Jarník’s Notes of the Lecture Course Allgemeine Idealtheorie by B. L. van der Waerden (Göttingen 1927/1928)

Bartel Leendert van der Waerden (1903-1996)

In: Jindřich Bečvář (author); Martina Bečvářová (author): Jarník’s Notes of the Lecture Course Allgemeine Idealtheorie by B. L. van der Waerden (Göttingen 1927/1928). (English). Praha: Matfyzpress, 2020. pp. 7–[32].

Persistent URL: `http://dml.cz/dmlcz/404377`

BARTEL LEENDERT VAN DER WAERDEN (1903 – 1996)

Family, childhood and studies

Bartel Leendert van der Waerden was born on February 2, 1903 in Amsterdam in the Netherlands. His father Theodorus van der Waerden (1876–1940) studied civil engineering at the Delft Technical University and then he taught mathematics and mechanics in Leewarden and Dordrecht. On August 28, 1901, he married Dorothea Adriana Endt (?–1942), daughter of Dutch Protestants Coenraad Endt and Maria Anna Kleij. In 1902, the young family moved to Amsterdam and Theodorus van der Waerden continued teaching mathematics and mechanics at the University of Amsterdam where he had become interested in politics; all his life he was a left wing Socialist. In 1910, he was elected as a member of the Sociaal-Democratische Arbeiderspartij to the Provincial government of North Holland. In 1911, he was awarded the degree of Doctor of Technical Sciences.

Bartel Leendert van der Waerden was the oldest son of the family. He had two younger brothers, Coenraad (born on December 29, 1904) and Benno (born on October 2, 1909). Bartel Leendert van der Waerden studied at an elementary school in Amsterdam. In 1914, he entered the Hogere Burger School in Amsterdam. It can be surmised that he was not allowed to read his father's books on mathematics and that he was not to enjoy his father's support in studying mathematics. As a pupil at the Hogere Burger School he showed remarkable talent for mathematics (for example he developed the laws of trigonometry by himself). In 1919, at the age of sixteen, he began his studies at the University of Amsterdam. He learnt almost every mathematical course, but he was mostly interested in topology taught by Gerrit Mannoury[1] and invariant theory taught by Roland Weitzenböck.[2]

---

[1] Gerrit Mannoury (1867–1956) was a Dutch philosopher and mathematician, one of the central figures of the "Vienna circle". In 1885, he graduated from the Hoogere Burger School in Amsterdam. In the same year he received a teacher's degree in accounting and in mechanics. In 1902, he also obtained a teacher's degree in mathematics. At the beginning of his career, he taught at primary schools in Amsterdam, Bloemendaal and Helmond. From 1910 to 1917, he was a teacher at the Hoogere Burger School in Vlissingen. In 1902, he had been appointed Privatdocent at the University of Amsterdam and after fifteen years he became a professor of mathematics there. He lectured on mechanics, analytic, descriptive and projective geometry, topology, logic and philosophy of mathematics.

[2] Roland Weitzenböck (1885–1955) after studies at secondary military schools in Eisenstadt and Hranice na Moravě (1897–1902), attended the Military Academy in Vienna and

Dirk van Dalen (* 1932), a Dutch mathematician and historian of science, described van der Waerden's studies in Amsterdam:

The study of mathematics was for him the proverbial 'piece of cake'. Reminiscing about his studies he said: "I heard Brouwer's lectures, together with Max Euwe and Lucas Smidt. The three of us listened to the lectures, which were very difficult." ... Van der Waerden meticulously took notes in class, and usually that was enough to master all the material. Brouwer's class was an exception. Van der Waerden recalled that at night he actually had to think over the material for half an hour and then he had in the end understood it.

Van der Waerden was an extremely bright student, and he was well aware of this fact. He made his presence in class known through bright and sometimes irreverent remarks. Being quick and sharp (much more so than most of his professors) he could make life miserable for the poor teachers in front of the blackboard. During the, rather mediocre, lectures of Van der Waals Jr. he could suddenly, with his characteristic stutter, call out: "Professor, what kind of nonsense are you writing down now?" He did not pull such tricks during Brouwer's lectures, but he was one of the few who dared to ask questions.[3]

In 1924, van der Waerden took his last examination at the University of Amsterdam. It is described in [DS1]:

... van der Waerden took his final examination with de Vries, whose course in classical algebra he had very much liked. It included subject like determinants and linear equations, symmetric functions, resultants and discriminants, Sturm's theorem on real roots, Sylvester's "index of inertia" for real quadratic forms, and the solution of cubic and biquadratic equations by radicals. Van der Waerden supplemented this course by studying Galois theory and other subjects from Heinrich Weber's textbook on algebra. He also read Felix Klein's Studien über das Ikosaeder and thoroughly studied the theory of invariants.[4]

In 1924, van der Waerden received his first degree in Amsterdam and he went for seven months to Göttingen to extend and deepen his mathematical knowledge and skills under Emmy Noether.[5]

Mödling (1902–1905). In the following five years he was engaged as a military serviceman and from 1910 he taught mathematics at the Military Academy in Mödling. During that time, he studied mathematics at the University of Vienna where he obtained his doctorate in 1910. In 1913, after a short time studies in Bonn and Göttingen, he was named Privatdocent at the University of Vienna and next year he moved to Graz as an assistant and a Privatdocent at the Polytechnic. During the First World War he served as a professional officer in the Austro-Hungarian army. From 1918 to 1920, he was a professor of mathematics at the German Polytechnic in Prague. In 1920, he obtained professorships at the Polytechnic in Graz and in 1921 he became a professor at the University of Amsterdam where he taught up to 1945. He was interested in the theory of invariants.

[3] See [W1] and [Da].

[4] [DS1], p. 125.

[5] Emmy Amalie Noether (1882–1935) was a daughter of Max Noether (1884–1921), professor of mathematics in Erlangen. After her studies at the "Höhere Töchter Schule in Erlangen" (1889–1897) she took the examinations of the State of Bavaria and became

Studies in Göttingen

Because van der Waerden finished his basic studies in Amsterdam in a very short period of time, his father decided to support him in his continued studies in some renowned European mathematical center. He paid for van der Waerden's stay abroad.

On October 21, 1924, Luitzen Egbertus Jan Brouwer,[6] one of van der Waerden's teachers, wrote to Hellmuth Kneser[7] who lectured in Göttingen before van der Waerden went there:

---

a certificated teacher of English and French at girls schools (from 1900). She never accepted this position as she decided to study mathematics at Erlangen University (1900–1902). She then continued in Nürnberg (1903) and finally completed her studies at Göttingen University (1903–1904) where she attended lectures by Otto Blumenthal (1876–1944), David Hilbert (1862–1943), Felix Klein (1849–1925) and Hermann Minkowski (1864–1909). In 1907, she obtained a doctorate in Erlangen under Paul Albert Gordan (1837–1912). From 1907 to 1915, she helped her father with his lectures at Erlangen University but was not named an assistant; for a woman it was impossible in this time. In 1915, she moved to Göttigen where she lectured thanks to the support of Hilbert. After a long battle with the university authorities, she was appointed professor of mathematics (1922) and she taught there up to 1933. During the school year 1928/1929 she gave some special courses on abstract algebra at Moscow University and she organised a research seminar on algebraic geometry which took place at the Academy in Moscow. In 1933, she had to emigrate to the USA; Nazi laws made her academic career no longer possible. She obtained a position at Bryn Mawr College in Pennsylvania and she also lectured at the Institute for Advanced Study in Princeton. Noether was incredibly influential for modern abstract algebra. From 1907 to 1919 she was interested in solving Jordans and Hilberts problems, from 1920 to 1926 she worked on ideal theory and from 1927 she studied and solved many problems on non-commutative algebra. She opened new and modern directions in abstract algebra which influenced the development of mathematical thinking. Her fundamental results were extended, generalized and popularized by her pupils and co-workers, for instance, B.L. van der Waerden, who wrote about Noether his memoire in which he described her mathematical achievements: *Nachruf auf Emmy Noether*, Mathematische Annalen 111(1935), pp. 469–476.

[6] Luitzen Egbertus Jan Brouwer (1881–1966) was a Dutch mathematician. He attended the famous high school in Hoorn (a town on the Zuiderzee north of Amsterdam). He completed his secondary school by the age of fourteen. He then spent the next two years studying Greek and Latin and in 1897 he passed the entrance examinations for the University of Amsterdam where he obtained his masters degree (1904) and finished his doctoral dissertation (1907). In 1909, he was appointed Privatdocent and in 1912 a professor of set theory, function theory and axiomatic theory at the University in Amsterdam. He held the post until his retirement in 1951. Then he lectured in South Africa, the United States and Canada till his death. During 1911–1913 he obtained almost all his fundamental results on topology and he was considered by many mathematicians to be its founder. Later he was interested in group theory, set theory, functional analysis, conceptual problems of modern mathematics and logical foundations and philosophy of mathematics.

[7] Hellmuth Kneser (1898–1973) after studies at the secondary school in Breslau attended classes in mathematics at Göttingen University (1916–1921) where he obtained his doctorate under Hilbert's supervision and he became his assistant. In 1922, he was named private docent at the university and taught there until 1925 when he moved to Greifswald as an ordinary professor of mathematics. From 1937, he lectured at Tübingen University. He was one of the founders of the German mathematical journal Archiv der Mathematik which was firstly published in 1952. He was interested in topology, function theory, differential geometry and algebra.

In some days my student (or actually Weitzenböck's) will come to Göttingen for the winter semester. His name is Van der Waerden, he is very clever and has already published (namely in Invariant Theory).[8]

Van der Waerden spent the winter semester 1924/1925 in Göttingen; he studied topology with Hellmuth Kneser. He talked about it in his interview:

... from the beginning I was in contact with him, and from him I really learned topology. Kneser and I used to have lunch together; after having eaten he went home, but on occasion we first took a brief walk. We strolled through the woods of Göttingen, and he taught me many things. It always went like this: he made some observations which I did not completely understand, so I then went into the library to find out what he was really saying. The next day I asked him if the interpretation was correct. Thus I learned for example topology.[9]

On the atmosphere in the Göttingen mathematical community, van der Waerden wrote:

... I came to Göttingen in 1924. ...

