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Proc. Nat. Acad. Sci. U.S.A. 22 (1936), 110-111

Persistent URL: http://dml.cz/dmlcz/501041

particular interest. Let X, Y be two symbols of a jacobian system. If a is a function of the independent variables, we have

$$(aX, aY) = (aXa)Y - (aYa)X.$$

Hence, *if the equations of a jacobian system are all multiplied by the same non-zero function, the system remains in nested form.* In fact, every one of its subsystems constitutes a complete system. Therefore, *this nested form does not depend upon the order in which the equations are written.*

Now let a be a non-vanishing determinant of order r formed from the coefficients of the X's. If (1) is solved for the corresponding set of r derivatives and the resulting equations are multiplied by a, we have a simple reduction to nested form in the coefficient *ring*, whereas reduction to jacobian form by solution is performed in the coefficient *field*. In this way, cumbersome denominators and the resulting singularities may be avoided.

It would be of interest to know whether a complete system can be put in jacobian form in the coefficient ring.

[1] Pfeiffer, G., "La généralization de la méthode de Jacobi," *Acta Math.*, **61**, 203–238 (1933). This paper had previously appeared in Russian, cf. *Zentralblatt für Math.*, **3**, 397 (1932). In a companion paper, *Acta Math.*, **61**, 239–261 (1933), Pfeiffer shows how the application of Jacobi's second method to non-linear systems in a special form leads to a nested system.

[2] Hoborski, A., "Über vollständige Systeme," *Prace Mat.*, **41**, 55–63 (1934).

[3] Hoborski's treatment in this particular is much the simpler.

ON GENERAL MANIFOLDS

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Communicated December 26, 1935

Let G be an abelian group. Let $0 \leqq p \leqq n$. A topological space R will be called an absolute orientable n-manifold of rank p over G, if it satisfies the following axioms:

I. R is a bicompact space.

II. $\dim R = n$.

III. There exists an absolute (n, R)-cycle over G, which is not ~ 0.

IV. If $S \neq R$ is a closed subset of R, then every absolute (n, S)-cycle over G is ~ 0 over G.

V. If U is a given neighborhood of a given point x of R, there exists a neighborhood $V \subset U$ of x having the following property: If C^n is an

(n, R)-cycle mod $R - U$ over G, then there exists an absolute (n, R)-cycle Ω^n over G such that $C^n \sim \Omega^n$ mod $R - V$.

VI^q ($p \leqq q \leqq n - 1$). If U is a given neighborhood of a given point x of R, there exists a neighborhood $V \subset U$ of x such that every (q, R)-cycle mod $R - U$ over G is ~ 0 mod $R - V$.

This definition is complete if G is either a bicompact group or a field. In other cases the cycles over G may have paradoxical properties and it is necessary to add a further axiom excluding this; as a matter of fact, it is sufficient to add an axiom excluding paradoxical properties of absolute (n, R)-cycles.

The most important cases arise when G is the group of all real numbers mod 1; if our axioms hold true for this particular group, they automatically hold true for any G whatever. Besides, in this case axiom III is a consequence of the remaining ones.

An absolute n-manifold (orientable or not) of rank p over G is a bicompact space R having the following property: Any point x possesses a neighborhood U such that the space obtained from R by considering the whole set $R - U$ as a single point is an absolute orientable n-manifold of rank p over G.

In my earlier theory of manifolds (*Annals of Mathematics*, **1933,** 621–730 and **1934,** 685–693) I was obliged to assume that G is a field. Moreover, I had three more axioms. *Firstly,* I had supposed that R has the property that every closed subset is a G_δ, which was a strong restriction. *Secondly,* I had assumed that the highest Betti number is equal to 1, which can be *proved* in the case $p = 0$ and is an unnecessary restriction in the general case. *Thirdly,* I had the following axiom:

VII^q ($p \leqq q \leqq n$). If U is a given neighborhood of a given point x of R, there exists a neighborhood $V \subset U$ of x such that every absolute (n, R)-cycle situated in $\overline{V}$ is ~ 0 in $\overline{U}$; now I can prove that this follows from other axioms.

The basic duality theorem for an absolute orientable n-manifold of rank p over G is: The pth absolute Betti group over G of R is isomorphic with the $(n - p)$th absolute dual Betti group over H of R, where H designates the nth absolute Betti group over G of R. The dual $(n - p)$ th Betti group over H is the character group of the ordinary $(n - p)$th Betti group over the character group of H, but it may be easily defined directly.