagogical seminars organized from 1938 in Brno, Professor Čech devoted much of his time and energy to the problems of school mathematics. After 1945 these seminars on elementary mathematics were held in Prague and in Brno.

Professor Čech took part in a number of international mathematical congresses where he represented Czechoslovak mathematics. He lectured as visiting professor at numerous foreign universities such as Warsaw, Lvov, Moscow, Princeton, Ann Arbor, New York, Harvard University etc. He was member of the Czech Academy of Sciences and Arts, of the Royal Society of Sciences, of the Moravian Society of Sciences, honorary member of the Association of Czechoslovak Mathematicians and Physicists, doctor honoris causa of Warsaw university, member of the Polish Academy of Sciences, member of the learned society "Towarzystwo Naukowe" of Wrocław, doctor honoris causa of Bologna university etc. His scientific activity includes 94 papers and 9 books. Besides these he published 7 textbooks for secondary schools. Professor Čech had a considerable influence on a number of Czechoslovak mathematicians. He brought up a number of disciples, among them Bedřich Pospíšil, Miroslav Katětov

and Josef Novák. He founded a mathematical school both in topology and in differential geometry. Numerous mathematicians throughout the world were influenced by his ideas and made use of the results obtained by him. The significance of his work was acknowledged by the granting of two State Prizes in 1951 and 1954. He always took the part of progress and prepared the Czechoslovak science for the tasks connected with the building up of socialism. For his merits he was decorated with the Order of the Republic. He died after a long and serious illness on March 15, 1960.

The scientific activity of Professor Čech was extremely rich. From 1925 he concentrated on topology, both general and algebraic — a closer connection of the two branches constituted a substantial part of the program he set himself — and in 1930 his first paper on topology appeared. By 1938 he had published about 30 topological papers. Later on, after a certain interval in publishing original results, he again took up his research in differential geometry. Nevertheless, even in that period he was interested in topology and in addition to a paper written in 1947 (jointly with J. Novák) he published the book “Topological Spaces” in 1959.

Čech wrote 12 papers on general topology or rather, topological papers not using algebraic methods; in fact, most of his papers on algebraic topology also refer to very general spaces; this is one of their characteristic features. Among them let us mention in the first place the paper [29] on compact spaces (instead of the original designation “bicomact”, “compact” is used here being more common now). For the first time the so-called maximal compactification $\beta(S)$ of a completely regular space S is systematically studied in this paper, i.e. a compact space containing S as a dense subset and such that each bounded continuous function on S can be extended to $\beta(S)$. The existence of such a space had already been probed by Tichonov in 1930; certain properties of the space $\beta(S)$ were investigated almost simultaneously with Čech but from another point of view by M. H. Stone, nevertheless Čech was the first to show the importance of this space and the potentialities of its use. The compactification $\beta(S)$, called the Čech or Stone-Čech compactification in the literature, has become a very important instrument in general topology and in certain fields of functional analysis. Numerous other concepts of general topology (Q-spaces etc.) have their origin in the theory of β -compactification; one of them, namely absolute G_δ - spaces, was studied (under the designation of topologically complete spaces) by Čech in the above paper [29]. The papers [28], [30], [31] are allied to this paper in general character. The paper “Topological spaces” [28] originated in Čech’s lectures at the topological seminar of Brno; it contains fundamental concepts of the theory of topological spaces presented according to an original and very general conception. The paper [30] (written jointly with B. Pospíšil) is concerned with various questions of general topology, especially with the character of points in spaces of continuous functions and with the number of incomparable L-topologies having certain other properties. The paper [31] (written jointly with J. Novák) analyses in detail some concepts connected with the Wallman compactification (which for a normal space coincides with that of Čech).

Papers [6] and [11] concern dimension theory as does the preliminary communication [3]. In the first of them the concept now called “large” inductive dimension (designated by Ind) was studied; for completely normal spaces the so-called additive theorem (the above-mentioned dimension of a countable union of closed sets is equal to the limit superior of their dimensions), the theorem on monotony and that on decomposition (which gives the inequality $\text{dim} \leq \text{Ind}$) have been proved. In the second paper the dimension defined by means of the covering (designation dim) was studied; in particular the additive theorem for normal spaces was proved.