In these years Göttingen became an international centre of mathematics. From all over the world mathematicians and physicists came to Göttingen to learn from Klein, Hilbert and Minkowski.[10]

... it was splendid; you could take the books from the shelves yourself. This was really not possible anywhere else. At Amsterdam, when you went into the university library, first you had to look in the catalogue, fill out a form, and put it in a box. And then, after half an hour, you obtained the book requested. At Göttingen, instead, where you could get the books from the shelves by yourself, it often happened that right near the book you were looking for there was another interesting one.[11]

In Göttingen, van der Waerden was very fascinated by modern algebra lectured by Emmy Noether who opened up a new mathematical world before him. She told him what to study – Steinitz's paper named *Algebraische Theorie der Körper*, Macaulay's tractat on polynomial ideals named *Algebraic Theory of Modular Systems*, Dedekind's and Weber's famous contribution named *Theorie der algebraischen Funktionen einer Veränderlichen* and her own papers *Idealtheorie in Ringbereichen* and *Eliminationstheorie und allgemeine*

---

[8] See [W1], [Da]. It should be mentioned that Brouwer could foresee the future of van der Waerden's article named *Über die fundamentalen Identitäten der Invariantentheorie*, Mathematische Annalen 95(1926), pp. 706–735 (In Amsterdam. Eingegangen am 29. 5. 1925). It can be added that van der Waerden published his first mathematical works named *Determinanten aus Formenkoeffizienten*, Verslagen van gewone Vergaderingen der wis- en natuurkundige Afdeeling der Koninklijke Akademie van Wetenschappen 25(1922), pp. 354–358, and *Über das Kominantensystem zweier und dreier ternären quadratischen Formen*, ibid., 26(1923), pp. 2–11, during his studies at the University of Amsterdam. For more information see [Gr].

[9] [DS3], p. 315.

[10] [Wa4], p. 4.

[11] [DS3], p. 315.

Idealtheorie. He also regularly attended Göttingen's well-stocked mathematical library and he took part in Noether's seminars. He was in close touch with her and her students. In his article *The School of Hilbert and Emmy Noether*, he mentioned his personal Göttingen's experience:

> *... In 1924 I had the pleasure to be an auditor in her course on "Hypercomplex Numbers". In 1926/27 she again lectured on the same subject under the title "Hyperkomplexe Größen und Darstellungstheorie". I took notes, and these notes were used by herself to compose a fundamental paper on the same subject.*[12]

Dirk van Dalen described van der Waerden's studies in Göttingen:

> *Once in Göttingen under Emmy's wings, Van der Waerden became a leading algebraist. Emmy was very pleased with the young Dutchman, "That Van der Waerden would give us much pleasure was correctly foreseen by you. The paper he submitted in August to the Annalen is most excellent (Zeros of polynomial ideals) ...", she wrote to Brouwer on 14 November 1925.*[13]

Second studies in Amsterdam

In 1925, van der Waerden returned to the Netherlands where he undertook mandatory military duty; he was on service at the marine base in Den Helder. During this time, he was also concerned with writing his doctoral dissertation under Hendrik de Vries's[14] supervision. Dirk van Dalen wrote about this period of van der Waerden's life:

> *In mathematics Van der Waerden was easily recognised as an outstanding scholar, but in the 'real world' he apparently did not make such a strong impression. When Van der Waerden spent his period of military service at the naval base in Den Helder, a town at the northern tip of North-Holland, his Ph.D. advisor* [Hendrik de Vries] *visited him one day. He said that the commander was not impressed by the young man, "he is a nice guy but not very bright".*[15]

---

[12] [Wa4], p. 6.

[13] [W1]. See also [Da]. It can be mentioned that Noether intended van der Waerden's article named *Zur Nullstellentheorie der Polynomideale*, Mathematische Annalen 96(1927), pp. 183–208 (In Amsterdam. Eingegangen am 14. 8. 1925).

[14] Hendrik de Vries (1867–1954) after studies at the secondary schools in Rotterdam and Frauenfeld in Switzerland entered the Eidgenossische Polytechnicum in Zurich in 1886. He graduated after four years there and he was appointed as an assistant of professor Wilhelm Fiedler (1832–1912) who lectured on descriptive and projective geometry. In 1894, de Vries returned to the Netherlands and he became a teacher of mathematics at the Hoogere Burger School in Amsterdam. At the same time, he attended the University of Amsterdam where he studied mathematics under Diederik Korteweg (1848–1941) who supervised his doctoral thesis. In 1901, de Vries was awarded his doctorate and next year he was appointed professor of mathematics at the Polytechnic School in Delft. In 1906, he was named professor of mathematics at the Municipal University of Amsterdam. He held this position until he retired in 1937. He was interested in projective and algebraic geometry and history of mathematics.

[15] [W1]. See also [Da].

On March 24, 1926, van der Waerden defended his doctoral thesis *De algebraise grondslagen der meetkunde van het aantal* [The Algebraic Foundations of the Geometry of Numbers] at the University of Amsterdam.[16] His work written in Dutch presented a program for the new foundation of algebraic geometry.

On his work on the doctoral thesis, van der Waerden wrote:

I wrote the thesis during my service as a marine at den Helder. Naturally, I was not free to go to Amsterdam to discuss my thesis, and I did my thesis practically by myself. At Göttingen I had above all made the acquaintance of Emmy Noether. She had completely redone algebra, much more general than any study made until then, and she was in fact my teacher at Göttingen. Thus I proved my theorems with the methods she had developed.[17]

Second studies in Göttingen and studies in Hamburg

The mathematicians at the University of Göttingen as well as in the Göttingen Mathematical Society[18] were impressed by van der Waerden's mathematical abilities and his results. It is not surprising that they helped him to develop his skills and knowledge. Thanks to the support of Richard Courant[19] and Emmy Noether he obtained the prestigious Rockefeller fellowship for one year.[20] Van der Waedern recalls:

---

[16] B.L. van der Waerden: *De algebraise grondslagen der meetkunde van het aantal*, Amsterdam, 1926, ix + 37 + 3 pages.

[17] [DS3], p. 315.

[18] The Göttingen Mathematical Society was founded in 1892 by Felix Klein and Heinrich Weber (1842–1913). At the beginning of the 20th century it became one of the most famous and influential world mathematical centers. For more information see http://www-groups.dcs.st-andrews.ac.uk/~history/Societies/Gottingen.html [9.10.2019].

[19] Richard Courant (1888–1972) after his studies at the König Wilhelm Gymnasium in Breslau attended classes in mathematics and physics at the University of Breslau but found them lacking in excitement and interest. In the spring of 1907, he left Breslau and spent one semester in Zurich. Then he moved to Göttingen which he found to be full of outstanding mathematicians. There he attended courses by Hilbert and Minkowski and their joint seminars. In 1908, he became Hilbert's assistant and under his supervision he obtained his doctorate in 1910. Following this, he collaborated with Hilbert and became a mathematics lecturer at Göttingen University where he taught until the start of First World War. After the war, which interrupted his career, he returned to Göttingen and was appointed professor of mathematics there. In 1922, he founded the University's Mathematics Institute and taught there until 1933 when he had to emigrate because of the Nazi regime. He then spent a short time in England and finally moved to New York. After some difficulties, he was appointed professor of mathematics at the University in New York (1936) where he built up an applied mathematics research center based on the Göttingen model. Thanks to his excellent reputation he helped many mathematicians who were forced to leave Germany to obtain positions in the USA. His outstanding mathematical results are connected with the Dirichlet problem, theory of conformal mapping, mathematical physics, partial differential equations. He is the co-author (with David Hilbert) the famous book, which was used for studies for more then fifty years: *Methoden der mathematischen Physik I., II.*, Springer, Belin, 1924, 1937, xiii + 450, xvi + 549 pages (later editions: I. 1931, 1943, 1968, 1993, II. 1943, 1968); Russian translation: 1933, 1964; English translation: 1953, 1962.

[20] Waerden's second stay was supported by the International Education Board from which

... after one semester at Göttingen, Courant started to take notice of me. He procured for me, on the recommendation of Emmy Noether, a Rockefeller grant for one year.[21]

He spent the winter semester 1926/1927 in Göttingen and collaborated again with Emmy Noether. He studied her original papers, he regularly attended her lectures and seminars. He also studied the methods of mathematical physics lectured by Richard Courant.

The Mathematical Institute of the University of Göttingen was not the only mathematical center where algebra flourished. The Mathematical Institute of the University of Hamburg was another very important and inspiring place. In the summer semester 1926/1927, van der Waerden was in Hamburg where he studied with Emil Artin,[22] Erich Hecke[23] and Otto Schreier.[24] Van der

---

he received a scholarship to spend seven months in Göttingen to work on abstract algebra with Noether. He was strongly recommended and supported by Noether and Brouwer. For more information see [SSr1], p. 299.

[21] [DS3], p. 315.

[22] Emil Artin (1898–1962) after his studies at the secondary school in Liberec (in his time, this town situated in northern Bohemia was called Reichenberg) enrolled at the University of Vienna (1916). After the first semester he had to serve the Austro-Hungarian army until the end of First World War. In 1919, he entered the University of Leipzig where he received his doctorate in 1921. Then he spent the accademic year 1921/1922 at the University of Göttingen. In 1922, he became an assistant at the University of Hamburg and he started to teach there from the winter semester 1922/1923. In 1923, he finished his habilitation and he was appointed Privatdocent of mathematics in Hamburg. After two years, he was promoted to an extraordinary professor and in the following year an ordinary professor. In 1933, when Hitler came to power, the Nazis legalized the "Civil Service Law" which removed Jewish teachers from the universities. Artin was not a Jew and he could continue in his teaching, but his wife Natalie Jasny was a Russian Jew and when the "New Official's Law" was passed in 1937 (relating to Jews by marriage) the Artins had do leave Germany. In 1937, they emigrated to the United States of America. From 1937 to 1938, Artin taught at Notre Dame, from 1938 to 1946 at the Indiana University in Bloomington and from 1946 to 1958 at Princeton. In 1958, he returned to Germany and was appointed again to the University of Hamburg where he taught up to his death in 1962. Artin made the important contributions to theory of noncomutative rings, rings with the minimum condition on right ideals (now so called Artinian rings), field theory, theory of braids, finite simple groups and their classifications, semi-simple algebras over the rationals etc. His results played a large role in the development of modern mathematics.