Further Čech’s papers on general topology are concerned with connected spaces. The paper [4] studies the irreducible connectedness between several points and the generalized concept of “dendrite” for arbitrary topological spaces. A short paper [5] deals with continua that can be mapped onto a segment in such a way that the inverse images of points are finite sets; the paper [18] based on the results obtained by Menger and Nöbeling treats the problems connected with the so-called “ n -Bogensatz”. Finally the paper [1], the first topological paper of E. Čech from the chronological point of view, contains a new demonstration of the Jordan theorem.

Of considerable importance for Czechoslovak mathematics was Čech’s book [IV] entitled “Bodové množiny I” (Point sets) with a supplement by V. Jarník. Published in 1936, the book was a pioneer work in Czech mathematical literature and even now has not become obsolete. Its first part is devoted to the topology of metric spaces, especially to complete spaces and to compactification; this nowadays standard subject is treated in an outstanding manner with admirable exactness and is presented according to an original conception from the methodological point of view. Čech’s last book “Topologické prostory” (Topological spaces) with two supplements by J. Novák and M. Katětov, appeared in 1959 but had been ready in fact for several years. The theory of topological spaces is treated in a considerably more general way than is common; special attention is given to problems studied at Brno seminar. Among the characteristic features of this book which is written in a precise manner peculiar to Čech, let us mention only two: the spaces for which the axiom $\overline{\overline{A}} = \overline{A}$ is not necessarily valid are studied in a fairly detailed manner; various properties of the mappings such as “exact continuity” and “inverse continuity” are studied in a general situation. (It should be mentioned here that a revised edition of this book appeared in English in 1966.) Among unpublished Čech’s papers was found a complete manuscript of “Bodové množiny II”. This book with some parts (Chapter I, II, III) of “Bodové množiny I” was published under the title “Bodové množiny” (Point sets) in 1966; English translation “Point Sets” will be published in 1968.

Čech’s papers on algebraic (combinatorial) topology deal in the first place with the theory of homology and general manifolds. As indicated by Čech himself in the introductory part to the report [20] the aim was to unite the methods and the way of reasoning used in the set topology and in the classical combinatorial topology, or rather, to discover the general substance of the classical theory of homology, of the theory of manifolds etc. and to incorporate it organically in the general theory

of topological spaces; at the same time it was of course desirable to exclude such means as the use of polyhedra. It may be stated that Čech made an important contribution to this program which determines the development of a considerable part of contemporary algebraic topology.

In his fundamental paper [7] Čech formulated in detail, for completely general spaces, the theory of homology based on finite open coverings. As a matter of fact he does not even suppose, at least at the beginning, that a topological space is considered; the concepts studied are, in fact, in modern terminology, projective limits of homological objects on finite complexes. The results achieved in this paper are probably the best known of the whole of Čech's work on topology. The theory formulated in [7] constitutes a part of the "basic fund" of contemporary algebraic topology; later it appeared that it is especially appropriate for compact spaces and in the literature it is commonly designated by Čech's name. However, it should be noted that the idea of the so-called projective sequence of complexes and particularly that of the nerve of a finite open covering of a compact space was introduced by P. S. Alexandrov as early as 1925 and was treated in detail by him in his paper of 1929.

To the paper [7] is allied the paper [14] in which some results of [7] are improved, while also the study of local Betti numbers (introduced independently also by P. S. Alexandrov in his paper of 1934) and of other concepts also considered in [19] and [21] is started. The second of these papers is devoted to a detailed study of local connectedness (or local acyclicity) of higher orders defined in terms of the theory of homology (local connectedness in this sense was also introduced by P. S. Alexandrov in 1929, but was not studied in detail until Čech's paper appeared). Local Betti numbers and local acyclicity are studied in different relations in the paper [8] resting equally on the fundamental treatise [7]; it contains a methodological novelty with a bearing on the papers on manifolds, namely the deduction of a number of theorems on the sphere without triangulation by means of a certain theorem concerning the relationship of homotopical and homological concepts. Finally, in the paper [10] the relation between unicoherence (defined in set theoretical terms) and the first Betti number are studied by means used in the paper [7].