[23] Erich Hecke (1887–1947) studied at the University in Breslau, then he moved to Berlin and worked under Edmund Landau (1877–1938). From there he went to Göttingen where he collaborated with David Hilbert and Felix Klein. In 1910 in Göttingen, he was awarded his doctorate. He remained there as an assistant to Hilbert and Klein. In 1912, he became a Privatdocent and he earned the right to lecture at the University of Göttingen. From 1915 to 1918, he taught at the University of Basel, from 1918 to 1919 he was again in Göttingen. In 1919, he accepted the chair of mathematics at the newly established University in Hamburg and he obtained the great opportunity to influence the creation and research tendencies of the new mathematical institute. Hecke was interested in analytic number theory, the algebra of Hecke operators, Hilbert modular functions, Dedekind zeta function, theory of algebraic functions, Dirichlet series with functional equation and kinematic theory of gases.

[24] Otto Schreier (1901–1929) after graduation from the high school in Vienna entered the University of Vienna and he studied there from 1920 up to 1923 when he was awarded his doctorate. After receiving it, he went to Hamburg as an assistant of mathematics at

Waerden attended the well-known Artin's courses on algebra[25] and he took notes and inspirations for writing a joint book with him which was planned for Springer-Verlag's "Yellow Series". But later Artin, when he saw some of the first parts of van der Waerden's text, suggested him to write the whole book alone, that is without any chapter prepared by Artin. This text became van der Waerden's famous textbook *Moderne Algebra*.[26]

Thanks to the collaboration with Noether and Artin van der Waerden finished two papers which were published in the Mathematische Annalen.[27]

First career in Amsterdam and Göttingen

On February 26, 1927, van der Waerden became a university lecturer at the University of Göttingen (that is "Privatdocent" according to the German academic system). He spent the years 1927 and 1928 there and lectured on modern algebra and number theory, prepared his papers and collaborated with Emmy Noether.

Thanks to the discussions with colleagues in Göttingen and great inspiration given by Emmy Noether van der Waerden finished two papers during 1928 which immediately appeared in the Mathematische Annalen.[28]

In early 1928, van der Waerden was offered a position at the University of Rostock, but at the same time, he was offered a lectureship at Groningen which he considered better for his future career. On May 6, 1928, he accepted a full professorship at the University of Groningen.

In 1929, he returned to Göttingen as a visiting professor once more.[29] He lectured on group theory and his lecture course was published under the name *Vorlesungen über kontinuerliche Gruppen*.[30] After his return to the Netherlands, he finished two papers published in the Mathematische Annalen.[31]

---

the "Mathematische Seminar". In 1926, he finished his habilitation thesis and two years later he was offered a post as professor of mathematics at the University of Rostock which he decided to accept from the summer semester of 1928/1929. During the winter semester 1928/1929 he gave parallel courses on function theory in Hamburg and Rostock. But on June 2, 1929, he died at the age of 28 of a general sepsis. He was interested in group theory (combinatorial group theory, subgroups of free groups, presentation for normal subgroups, Lie groups, Abelian groups, knot groups etc.).

[25] In [FTW], p. 138, there is written that van der Waerden was an assistant at the University of Hamburg during the summer semester 1926/1927.

[26] For more information about van der Waerden's inspirations see [Wa3].

[27] *Der Multiplizitätsbegriff der algebraischen Geometrie*, Mathematische Annalen 97(1927), pp. 756–774 (In Göttingen. Eingegangen am 30. 6. 1926); *Eine Verallgemeinerung des Bézoutschen Theorems*, ibid., 99(1928), pp. 497–541 (In Göttingen. Eingegangen am 3. 6. 1927).

[28] *Zur Produktzerlegung der Ideale in ganz-abgeschlossenen Ringen*, Mathematische Annalen 101(1929), pp. 293–308 (In Groningen. Eingegangen am 12. 6. 1928); *Zur Idealtheorie der ganz-abgeschlossenen Ringe*, ibid., 101(1929), pp. 309–311 (In Groningen. Eingegangen am 17. 11. 1928).

[29] For van der Waerden's memory of his stay in Göttingen see [Wa1].

[30] Göttingen, Als Ms. vervielfältigt, 1929, ii + 203 pages.

[31] *Topologische Begründung des Kalküls der abzählenden Geometrie*, Mathematische An-

In July 1929, he met Camilla Rellich, sister of Franz Rellich,[32] a mathematician who completed his doctoral thesis under Richard Courant. Bartel Leendert van der Waerden and Camilla Reich married in September 1929 and returned to Groningen where he continued working on his *Moderne Algebra*. Its first volume was published in 1930 and contained much material from Noether's and Artin's lectures and works. Its second volume was published in 1931 and contained much of van der Waerdens' own works and results.[33]

Wouter Peremans (1921–1999), a Dutch mathematician and one of van der Waerden's student, wrote on the influence of *Moderne Algebra* in 1994:

We should like now to devote some words to the impact of MA. We have remarked already that it was important and long-lasting. It is difficult to explain this to a younger generation of mathematicians. What was new at that time is now part of the standard curriculum for us all. This does not mean that this holds for all of the contents of MA. The algebra curriculum of the thirties was quite different from what it is nowadays. It is perhaps amazing that group theory was not a standard subject in Dutch universities in the early thirties.

The influence of MA did not only concern its contents but certainly also its style. As to the contents not much comment is needed. A considerable part of it has turned into standard baggage of every mathematician. Its style may be characterized as concise and still clear. It had the appeal of something new and fresh. My personal remembrance is that at my first acquaintance with the book as a young student I was thoroughly impressed by its beauty and cogency. I am convinced that this experience will have been shared by many other people of my generation. Later I have had the opportunity to observe that Van der Waerden also was an excellent oral teacher. At any rate I remember that in the after war years whoever whished to count in the mathematical community should be acquainted with the contents of MA.[34]

During the years 1930 and 1931 he also wrote three short (but interesting) notes published in the Mathematische Annalen.[35]

---

nalen 102(1930), pp. 337–362 (In Groningen. Eingegangen am 23. 2. 1929); *Eine Bemerkung über die Unzerlegbarkeit von Polynomen*, ibid., 102(1930), pp. 738–739 (In Groningen. Eingegangen am 14. 10. 1929).

[32] Franz Rellich (1906–1955) was an Austrian-Italian-German mathematician. After studies at the universities in Graz and Göttingen (1924–1929) he received his doctorate in 1929 at the University of Göttingen where he worked under Courant's supervising. At the beginning of the thirties, he took an active position against Nazism and he had to leave Göttingen. In 1934, he became a Privatdocent in Marburg. In 1942, he was appointed as a professor of mathematics in Dresden and after the World War Second he became the director of the Mathematische Institute in Göttingen. He obtained important results in mathematical physics (foundations of quantum mechanics and theory of partial differential equations).

[33] For reviews see [HT], [Ma] and [Ta].

[34] [Pe], p. 136.

[35] *Der Zusammenhang zwischen den Darstellungen der symmetrischen und der linearen Gruppen*, Mathematische Annalen 104(1931), pp. 92–95 (In Groningen. Eingegangen am 23. 4. 1930); *Zur Begründung des Restsatzes mit dem Noetherschen Fundamentalsatz*, ibid., 104(1931), pp. 472–475 (In Groningen. Eingegangen am 19. 11. 1930); *Der Zusammenhang*

Career in Leipzig

In 1931, van der Waerden was appointed professor of mathematics at the University of Leipzig. He took this position to be closer to the modern German mathematics. He accepted a position as a full professor and co-director of the mathematical seminar and the mathematical institute there.

In Leipzig, he became a colleague and good friend of Werner Heisenberg[36] and other theoretical physicists as for example Karl-Friedrich Bonhoeffer[37] and Peter Debye.[38] He attended seminars of physics held by Werner Heisenberg and Friedrich Hund[39] and he learnt modern theoretical physics there. Under their influence he became interested in the application of modern algebra in quantum mechanics. In 1932, he published the book *Die gruppentheoretische Methode in der Quantenmechanik*. In his interview, van der Waerden said about his book:

---

zwischen den Darstellungen der symmetrischen und der linearen Gruppe. Nachtrag zu meiner Arbeit in diesem Band, ibid., 104(1931), p. 800 (In Groningen. Eingegangen am 25. 2. 1931). For more information on van der Waerden's publications from the end of the 20th and the beginnig of the 30th see [Gr].

[36] Werner Karl Heisenberg (1901–1976) after studies at the Elisabethenschule and Maximilians Gymnasium in Munich entered the University of Munich where he wanted to study pure mathematics. But thanks to interviews with Arnold Johannes Wilhelm Sommerfeld (1868–1951) he began to engage in theoretical physics. He also attended many courses of mathematics (from number theory to geometry) and experimental physics. In the summer of 1922, he attended the letures given by Niels Bohr (1885–1962) in Göttingen. The accademic year 1922/1923 he spent in the United States of America and again in Göttingen. In 1923, he received his doctorate in Munich and he was named as Max Born's assistant in Göttingen. Next year he delivered his habilitation lecture and received the qualification to teach in German universities. From 1924 to 1925, he worked thanks to the scholarship of the Rockefeller grant at the University of Copenhagen where he collaborated with N. Bohr. In 1926, he was appointed lecturer of theoretical physics in Copenhagen and next year he was named professor at the University of Leipzig. He held this post until 1941 when he was made the director of the Kaiser Wilhelm Institute for Physics in Berlin. After the war, he was arrested and interned in England with other leading German scientists because during the war he headed the German nuclear weapons project "Uranverein". In 1946, he returned to Germany and was appointed director of the Max Planck Institute for Physics and Astrophysics in Göttingen (later moving to Munich). He held this position until he resigned in 1970. He made very important contributions to quantum mechanics and nuclear physics. In 1932, he was awarded the Nobel Prize in physics for the creation of quantum mechanics and the discovery of the allotropic forms of hydrogen.

[37] Karl-Friedrich Bonhoeffer (1899–1957) was a German physical chemist, a specialist in physical chemistry and electrochemistry. Togehther with Paul Harteck (1902–1985), he discovered the spin isomers of hydrogen, orthohydrogen and parahydrogen.

[38] Petrus Josephus Wilhelmus Debye (1884–1966) was a Dutch physicist and chemist who developed the theory of the specific heats and the method to determine the atomic structure of crystals by means of X-rays (so called Debye-Scherrer method). In 1936, he was awarded the Nobel Prize in chemistry for his work on the structure of molecules. He taughted at the universities in Aachen, Munich, Zurich, Utrecht, Leipzig and Berlin. But in 1940, he was forced to leave Germany by the Nazis. He emigrated to the USA and became professor of chemistry at the Cornell University in Ithaca.

[39] Friedrich Hermann Hund (1896–1997) was a German physicist. He became well-known for his work on the electronic structure of atoms and molecules. He taught and did research at German universities (Rostock, Leipzig, Jena, Frankfurt am Main, and Göttingen).

I wrote a book on group theory and quantum mechanics. There are applications of group theory to quantum mechanics, made at that time by John von Neumann and Wigner. Hermann Weyl had written a book on the subject entitled – I think – Group Theory and Quantum Mechanics. However, his book was so difficult that no one understood it. Hermann Weyl wanted to write mathematics for beauty's sake, but I did not find it very beautiful. Thus I wrote a new book on the method of group theory in quantum mechanics. The book was well received by physicists and was rapidly sold out. Later I rewrote it in English; it is still available.[40]

During his first year in Leipzig, he began publishing a series of scientific articles named *Zur algebraischen Geometrie* in the famous German mathematical journal *Mathematische Annalen*, which dealt with algebraic geometry using the ideal theory in polynomial rings created by Artin and Noether. He also made considerable use of the algebraic theory of fields.[41]

---

[40] [DS3], p. 316.

[41] *Zur algebraischen Geometrie. I. Gradbestimmung von Schnittmannigfaltigkeiten einer beliebigen Mannigfaltigkeit mit Hyperflächen*, Mathematische Annalen 108(1933), pp. 113–125 (In Leipzig. Eingegangen am 12. 7. 1932); *Zur algebraischen Geometrie. II. Die geraden Linien auf den Hyperflächen des P_n*, ibid., 108(1933), pp. 253–259 (In Leipzig. Eingegangen am 27. 7. 1932); *Zur algebraischen Geometrie. III. Über irreduzible algebraische Mannigfaltigkeiten*, ibid., 108(1933), pp. 694–698 (In Leipzig. Eingegangen am 27. 10. 1932); *Zur algebraischen Geometrie. IV. Die Homologiezahlen der Quadriken und die Formeln von Halphen der Liniengeometrie*, ibid., 109(1934), pp. 7–12 (In Leipzig. Eingegangen am 27. 10. 1932); *Zur algebraischen Geometrie. V. Ein Kriterium für die Einfachheit von Schnittpunkten*, ibid., 110(1935), pp. 128–133 (In Leipzig. Eingegangen am 8. 10. 1933); *Zur algebraischen Geometrie. VI. Algebraische Korrespondenzen und rationale Abbildungen*, ibid., 110(1935), pp. 134–160 (In Leipzig. Eingegangen am 11. 10. 1933); *Zur algebraischen Geometrie. VII. Ein neuer Beweis der Restsatzes*, ibid., 111(1935), pp. 432–437 (In Leipzig. Eingegangen am 29. 3. 1935); *Zum algebraischen Geometrie. Berichtigung und Ergänzungen*, ibid., 113(1937), pp. 36–39 (In Leipzig. Eingegangen am 13. 4. 1936); *Zur algebraischen Geometrie. VIII. Der Grad der Graßmannschen Mannigfaltigkeit der linearen Räume S_m in S_n*, ibid., 113(1937), pp. 199–205 (In Leipzig. Eingegangen am 26. 3. 1936); W.-L. Chow, B.L. van der Waerden: *Zum algebraischen Geometrie. IX. Über zugeordnete Formen und algebraische Systeme von algebraischen Mannigfaltigkeiten*, ibid., 113(1937), pp. 692–704 (In Leipzig. Eingegangen am 24. 5. 1936); *Zur algebraischen Geometrie. X. Über lineare Scharen von reduziblen Mannigfaltigkeiten*, ibid., 113(1937), pp. 705–712 (In Leipzig. Eingegangen am 29. 5. 1936); *Zur algebraischen Geometrie. XI. Projektive und birationale Äquivalenz und Moduln von ebenen Kurven*, ibid., 114(1937), pp. 683–699 (In Leipzig. Eingegangen am 19. 3. 1937); *Zur algebraischen Geometrie. XII. Ein Satz über Korrespondenzen und die Dimension einer Schnittmannigfaltigkeit*, ibid., 115(1938), pp. 330–332 (In Leipzig. Eingegangen am 5. 8. 1937); *Zur algebraischen Geometrie. XIII. Vereinfachte Grundlagen der algebraischen Geometrie*, ibid., 115(1938), pp. 359–378 (In Leipzig. Eingegangen am 30. 10. 1937); *Zur algebraischen Geometrie. XIV. Schnittpunktszahlen von algebraischen Mannigfaltigkeiten*, ibid., 115(1938), pp. 619–642 (In Leipzig. Eingegangen am 30. 10. 1937); *Zur algebraischen Geometrie. XV. Lösung des Charakteristikenproblems für Kegelschnitte*, ibid., 115(1938), pp. 645–655 (In Leipzig. Eingegangen am 10. 12. 1937).
In the 1950s and 1970s, van der Waerden dealt once more with algebraic geometry and he published his new series of his results: *Zur algebraischen Geometrie 16. Vielfältigkeiten von abstrakten Ketten*, Mathematische Annalen 125(1952), pp. 314–324 (In Zürich. Eingegangen 6. 6. 1952); *Zur algebraischen Geometrie 17. Lokale Dimension und Satz von Eckmann,*

It is not without interest that later he changed his approach to this topic as it can be evident in his book *Einführung in die algebraische Geometrie* (1939).[42] Dan Pedoe (1910–1998), an English mathematician, wrote in a review:

About ten years ago, van der Waerden, already eminent as an algebraist, began, in a series of papers in the Mathematische Annalen, to create rigorous foundations for algebraic geometry. The implication-that there was something unsound in the magnificent structure of Italian geometry-was vigorously contested by Severi. Fortunately, van der Waerden continued his researches, but with the implicit sub-title, "An algebraist looks at algebraic geometry". With increasing knowledge of the powerful methods of the Italian school, he has gladly modified his own methods. Ideal-theory, the weapon of attack in his first papers, he has found almost completely unnecessary … As a result of the experience gained in writing these papers, and in giving various courses of lectures, Professor van der Waerden has produced a work which must sooner or later find a place on every geometer's bookshelves.[43]

In 1934, van der Waerden became one of the members of the editorial board of *Mathematische Annalen.*[44] It was not an easy time for him because he came under pressure from the Nazis not to take and publish any work written by Jewish mathematicians. He started to think about his resignation but was informed that he could be replaced by the fanatic Nazis Wilhelm Blaschke[45] and Ludwig Bieberbach.[46]

---

ibid., 128(1954/1955), pp. 128–134 (In Zürich. Eingegangen 11. 3. 1954); *Zur algebraischen Geometrie 18. Ketten in mehrfach-projektiven Räumen*, ibid., 128(1954/1955), pp. 135–137 (In Zürich. Eingegangen 11. 3. 1954); *Zur algebraischen Geometrie 19. Grundpolynom und zugeordnete Form*, ibid., 136(1958), pp. 139–155 (In Zürich. Eingegangen 30. 4. 1958); *Zur algebraischen Geometrie 20. Der Zusammenhangssatz und der Multiplizitätsbegriff*, ibid., 193(1971), pp. 89–108 (In Zürich. Eingegangen 20. 8. 1970).

[42] For more information see [Sch] and [Wa2].

[43] See [W1].

[44] For the first time, his name appeared on the front page of the 109th volume of the journal (1934) and he held this position up to the 177th volume (1968).

[45] Wilhelm Johann Eugen Blaschke (1885–1962) was an Austrian differential and integral geometer. After his studies at the Technical Hochschule in Graz and at the University of Vienna where he was awarded his doctorate (1908), he visited many different universities to learn modern geometry from the leading mathematicians (Pisa, Göttingen, Bonn, Greifswald etc.). In 1910, he submitted his habilitation thesis and in 1913 he became an extraordinary professor of mathematics at the Deutsche Technische Hochschule in Prague. Two years later he accepted a post at the University of Leipzig. In 1917, he was appointed professor of mathematics at the University of Königsberg and from 1919 he held a post at the University of Hamburg. From 1936, he demonstrated his strong fascist sympathies and his support to Nazi politics and ideology. He took a leading role in German mathematics until the end of Second World War. In the September 1945, he was dismissed from his chair at Hamburg on the recommendations of several mathematicians who criticised his activities during war years. But in the October 1946, he was reinstated and he continued to hold his chair in Hamburg until he retired in 1953. His work was devoted to axiomatic foundations of various geometries, fundamental technics in kinematics, topological differential geometry, integral geometry and the study of invariant differentiable mappings.