The papers [8], [13], [15], [17], [19], [23], [25] are devoted to manifolds in a sense that varies in different papers. The main aim of these papers, which as a whole constitute an important chapter of algebraic topology and are one of the most important achievements of Czechoslovak mathematics, is the introduction of the general concept of manifold in such a way as to include connected spaces homeomorphic with E_n and to be defined uniquely by general topological properties as well as by assumptions expressed in terms of general theory of homology; at the same time it is of course desirable that the theorems on duality be true with necessary modifications for these general manifolds. This aim was in fact attained in Čech's papers (S. Lefschetz obtained the same results independently and at about the same time); moreover a number of Čech's theorems were new even for the classical case of duality for sets in E_n or

in S_n . Later on, R. Wilder and other authors started developing the results obtained by Čech and succeeded in simplifying them considerably by use of new means; however, it seems that the theory of general manifolds in Čech's sense is far from being closed.

The papers [2], [22], [26], are in a loose relationship to the above main directions of Čech's work in algebraic topology. In the important paper [26] were studied cohomological concepts (in the terminology of that time dual cycles etc.). A short time after they were formulated explicitly by J. W. Alexander and A. N. Kolmogorov in 1935. The multiplication of cocycles and of a cycle and cocycle were introduced here. In the paper [22] theorems concerning the unique determination of Betti groups with arbitrary coefficients by means of ordinary Betti groups are proved for infinite complexes. The paper [2] is the first of Čech's papers on algebraic topology. It contains theorems of considerable generality concerning among other things the decomposition of a space between two points; an entirely special case of these theorems are some classical theorems of the topology of surfaces.

We should mention in addition two papers containing only results without proofs. The paper [25] on Betti groups of compact spaces (these are in general continuous groups) and [27] on the accessibility of the points of a closed set in E_n . Finally, at the International Congress of Mathematicians held in Zürich in 1932, Čech presented a communication on higher homotopy groups. Unfortunately, he never came back to this subject afterwards and in the Proceedings of the Congress his communication (see [9]) was formulated in a very brief and not completely clear manner; nevertheless, W. Hurewicz who formulated in an outstanding way a systematic theory of higher groups of homotopy in his publications from 1935 onwards, says in one of his papers (Akad. Wetensch. Amsterdam. Proc. 38 (1935), p. 521) that Čech's definition of these groups is equivalent to his.

At the Topological Symposium held in Prague in 1961, Professor P. S. Alexandrov said the following about Čech's definition:

“This definition did not meet with the attention it merited; in fact, the commutativity of these groups for dimensions exceeding one was criticised (this was unfounded, as we now know).

Thus, Prof. Čech's definition of the homotopy groups was, in 1932, simply not understood — a situation extremely rare in modern mathematics. We must express our admiration at the intuition and talent of Prof. Čech, who defined the homotopy groups several years before W. Hurewicz.”

Čech's papers on mathematical analysis are connected to a considerable extent with his teaching activity at the University and have the character of brief notes. In the paper [90] he derived by an original method properties of the functions x^x , e^x , $\log x$, $\sin x$, $\cos x$; in the paper [91] he generalized the elementary method of K. Petr for the examination of Fourier series for the functions of bounded variation; in the papers [92] and [93] he gave a simple proof of Cauchy's theorem and of Gauss'

formula. From the point of view of method the paper [94] is allied to papers on general topology. It deals with continuous functions on an interval being non-constant on any infinite set. Mathematical analysis is also concerned in the second part of the book [IV] "Bodové množiny I" dealing with measure and integral; the approach to these subjects is remarkably original in this book and some particular results were probably new when it appeared.