[46] Ludwig Georg Elias Moses Bieberbach (1886–1982) after studies at the Humanistic

In the second half of the thirties, van der Waerden, as a foreigner, had problems with the Nazis because he refused to give up his Dutch citizenship and becuase he was in close contacts with Jewish mathematicians and physicists.[47] Although Germany occupied the Netherlands from 1940, van der Waerden's homeland, he did not return home despite being offered to become a professor of mathematics at the University of Utrecht twice. And moreover he went ahead with teaching and scientific activities in Leipzig. But he had no sympathies at all for the Nazis. He only found excellent conditions for his mathematical research in Germany although many of his Jewish friends and colleagues had been pursued and murdered. He made his mathematics and he was not interested in politics.[48]

The situation during wartime was for the Wardens difficult as Hans-Georg Gadamer (1900–2002), a German philosopher and van der Waerden's good friend, recalled:

... The time at Leipzig, those awful years, created above all precious links of friendship. I had the joy of stimulating van der Waerden's interest in the birth of science. Since I was an old friend of Franz Rellich, our interactions were wonderful from the beginning. When the war began, I had the opportunity to perform a little act of heroism. When van der Waerden was arrested for being a Dutchman, an expedient came to my mind. Once I had helped the wife of the then chief of police on her philosophical travails and thus I also had the opportunity to make the acquaintance of her husband. I wrote to him, and van der Waerden was released, and the chief of police thanked me, for, after a few days, the liberation of all the Dutch citizens was ordered.[49]

Gymnasium in Bensheim in Germany entered the University of Heidelberg where he attended the courses given by Leo Königsberger (1837–1921). Then he moved to Göttingen and he visited the courses given by Hermann Minkovski, Felix Klein and Paul Koebe (1882–1945). In 1910, he received his doctorate under the supervision of Klein. At the same time he began working as a "Privatdocent" at the University of Königsberg. In 1913, he was appointed professor of mathematics at the University of Basel, from 1915 to 1921, he taught at the University of Frankfurt and from 1921 to 1945 at the University of Berlin. At the beginning of the thirties, he converted to the views of the Nazis and he energetically persecuted his Jewish colleagues not only in Germany. Very soon, he became the leading Nazi mathematician. As an editor of the *Jarbuch über die Fortschritte der Mathematik*, the *Jahresbericht der Deutschen Mathematiker-Vereinigung* and the *Deutsche Mathematik* he encouraged and promoted the "German mathematics" and he also wrote many papers expressing his racist views. After the war, he lost all his positions and he was dismissed and arrested. But in 1949, Alexander Markovich Ostrowski (1893–1986) invited him to lecture at the University of Basel because he respected Bieberbach's mathematical results and he considered Bieberbach's political views irrelevant to his contributions to mathematics. Bieberbach was interested in theory of functions, differential equations, analytical, projective, differential and integral geometry, modern algebra etc.

[47] For example, van der Waerden wrote his letter (6. 2. 1934) of support for Edmund Landau who lost his position at the University of Göttingen. See Zbl 062343963.

[48] More information about mathematics and resistences of mathematicians under the Nazis can be found in [Se] and [SSr2].

[49] [DS3], p. 317.

On December 4, 1943, van der Waerden's home in Leipzig was bombed and the van der Waerdens had to be moved with their three children (Helga, Ilse and Hans) to Camilla's brother Franz Rellich's home who was a professor of mathematics in Dresden. But their situation was very bad and van der Waerden accepted an invitation from one of his students to live with his family in Bischofswerda, a small town situated near Dresden. They stayed there for nearly one year and then they returned to Dresden. At the end of World War II, Dresden was under continual air attacks from the British air force and therefore they moved to Austria to live with Camilla's mother in the small country town of Tauplitz in Styria. After the war, the van der Waerdens returned to the Netherlands and they started to live in Laren in the house built by Theodor van der Waerden, van der Waerden's father.[50]

From 1931 to 1945, van der Waerden published 5 mathematical books and 57 scientific papers on algebra, algebraic geometry, statistics, quantum mechanics and history of mathematics and astronomy. He had supervised 4 doctoral theses.[51]

Life in the Netherlands after the war

When the Americans arrived to Graz in July 1945, the van der Waerdens became "displaced persons" and they were taken back to their homeland by bus. After returning to the Netherlands, van der Waerden could not find any job and he had hardly any money for his family. His friend Hans Freudenthal[52] arranged him an offer of a post of professor of mathematics at the University of Utrecht. The Dutch government refused to allow him to take up the position because of his teaching in Germany during the war and because of his collaboration with

[50] For more information on van der Waerden's pedagogical and scientific activities and his life in Leizpig from 1931 to 1945 see [Ei], [So1]–[So7], [Th1], [Th2] and [Wa1].

[51] For more information see [Ne], [So5]–[So7], [SSr2]–[SSr4], [Th1], [Th2], [W1]–[W3] and [Wa1].

[52] Hans Freudenthal (1905–1990) studied at the Gymnasium in Luckenwalde in Germany where he was interested in mathematics, natural sciences, literature and poetry. In 1923, he entered the University of Berlin to study mathematics and physics. In 1927, he spent the summer semester at the University of Paris and in 1931 he was awarded his doctorate for a thesis on the theory of groups supervising by Heinrich Hopf (1894–1971). From 1930, he was an assistant of L.E.J. Brouwer in Amsterdam; very soon he became a lecturer at the Mathematical Institute of the University of Amsterdam. Because he was a Jew living outside Germany he could continue with his teaching and research up to 1940 when Germany invaded the Netherlands. He lived in Amsterdam during the war under very difficult circumstances. In 1946, he was offered the chair of pure and applied mathematics and foundations of mathematics at the University of Utrecht. He held this chair until he retired in 1975. In 1971, he was also appointed the director of the Institute for the Development of Mathematical Education in Utrecht which became a part of the Faculty of Mathematics and Computer Science at the University of Utrecht in 1981. Freudenthal worked on topology (algebraic characterisation of the topology, spectral theorem for Riesz spaces), algebra (theory of groups, real semisimple Lie groups), logic (relation between axiomatic mathematics and reality, intuitionism), applications of mathematics to linguistics, history of mathematics (many articles to the *Dictionary of Scientific Biography*), history of geometry and mathematical education (textbooks, lectures on the development of mathematical instruction).

German mathematicians. In the difficult living situation, Freudenthal managed to obtain a position for him at Shell, the Royal Dutch Petroleum Company, in Amsterdam where he worked as an applied mathematician.

On his work, van der Waerden remembered:

At Shell I solved some problems which the engineers found too difficult. It was entertaining. They had quite different problems: for example, what is the best circuit for regulation devices? Problems of optimization, in a word. At Shell there was another mathematician with whom I worked on questions of optimization, and together we found beautiful solutions.[53]

In 1947, van der Waerden visited the Johns Hopkins University in Baltimore in the United States of America. Thanks to his mathematical results he was offered a permanent post as a professor of mathematics there. Despite his bad experience in the Netherlands, he refused the American career and returned to Amsterdam. In 1948, he was named chair of mathematics at the University of Amsterdam, he became an extraordinary professor here. In 1950, he was appointed an ordinary professor and he remained until 1951.

Career in Switzerland

In 1951, van der Waerden was appointed to fill the chair at the Polytechnic in Zurich which became vacant due to Fueter's death.[54] He remained in Zurich for the rest of his life. There he greatly stimulated research interests in a very broad range of mathematics. For example he supervised over 40 doctoral students who did their researches in a wide range of topics.[55]

Thanks to his regular lecture courses in mathematics for natural scientists and school teachers from secondary schools, he also promoted exact mathematical thinking in a wider circle. As an editor of the "yellow" series *Grundlagen der mathematischen Wissenschaften*, and of the journals *Mathematische Annalen* and *Archive for History of Exact Sciences* he established very high standards for mathematical and historical research in the world-wide scientific community.

---

[53] [DS3], p. 318.

[54] Karl Rudolf Fueter (1880–1950) studied at a secondary school in Basel. From 1899 to 1903, he entered the University of Göttingen where he was awarded his doctorate suprevised by Hilbert. After obtaining his doctorate, he visited various European mathematical centers (for example Paris, Vienna, London). From 1907, he taught at the University of Marburg and at the Mining Academy in Clausthal. In 1908, he was appointed professor of mathematics at the University of Basel and five years later he moved to Karlsruhe where he took the chair of mathematics at the Technical University. After three years he became a professor of mathematics at the University of Zurich where he remained up to his death. He was a co-founder of the Swiss Mathematical Society, he also served as an editor on the famous project to publish the complete works of Leonhard Euler and he undertook editorial work for the Comentarii Mathematici Helvetici. Fueter dealt with number theory, elliptic functions, applications of mathematics in chemistry, biology and statistics.

[55] For more information see famous www pages called *Mathematics Genealogy Project* (http://www.genealogy.ams.org [9.9.2019]).

Van der Waerden retired from his chair in 1973. On June 12, 1985, he received the honorary doctor degree from the University of Leipzig. He died on January 12, 1996 in Zurich.[56]

Van der Waerden's mathematical achievements

Bartel Leendert van der Waerden was an outstanding mathematician who developed and contributed to nearly all modern mathematical disciplines as well as he was an outstanding historian of mathematics. During his long career, he wrote about 300 scientific works.[57] He was devoted to problems of algebraic geometry, abstract algebra, group theory, topology, number theory, elementary geometry, combinatorics, analysis, probability, mathematical statistics, quantum mechanics, history of mathematics, modern physics and astronomy, history of natural sciences and philosophy of ancient civilizations.

In [FTW], we can read these concise words on van der Waerden's mathematical results:

... He may well be the last scientist who has an overall understanding of all areas of mathematics and who has enriched many of these with significant contributions. In fact, his scientific work extends far beyond the field of mathematics; it also deals with fields such as physics, history of mathematics and history of astronomy.

Many people know van der Waerden's name mainly from his book Algebra, which appeared in two volumes and in many translations throughout this century ... The emergence of this work marked a new era in algebra; it introduced several generations of mathematicians to the modern theory of abstract algebra, an area of mathematics that finds its origins in the work of R. Dedekind, D. Hilbert, E. Artin and E. Noether. Even today, after more than 60 years, textbooks on higher algebra basically follow the exposition given by van der Waerden.

Still and all, van der Waerden's main field of research was algebraic geometry, a part of mathematics emanating from the theory of algebraic curves and surfaces and currently a central area of mathematical research. Algebra, geometry, number theory, topology and mathematical analysis have found close and very deep connections through algebraic geometry. It plays a prominent rôle in current mathematics as well as modern physics; examples are provided by string theory in the theory of elementary particles and twistor theory in the theory of relativity. Much of what we currently know about algebraic geometry we owe to foundational work of van der Waerden. He laid down the solid foundation for this field by means of the modern abstract algebra and made hence possible later work by Zariski, Weil, Serre and Grothendieck.[58]

---

[56] Information on B.L. van der Waerden's life can be found for example in [Be], [CR], [DS1]–[DS3], [Ed], [Ei], [Fr1], [Fr2], [FTW], [Ge], [Gr], [Hl], [Ne], [Pe], [Ro], [Sc1], [Sc2], [So1]–[So7] and [W1]–[W3].

[57] For more information on van der Waerden's bibliography see [Gr], [Ho2], [Ne], [TW].

[58] [FTW], pp. 137–138.

We describe very briefly van der Waerden's most important mathematical results and achievements.[59]

Van der Waerden produced important results in linear groups (theory of linear groups over arbitrary fields, general representation theory of groups and rings, etc.), Lie groups, theory of transformations and invariant theory. He generalised some of Emmy Noether's results on ideals theory to polynomial rings having the property that every ascending chain of ideals is finite and then he carried it over to rings that are integrally closed in their field of fractions. In Galois theory by using the method of factoring an algebraic equation over residue class fields modulo a prime number p, van der Waerden showed that almost all algebraic equations with integer coefficients have the symmetric group as their Galois group over the rational numbers. He also found an example of a group generated by two elements that contains a non-finitely generated abelian subgroup.

In the thirties and forties, van der Waerden explained in his many studies a principal goal of algebraic geometry and by using new methods of abstract algebra (the theory of ideals in polynomial rings, the elimination theory, the algebraic theory of fields, the algebraic varieties, etc.) he presented quite a new concept and some original methods of algebraic geometry and showed their new applications.

In number theory, van der Waerden proved Dirichlet's unit theorem without classical use of logarithm; his proof was newly based on the valuation theory. He also succeeded in founding an elementary proof of the existence theorem in number theory and obtained new results on the reduction theory of positive definite quadratic forms which were developed by his PhD students at the University of Zurich.

Van der Waerden also worked on modern problems in topology. For example, with David van Dantzig,[60] they investigated and described the conditions under which a topological space is homogeneously metrizable. His other important contributions to topology were connected with combinatorial topology, the cohomology of polyhedra and the topology and uniformization of Riemannian surfaces.

In probability and mathematical statistics, van der Waerden developed practical tests for a small number of samples when the law of large numbers cannot be used. He examined the validity of the χ^2 criterion and his results received many applications in the natural and medical sciences. He also earned particular regard for the X test (today the so-called van der Waerden's test) and explained the applications of mathematical statistics by giving many examples from different fields.

[59] For a detailed analyses see [Bo], [Ei], [FTW], [Ne], [Sch], [So1]–[So7] and [Wa2].

[60] David van Dantzig (1900–1959), a Dutch mathematician, is well known for his results in topology (the construction of the dyadic solenoid). He was interested in differential geometry, topology, probability and mathematical statistics.

Van der Waerden was also concerned with combinatorics. He proved for example a generalization of a conjecture of Baudet[61] and thanks to the conjecture of van der Waerden (dealt with the minimal value of the permanent of doubly-stochastic $n \times n$ matrices) opened the starting point of a completely new area of reserch in combinatorics.[62]

In physics, van der Waerden was primarily interested in mathematical questions of quantum mechanics, he worked on the theory of electron spin and on the theory of general relativity (the studies of the wave equation of the electron). His works were stimulated by Paul Ehrenfest[63] and Leopold Infeld.[64]

From the beginning of the thirties, van der Waerden wrote and published newer and more modern textbooks and monographs on algebra, algebraic geometry, algebraic applications in physics, mathematical statistics and differential calculus which were very highly esteemed and translated into foreign languages and thanks to them several generations learned modern algebra and its applications from van der Waerden. His most important books are:

Moderne Algebra I., II.,[65]

---

[61] Pierre Joseph Henry Baudet (1891–1921) was a gifted Dutch mathematician. For more information see E. Arrias: *In Memoriam Prof. P.J.H. Baudet*, Eigen Haard, wekeljksch tijdschrift voor het gezin 48(1922), pp. 92–94. The Baudet's conjecture in its most simple form states that if l is a natural number and if the set of natural numbers is divided over two classes, then at least one of these classes contains an arithmetic progression of length l. This problem circulated in the German mathematical community in the twenties and B.L. van der Waerden became famous in the European mathematical circles thank to his elegant solution of Baudet's conjecture.

[62] For more information see B.L. van der Waerden: *Beweis einer Baudetschen Vermutung*, Nieuw Archief voor Wiskunde 15(1927), pp. 212–216; B.L. van der Waerden: *Wie der Beweis der Vermutung von Baudet gefunden wurde*, Abhandlungen aus dem Mathematischen Seminar der Hamburgischen Universität 28(1965), pp. 6–15.

[63] Paul Ehrenfest (1880–1933), an Austrian physicist and mathematician, was one of the founders of statistical and quantum mechanics.

[64] Leopold Infeld (1898–1968), a Polish physicist, was interested in the theory of relativity. He worked together with Albert Einstein (1878–1955) at Princeton University; they formulated the equation describing star movements.

[65] *Moderne Algebra. Unter Benutzung von Vorlesungen von E. Artin und E. Noether*, Erster Teil, Die Grundlehren der mathematischen Wissenschaften, Band 33, Springer, Berlin, 1930, viii + 243 pages; 2nd edition, 1937, x + 272 pages; 3rd edition, 1950, viii + 292 pages; 4th edition, 1955; 5th edition, 1960; 6th edition, 1964; 7th edition, Heidelberger Taschenbücher, Band 12, Springer, Berlin, Göttingen, Heidelberg, 1966, ix + 271 pages; 8th edition, 1971; 9th edition, 1993.
Moderne Algebra. Unter Benutzung von Vorlesungen von E. Artin und E. Noether, Zweiter Teil, Die Grundlehren der mathematischen Wissenschaften, Band 34, Springer, Berlin, 1931, vii + 216 pages; 2nd edition, 1940, viii + 224 pages; 3rd edition, 1955; 4th edition, 1959, ix + 275 pages; 5th edition, Heidelberger Taschenbücher, Band 23, Springer, Berlin, Göttingen, Heidelberg, 1967, x + 300 pages.
Russian translation: *Sovremennaja algebra* OGIZ, Moscow, 1947, 339 + 260 pages; 1st new edition, *Algebra*, Nauka, Moscow, 1976, 648 pages [Translated from the 8th and 5th German edition of Volume 1 and 2 by A.A. Bel'skij.]; 2nd edition, 1979, 623 pages.
English translation: *Modern Algebra*, Ungar, New York, 1949, 1950, xii + 264, ix + 222 pages [Volume 1 translated from the 2nd revised German edition by Fred Blum with revisions and

Die Gruppentheoretische Methode in der Quantenmechanik,[66]

Gruppen von linearen Transformationen,[67]

De logische grondslagen van de Euclidische Meetkunde,[68]

Einführung in die algebraische Geometrie,[69]

Zur algebraischen Geometrie. Selected Paper,[70]

Differentiaalrekening,[71]

Integraalrekening,[72]

Tafeln zum Vergleich zweier Stichproben mittels X-Test und Zeichentest,[73]

Mathematische Statistik,[74]

Sources of Quantum Mechanics,[75]

---

additions by the author. Volume 2 translated by Theodore J. Benac]; reprints: Volume 1, 1953; Volume 1 + 2, Ungar, New York, 1970; Springer, New York, 1991, xiv + 265 + xii + 284 pages [Volume 1. Translated from the 7th German edition by Fred Blum and John R. Schulenberger. Volume 2. Translated from the 5th German edition by John R. Schulenberger.]; 1st softcover print, New York, Berlin, Heidelberg, 2003, xiv + 265, xii + 284 pages.
Portuguese translation: Sociedale Portuguesa de Matemática, Lisboa, 1954 [Translated by Hugo B. Ribeiro]. Chinese translation: 1964.

[66] Die Grundlehren der mathematischen Wissenschaften, Band 36, Springer, Berlin, 1932, viii + 157 pages; reprint: J.W. Edwards, Ann Arbor, Michigan, 1944.
English translation: *Group Theory and Quantum Mechanics*, Die Grundlehren, Band 214, Springer, New York, Heidelberg, 1974, viii + 211 pages; 2nd edition, 1986.
For more information about Waerden's contributions to the development of quantum mechanics see [Sm].

[67] Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 4, Springer, Berlin, 1935, 91 pages; reprint: Chelsea Publishing Company, New York, 1948.

[68] Noordhoff, Groningen, 1937.

[69] Die Grundlehren der mathematischen Wissenschaften, Band 51, Springer, Berlin, 1939, vii + 247 pages; reprint: Dover Publications, Inc., New York, 1945; 2nd edition, 1973, xi + 280 pages.

[70] *Mit einem Geleitwort von F. Hirzebruch*, Springer, Berlin, Heidelberg, 1983, vii + 479 pages.

[71] Servire's encyclopaedie, A. Natuurwetenschappen, 1. Afdeling Wiskunde 5, Den Haag, 1951, 126 pages.

[72] Servire's encyclopaedie, A. Natuurwetenschappen, 1. Afdeling Wiskunde 5, Den Haag, 1958, 120 pages.

[73] With E. Nievergelt (1929–2018), Springer Verlag, Berlin, Göttingen, Heidelberg, 1956, iv + 34 pages.