The papers of Prof. Čech on differential geometry originated in two periods; from 1921 to 1930 and in the years after the World War II. Čech is one of the founders of projective differential geometry and his work not only brought many valuable results but also influenced substantially the whole development of this discipline. His work was continued in the first place in Italy, Roumania, Germany, and of course in this country; his papers received considerable attention in the U.S.S.R. Čech succeeded in finding three fundamental principles which appear distinctly in his work and are of essential importance for research in differential geometry: a systematic attention given to the contact of manifolds, the study of correspondences (in contradiction to the study of isolated manifolds) and a systematic use of duality in projective spaces. To appraise the value of Čech's work would be to write the history of projective differential geometry; here will be given only an account of concrete results achieved by him.

The first papers of Prof. Čech [32], [33] deal with the association of certain geometric objects and correspondences with the elements of lowest order of curves and surfaces in three-dimensional projective space; as a matter of fact, it is a geometric determination of these elements by a minimal number of objects. A similar problem is dealt with in the paper [36] studying the element of fourth order of a surface and [37] where the results previously obtained are applied to ruled surfaces and where the neighbourhood of the straight line generating the surface in question is considered. In the paper [42] Čech studies collineations of a projective space onto itself preserving the element of third order of the surface. Starting from these considerations he gave in the years after the war (in an unpublished paper) an absolute definition of the canonical straight lines of the surface. The paper [59] deals with the geometric significance of the index of Darboux quadrics.

In the paper [34] he proved, among other things, that the osculating planes of three curves of Segre have one canonical straight line in common. The paper [39] with a preliminary communication [38] discovers all surfaces for which all these straight lines go through a fixed point, in other words, for which the curves of Segre are plane; the paper [40] determines the surfaces with plane Darboux curves. It should be noted here that these computations required a very difficult integration of a system of partial differential equations.

It is well known that the study of the surfaces in an Euclidean three-dimensional space is equivalent to the study of two fundamental differential forms on the surface. The main idea of G. Fubini was to establish a similar procedure for a surface and hypersurface in a projective space using a quadratic and cubic form. To this theory

Čech contributed in his papers [41], [43], [45], [49], [50], [52], [62]. He found the geometric significance of different normalizations of homogenous coordinates of the points of the surface, the geometric significance of the projective linear element (playing the same role as ds^2 in Euclidean geometry) and a complete system of its invariants; he further studied its extremals (projective geodesics).

The papers [34], [35], [55], [60], [63], [64], [67] and [68] are devoted to the theory of correspondences between surfaces. Čech contributed here substantially to the theory of the projective deformation of surfaces in three-dimensional space. He gave a new characterization of projective deformation by means of osculating planes corresponding to each other and he further studied different generalizations of projective deformation as well as the general asymptotic or semi-asymptotic correspondence between surfaces; he found the solution to the main existence questions for different types of these asymptotic correspondences. He finally used all these techniques to study and to find the congruence of the straight lines the focal surfaces of which are in projective deformation or on which Darboux curves correspond to one another. Later on, the same problem was studied by different methods by S. P. Finikov. Of considerable importance for the theory of projective deformations is further the discovery of surfaces admitting ∞^1 projective deformations in themselves or on which there exist ∞^1 R-nets one of which has the same invariants.

A new method of study of ruled surfaces, applicable mainly to projective spaces of odd dimension, was introduced in the papers [48], [53], [54]. Other authors, in the first place Czechoslovak ones, exploited these results and proved the advantage of Čech's procedure.

Of fundamental importance are the papers [58] and [66] dealing with the contact of two curves in projective spaces of an arbitrary dimension and with the possibility of increasing this contact by projection from a suitably chosen centre. Čech came back to this problem in his last paper [87] where similar problems for two manifolds are studied. These papers not only contained concrete results of basic importance but also constituted a starting point for the formulation of the theory of correspondences which will be discussed farther.