[74] Die Grundlehren der mathematischen Wissenschaften, Band 87, Springer, Berlin, Göttingen, Heidelberg, 1957, ix + 360 pages; 2nd edition, 1965; 3rd edition, 1971.
Russian translation: *Matematičeskaja statistika*, Izdatel'stvo innostrannoj literatury, Moscow, 1960, 434 pages.
French translation: *Statistique mathématique*, Dunod, Paris, 1967, xi + 371 pages.
English translation: *Mathematical Statistics*, Die Grundlehren, Band 156, Springer, Berlin, Heidelberg, 1969, xi + 367 pages.

[75] B.L. van der Waerden (Edited with a historical introduction), North-Holland, Amsterdam, 1967, xi + 430 pages; reprint: Dover, New York, 1968.

Studien zur Theorie der quadratischen Formen,[76]

Mathematik für Naturwissenschaftler.[77]

Van der Waerden as a historian of sciences

After he retired, van der Waerden continued to undertake research in the history of mathematics and astronomy which he enjoyed from his studies at the University of Amsterdam and at the University of Göttingen. About the beginning of his interests in history of mathematics he said:

When I was a student, when Hendrik de Vries gave a course on history of mathematics. After that I read Euclid and some of Archimedes. At Göttingen – the first time I was there – I attended the lectures of Neugebauer, who gave a course on Greek mathematics.

... At that time, at Göttingen, Neugebauer worked above all on Egyptian mathematics and gave classes on it. His thesis was precisely on Egyptian mathematics. This was very stimulating. Later I visited him once at Copenhagen, and then he spoke to me of Babylonian astronomy. This was most interesting to me.[78]

Other inspirations on Greek mathematics, astronomy and philosophy van der Waerden discovered in Leipzig thanks to his contacts and discussions with his good friend – philosopher Gadamer. At the beginnig of the thirties, he was mainly interested in the history of Egyptian, Babylonian and Greek mathematics as well as in the birth and roots of science. From the end of the thirties up to the end of the forties, his keenness for the history of mathematics increased and he published more than 15 short articles. His first important paper entitled *Die Arithmetik der Pythagoreer*[79] appeared in 1947 followed by *Die Astronomie der Pythagoreer* in 1951.[80]

After the fifties, he published more than 60 papers on different historical topis and five extensive books. They dealt with Pythagorean number theory, Greek geometry, Greek geometrical algebra, Babylonian, Sanskrit, Aryabhata, Persian and Greek astronomy, Egyptian, Babylonian, Greek and Indian methods for computing planetary positions, the development of astrology and its influence on astronomy and the development of modern algebra.[81]

---

[76] B.L. van der Waerden (Hrsg.), Lehrbücher und Monographien aus dem Gebiete der exakten Wissenschaften, Mathematische Reihe, Band 34, Birkhäuser, Basel, Stuttgart, 1968, 254 pages.

[77] BI-Hochschultaschenbücher, Band 281, BI-Wissenschaftliche-Verlag, Mannheim, 1975, 280 pages; reprint 1990.

[78] [DS3], p. 319.

[79] Mathematische Annalen 120(1947/1948), pp. 127–153, 676–700.

[80] Verhandelingen der Koninklijke Akademie van Wetenschappen, Amsterdam 20(1951), No. 1, 80 pages.

[81] For a detailed analysis see [Ho1], [Ho2], [Kn], [Ne], [St] and [W2].

Jan P. Hogendijk (*1955), a great Dutch historian of mathematics and natural sciences, wrote in his analysis of van der Waerden's methodology and his contributions to history of mathematics and astronomy:

… By mathematical and logical analysis of ancient and medieval astronomical sources, Van der Waerden has been able to discover many mathematical and historical connections that had not been noticed before.[82]

Van der Waerden's most well-known, translated, studied and read books are:

Einfall und Überlegung,[83]

Ontwakende wetenschap I., II.,[84]

Die Pythagoreer. Religiöse Bruderschaft und Schule der Wissenschaft,[85]

Geometry and Algebra in Ancient Civilizations,[86]

A History of Algebra: From al-Khwārizmī to Emmy Noether,[87]

Die Astronomie der Griechen.[88]

Dirk Jan Struik (1894–2000), a great Dutch mathematician and historian of mathematics, reviewing the first of these books, wrote:

---

[82] [Ho1], p. 145.

[83] *Einfall und Überlegung. Drei kleine Beiträge zur Psychologie des mathematischen Denkens*, Birkhäuser, Basel, Suttgart, 1954, 28 pages; 2nd edition, 1968; 3rd editon, 1973, 36 pages.

[84] *Ontwakende wetenschap I. Egyptische, Babylonische en Griekse wiskunde*, Historische Bibliotheek voor de exacte wetenschappen 7, Noordhoff, Groningen, 1950, 332 pages; *Ontwakende wetenschap II.*, Noordhoff, Groningen, 1955.
German translation: *Erwachende Wissenschaft I. Ägyptische, babylonische und griechische Mathematik*, Wissenschaft und Kultur, Band 8, Birkhäuser, Basel, Stuttgart, 1956, iii + 488 pages; 2nd edition, 1966.
Erwachende Wissenschaft II. Die Anfänge der Astronomie, Wissenschaft und Kultur, Band 23, Noordhoff, Groningen, 1965, xii + 315 pages; Birkhäuser, Basel, Stuttgart, 1968, 1980, 2019.
English translation: *Science Awakening I. Egyptian, Babylonian and Greek Mathematics*, Noordhoff, Groningen, Leyden, 1954, 306 pages; Oxford University Press, New York, 1961; Reidel, Dordrecht, 1974; Noordhoff, Leyden, 1969, 1975; Kluwer, Dordrecht, 1988.
Science Awakening II. The Birth of Astronomy, Springer, New York, 1973, 368 pages; Noordhoff, Leyden, Oxford University Press, New York, 1974, xiv + 347 pages.
Russian translation: *Probuždajuščajasja nauka I., Matematika drevnego Egypta, Vavilona i Grecii*, GIFML, Moscow, 1959, 460 pages; *Probuždajuščajasja nauka II. Roždenie astronomii*, Nauka, Moscow, 1991, 384 pages.
Persian translation: 1994.
Greek translation: 2003, xxvi + 370 pages. – For more information see [Fh].

[85] Die Bibliothek der alten Welt. Reihe Forschung und Deutung, Artemis, Zürich & München, 1979, 505 pages.

[86] Springer, Berlin, Heidelberg, New York, Tokyo, 1983, xii + 223 pages.

[87] Springer, Berlin, Heidelberg, New York, Tokyo, 1985, xi + 271 pages.

[88] Die Altertumswissenschaft, Wissenschaftliche Buchgesellschaft, Darmstadt, 1988, x + 315 pages.

This is the first book which bases a full discussion of Greek mathematics on a solid discussion of pre-Greek mathematics. Carefully using the best sources available at present, the author acquaints the reader not only with the work of Neugebauer and Heath, but also with that of the philological critics who centered around the "Quellen und Studien".

... This book contains a wealth of material, critically arranged, and reads exceedingly well. It has an original approach and contains much novel material.[89]

Jeremy John Gray (*1947), one of the most important contemporary mathematicians and historians of mathematics, wrote in his review on van der Waerden's *A History of Algebra: From al-Khwārizmī to Emmy Noether*:

It is almost unfailingly clear. The arguments presented are summarized with a deftness that isolates and illuminates the main points, and as a result they are frequently exciting. Since nearly 200 pages of it are given over to modern developments which are only now receiving the attention of historians, this book should earn itself a place as an invaluable guide. Its second virtue is the zeal with which the author has attended to the current literature. Almost every section gives readers an indication of where they can go for a further discussion. As a result, many pieces of information are here presented in book form that might otherwise have languished in the scholarly journals. Since one must be cynical of the mathematicians' awareness of those journals, the breadth and generosity of van der Waerden's scholarship will do everyone a favour.[90]

After he went into retirement, van der Waerden took the great occasion to create the Institute for History of Mathematics with a big library which became a part of the University of Zurich. He donated his personal library built for many years to the Institute and directed it till 1979. He believed that the research in history of mathematics and astronomy would successfully continue. But at the beginning of the eighties, the Institute was abolished because the interest in the history of mathematics was very rare in Switzerland.

In [FTW], we can find these words on the van der Waerden's influence on the history of exact sciences:

Beyond this, van der Waerden earned great respect in the field of history of exact sciences – particularly history of mathematics and of astronomy of ancient civilizations – devoting more than 100 papers and 6 books to this field. He is one of the very few scholars working on the history of mathematics who was also extensively involved in current mathematical research. For that reason he was able to give new impulses to the filed of history of exact sciences.[91]

---

[89] See [W1] and [St].
[90] See [W1].
[91] [FTW], p. 138.

REFERENCES

Seventieth Birthday

[Gr] Gross H., *Herr Professor B.L. van der Waerden feierte seinen siebzigsten Geburtstag*, Elemente der Mathematik **28** (1973), pp. 25–32.

Ninetieth Birthday

[Fr1] Frei G., *Dedication: Bartel Leendert van der Waerden. Zum 90. Geburtstag*, Historia mathematica **20** (1993), pp. 5–11.

Commemorative Articles

[DS1] Dold-Samplonius Y., *In Memoriam: Bartel Leendert van der Waerden* (*1903–1996*), Historia mathematica **24** (1997), pp. 125–130.

[Fr2] Frei G., *Zum Gedenken an Bartel Leendert van der Waerden* (*2. 2. 1903 – 12. 1. 1996*), Elemente der Mathematik **53** (1998), No. 4, pp. 133–138.

[CR] O'Connor J., Robertson E., *Bartel van der Waerden, 1903–1996*, Lecturas Matemáticas **17** (1996), No. 1, pp. 89–91.

[Ro] Roquette P., *Bartel Leendert van der Waerden (1903–1996)*, Jahrbuch der Heidelberger Akademie der Wissenschaften für 1996, Heidelberg, 1997, pp. 138–141.

[Sc1] Scriba C.J., *Bartel Leendert van der Waerden* (*1903–1996*), Mitteilungen der Mathematischen Gesellschaft in Hamburg **15** (1996), pp. 13–18.

[Sc2] Scriba C.J., *Bartel Leendert van der Waerden* (*2. Februar 1903 – 12. Januar 1996*), Berichte zur Wissenschaftsgeschichte. Organ der Gesellschaft für Wissenschaftsgeschichte **19** (1996), No. 4, pp. 245–251.

Other Articles and Books

[Be] Beckert H., *Bartel L. van der Waerden: 2. 2. 1903 – 12. 1. 1996*, Jahrbuch der Sächsische Akademie der Wissenschaften zu Leipzig 1995/1996, Leipzig, 1997, pp. 435–449.

[Bo] de Boer J.H., *Van der Waerden's Foundations of Algebraic Geometry*, Nieuw Archief voor Wiskunde, 4th series, **12** (1994), No. 3, pp. 159–168.

[Da] van Dalen D., *Mystic, Geometer, and Intuitionist: The Life of L.E.J. Brouwer*, The Clarendon Press, Oxford University Press, Oxford, 1999, 440 pages.

[DS2] Dold-Samplonius Y., *Bartel Leendert van der Waerden befragt von Yvonne Dold-Samplonius*, NTM – International Journal of History and Ethics of Natural Sciences, Technology and Medicine, New Series **2** (1994), pp. 129–147.

[DS3] Dold-Samplonius Y., *Interview with Bartel Leendert van der Waerden*, Notices of the American Mathematical Society **44** (1997), No. 3, pp. 313–320.

[Ed] *Editorial on Van der Waerden Day*, Nieuw Archief voor Wiskunde, 4th series, **12** (1994), No. 3, pp. 133–134.

[Ei] Eisenreich G., *B.L. van der Waerden's Wirken von 1931 bis 1945 in Leipzig*, pp. 218–244, in H. Beckert, H. Schumann (Hrsg.), *100 Jahre Mathematisches Seminar der Karl-Marx-Universität, Leipzig*, VEB Deutscher Verlag der Wissenschaften, Berlin, 1981, 340 pages.

[FTW] Frei G., Top J., Walling L., *A Short Biography of B.L. van der Waerden*, Nieuw Archief voor Wiskunde, 4th series, **12** (1994), No. 3, pp. 137–144.

[Fh] Freudenthal H., *Review: B.L. van der Waerden, Science Awakening*, Bulletin of the American Mathematical Society **68** (1962), pp. 543–545.

[Ge] Geyer W.-D., *Bartel Leendert van der Waerden: 2. 2. 1903 – 12. 1. 1996*, Jahrbuch der Bayerischen Akademie der Wissenschaften, Jahr 1996, München, 1997, pp. 229–234.

[HT] Hahn H., Taussky O., *Moderne Algebra. B.L. van der Waerden (Unter Benützung von Vorlesungen von E. Artin und E. Noether.) I. Theil. (Die Grundlehren der mathematischen Wissenschaften, Band XXXIII.) J. Springer, Berlin 1930. VIII und 243 S.*, Monatshefte für Mathematik und Physik **39** (1932), Literaturberichte, pp. A11–A12.

[Hl] Hlawka E., *Bartel Leendert van der Waerden*, Almanach 1995/96, 146. Jahrgang, Österreichische Akademie der Wissenschaften, Wien, 1996, pp. 399–405.

[Ho1] Hogendijk J.P., *B.L. van der Waerden's Detective Work in Ancient and Medieval Mathematical Astronomy*, Nieuw Archief voor Wiskunde, 4th series, **12** (1994), pp. 145–158.

[Ho2] Hogendijk J.P., *Bibliopraphy of publications by B.L. van der Waerden since 1983 on ancient and medieval science*, Historia mathematica **21** (1994), No. 1, pp. 71–72.

[Kn] Knorr W.R., *The Geometer and the Archaeoastronomers: on the Prehistoric Origins of Mathematics*, British Journal for the History of Science **18** (1985), No. 2, pp. 197–212.

[Ma] Mac Lane S., *Van der Waerden's Modern Algebra*, Notices of the American Mathematical Society **44** (1997), No. 3, pp. 321–322.

[Ne] Neuenschwander E., *Waerden, Bartel van der*, pp. 547–551, in J.W. Dauben, C. Scriba (Hrsg.), *Writing the History of Mathematics*, Birkhäuser, Basel, Boston, Berlin, 2002, 689 pages.

[Pe] Peremans W., *Van der Waerden Day in Groningen, May 28, 1993. An Introduction*, Nieuw Archief voor Wiskunde, 4th series, **12** (1994), No. 3, pp. 135–136.

[Sch] Schappacher N., *A Historical Sketch of B.L. van der Waerden's Work on Algebraic Geometry: 1926–1946*, pp. 245–283, in J.J. Gray, K.H. Parshall (eds.), *Episodes in the History of Modern Algebra (1800–1950)*, History of Mathematics, Volume 32, American Mathematical Society, London Mathematical Society, Providence, London, 2007, viii + 335 pages.

[Sm] Schneider M.R., *Zwischen zwei Disziplinen. B.L. van der Waerden und die Entwicklung der Quantenmechanik*, Mathematik im Kontext, Springer, Heidelberg, Dordrecht, London, New York, 2011, xvi + 411 pages.

[Se] Segal S.L., *Mathematicians under the Nazis*, Princeton University Press, Princeton, NJ, 2003, 530 pages.

[SSr1] Siegmund-Schultze R., *Rockefeller and the Internationalization of Mathematics Between the Two World Wars*, Birkhäuser, Basel, 2001, 341 pages.

[SSr2] Siegmund-Schultze R., *Mathematicians Fleeing from Nazi Germany. Individual Fates and Global Impact*, Princeton University Press, Princeton, 2009, 504 pages.

[SSr3] Siegmund-Schultze R., *Bartel Leendert van der Waerden (1903–1996) in Dritten Reich: Moderne Algebra im Dienste des Anti-Modernismus?*, pp. 200–229, in D. Hoffmann, M. Walker (Hrsg.), *'Fremde' Wissenschaftler im Dritten Reich. Die Debye-Affäre im Kontext*, Wallstein Verlag, Göttingen, 2011, 512 pages.

[SSr4] Siegmund-Schultze R., *Van der Waerden in the Third Reich*, Notices American Mathematical Society **62** (2015), pp. 924–929 (Book Review of [So7].).

[So1] Soifer A., *In Search of van der Waerden, Leipzig and Amsterdam, 1931–1951. I. Leipzig*, Geombinatorics **14** (2004), No. 1, pp. 21–40.

[So2] Soifer A., *In Search of van der Waerden, Leipzig and Amsterdam, 1931–1951. II. Amsterdam, 1945*, Geombinatorics **14** (2004), No. 2, pp. 72–102.

[So3] Soifer A., *In Search of van der Waerden, Leipzig and Amsterdam, 1931–1951. III. Amsterdam, 1946–1951*, Geombinatorics **14** (2005), No. 3, pp. 124–161.

[So4] Soifer A., *In Search of van der Waerden, the Early Years*, Geombinatorics **16** (2007), No. 3, pp. 305–342.

[So5] Soifer A., *The Drama of Bartel Leendert van der Waerden*, Congressus Numerantium **192** (2008), pp. 53–63.

[So6] Soifer A., *The Mathematical Coloring Book: Mathematics of Coloring and the Colorful Life of Its Creators*, Springer, New York, 2009, 607 pages.

[So7] Soifer A., *The Scholar and the State: In Search of Van der Waerden*, Birkhäuser, Basel, 2015, 471 pages (See also Zbl 062343963.).

[St] Struik D.J., *Review of van der Waerden's Ontwakende wetenschap: Egyptische, Babylonische en Griekse wiskunde*, Mathematical Reviews **12** (1951), p. 381.

[Ta] Taussky O., *Moderne Algebra. B.L. van der Waerden. Bd. II.* (*Grundlehren der mathematischen Wissenschaft, Bd. 34, J. Springer, Berlin 1931*), Monatshefte für Mathematik und Physik **40** (1933), Literaturberichte, pp. A3–A4.

[Th1] Thiele R., *Van der Waerdens Leipziger Jahre 1931–1945*, Mitteilungen der Deutschen Mathematiker-Vereinigung, 2004, No. 1, pp. 8–20.

[Th2] Thiele R., *Van der Waerden in Leipzig*, EAGLE 36. EAGLE-Essay, Edition am Gutenbergplatz, Leipzig, 2009, 155 pages.

[TW] Top J., Walling L., *Bibliography of B.L. van der Waerden*, Nieuw Archief voor Wiskunde, 4th series, **12** (1994), No. 3, pp. 179–193.

[Wa1] van der Waerden B.L., *Meine Göttinger Lehrjahre. Gastvortrag in der Algebravorlesung gehalten am 26. 1. 1979 in Heidelberg. Mit einem Nachwort von Peter Roquette*, Mitteilungen der Deutschen Mathematiker-Vereinigung, 1997, No. 2, pp. 20–27.

[Wa2] van der Waerden B.L., *The Foundation of Algebraic Geometry from Severi to André Weil*, Archive for History of Exact Sciences **7** (1971), pp. 171–180.

[Wa3] van der Waerden B.L., *On the Sources of My Book Moderne Algebra*, Historia Mathematica **2** (1975), pp. 31–40.

[Wa4] van der Waerden B.L., *The School of Hilbert and Emmy Noether*, Bulletin of the London Mathematical Society **15** (1983), No. 1, pp. 1–7.

www pages

[W1] http://www-groups.dcs.st-andrews.ac.uk/~history/Biographies/Van_der_Waerden.html [9.9.2019].

[W2] http://de.wikipedia.org/wiki/Bartel-Leendert-van-der-Waerden [9.10.2019].

[W3] Neuenschwander E., *Bartel Leendert van der Waerden, Historisches Lexikon der Schweiz*, http://www.hls-dhs-dssch/textes/d/D43138.php [2.7.2019].