The papers [46], [47], [56] and [57] are devoted to the study of the stripes of contact elements on a surface in three-dimensional or affine space, i.e. of a system of plane elements in the points of a curve situated on the considered surface. Special attention is given to pairs of surfaces having contact of a certain order along the whole curve and the conditions are studied under which this curve is a curve of Darboux or Segre on both surfaces at the same time, as well as other problems of this character. Čech strongly emphasized the importance of his procedure considering a whole stripe of elements instead of a curve (which in practice we do in Euclidean geometry without being aware of it); these papers have not yet been exploited.

Finally, the papers [61] and [65] are devoted to the projective differential geometry of plane nets.

This first period of Čech's active interest in differential geometry culminates with the publication of three books [I], [II], [III] two of which were written in cooperation with G. Fubini. It should be mentioned here that [II] and [III] are the first systematic books on projective differential geometry. Both books originated in long written discussions on the conception of the whole matter; and a specialist who knows Čech's geometric limpidity combined with extremely complicated computations may easily trace the part of both authors in the whole work. Thanks to Čech's initiative, a chapter on the use of Cartan methods was put into the French book; nowadays we clearly see that at that time it was a very sagacious act. The Czech book [I] is an isolated work in the world literature; it deals in a perfectly exact, formal manner with one-parameter objects and so shows that differential geometry can be explained in a perfectly precise way.

After the World War II Čech continued to work in classical differential geometry. The results he achieved rank among the most important attained in the world in this field, which has become classical since then. These papers may be divided into three groups.

The papers [69] and [71] give a systematic theory of correspondences between projective spaces studied from the point of view of the possibility of their best approximation by means of tangent homographies. This determines the natural classification of special types of correspondences which are either constructed directly geometrically or at least their general character is given. In a very detailed way projective deformations of the layer of hypersurfaces are studied. Čech found a great number of secondary results (from the point of view of the theory of correspondences) which play, however, an important role in other theories. In this manner were found all asymptotic transformations of the congruence of the straight lines L (i.e. all transformations $S'_3 \rightarrow S_3$ for which every ruled surface in L passes asymptotically into the ruled surface of the corresponding congruence L'). It was further stated that this problem is in fact equivalent to the classical problem of Fubini concerning the discovery of projective deformations of the surface. Čech's theory met with an attention abroad and influenced substantially the group of Italian geometers in Bologna who had been working intensively in the geometry of correspondences under the leadership of Professor Villa.

It appeared that in the development of the theory of correspondences the congruences of straight lines are of essential importance. It is therefore natural that later on Čech started to study them systematically; the results were published in the papers [72], [75], [78], [84]. He paid a systematic attention to correspondences between congruences which transfer in themselves their developable surfaces and he analysed in detail the problem of their projective deformation; he achieved outstanding results especially for the W-congruences. In this field which has also been studied by P. S. Finikov and his Moscow school, existence questions and geometrical constructions solved by Čech rank among the best results. Our geometers achieved by these methods a number of very deep and sometimes definitive results in the theory of

Segre congruences and of congruences and surfaces with a conjugate net in higher-dimensional spaces.

The papers [80], [81], [82], [85], [86] are devoted to various subjects; there are studied relations between differential classes of the points of the curve and of associated objects, n -frame of Frenet and the osculating circle and sphere in Euclidean space of dimension three or four. These results are partly definitive and rather surprising. Nevertheless, much effort must be developed to formulate a systematic theory in this new part of differential geometry of curves and to find eventually more effective methods of investigation.

In conclusion let us recall the paper [83] dealing with projective deformations of developable surfaces and the papers [39] and [74] which have the character of summary reports on the theory of correspondences and on some fundamental questions of differential geometry.

The enumeration of Čech's papers in differential geometry is of course incomplete not only for being very brief but also for the fact that a number of Čech's ideas and methods were dealt with in the papers of his direct or indirect disciples. In addition, a number of manuscripts (often very incomplete) of new papers was found after his death. Some of them have been treated and published in [89].

Prague, April 1, 1968.

M. Katětov, J. Novák, A. Švec