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RIEMANN PROBLEM SOLUTIONS THAT ARE STABLE TO PERTURBATION

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ABSTRACT. For a system of two conservation laws in one space dimension, we consider Riemann problem solutions that are stable to perturbation of the Riemann data. In other words, if the left state, right state, and flux function are perturbed, the new Riemann problem solution should contain the same sequence of wave types as the old. A large class of such solutions is identified, some of which contain wave types that have not previously appeared in the literature.

A *system of two conservation laws in one space dimension* is a partial differential equations of the form

$$U_t + F(U)_x = 0 \tag{1}$$

with $U \in \mathbb{R}^2$, $F : \mathbb{R}^2 \to \mathbb{R}^2$ a smooth map, $t \in \mathbb{R}$, and $x \in \mathbb{R}$. Such equations arise in the study of many physical systems, for example, gas dynamics [CF], three-phase flow [ShSchaMP-L], elastic strings [KK], and phase transitions [J, Sl]. A good general reference is [Sm].

For both theoretical and numerical purposes, the most basic initial value problem for (1) is the *Riemann problem*, in which the initial data are piecewise constant with a single jump at $x = 0$:

$$U(x,0) = \begin{cases} U_L\,, & x < 0\,, \\ U_R\,, & x > 0\,. \end{cases} \tag{2}$$

The solution of a Riemann problem is constant on lines through the origin: it is a function $U\left(\frac{x}{t}\right)$ consisting of constant parts, smoothly changing parts (rarefaction waves), and jumps (shock waves); see Figure 1. Shock waves occur when

$$\lim_{\frac{x}{t} \to s-} U\left(\frac{x}{t}\right) = U_- \neq U_+ = \lim_{\frac{x}{t} \to s+} U\left(\frac{x}{t}\right), \tag{3}$$

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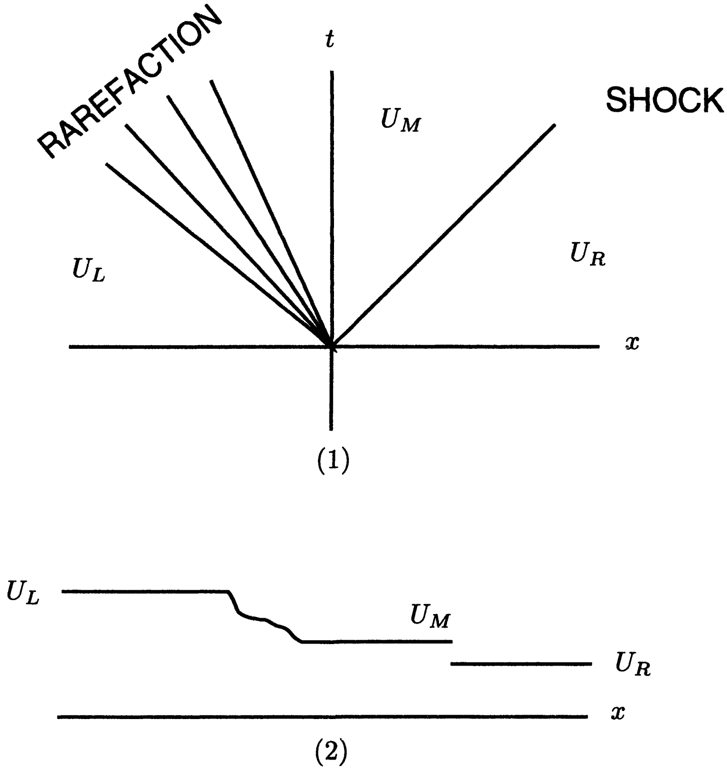


FIGURE 1. A Riemann problem solution (1) in the xt-plane, and (2) in profile for fixed t.

and one must decide which discontinuities (U_-, s, U_+) to allow. It is well-known that requiring only that $U\left(\frac{x}{t}\right)$ be a weak solution of (1), (2) allows multiple solutions of Riemann problems, including clearly nonphysical ones.

Various shock admissibility criteria are used to remedy this situation. Perhaps the most widely accepted is the *viscous profile criterion* [CF, G]. Suppose the equation (1) arises by ignoring the small viscous term in the parabolic equation

$$U_t + F(U)_x = \varepsilon D U_{xx} . \tag{4}$$

Then the viscous profile criterion states that the discontinuity (3) is admissible in solutions of (1), (2) if and only if the parabolic equation (4) has a traveling wave solution $U\left(\frac{x-st}{\varepsilon}\right)$ with

$$\lim_{\xi\to\pm\infty} U(\xi) = U_\pm , \quad \lim_{\xi\to\pm\infty} U'(\xi) = 0 .$$

This amounts to requiring that the ordinary differential equation

$$DU' = F(U) - F(U_-) - s(U - U_-) \tag{5}$$

have a heteroclinic solution $U(\xi)$ from the equilibrium U_- to a second equilibrium U_+.

In simple cases the viscous profiles criterion coincides with the more easily used entropy criterion of L a x [La], and with its generalization due to L i u [Li]. However, the viscous profile criterion allows, for example, shock waves that correspond to saddle-to-saddle connections of (5), which do not satisfy Lax's or Liu's criterion. Recent work strongly supports admitting these nonclassical shock waves: they are sometimes needed to solve Riemann problems [ShSchaMP-L, Sh, IMP, ScheSh]; they arise, apparently stably, in numerical calculations [ZPM]; and they can sometimes be proved to be time-asymptotically stable solutions of (4) [LZ]. We therefore adopt the viscous profile shock admissibility criterion, and make the further simplification $D = I$.

Let us discuss rarefactions and shocks in more detail.

Let

$$\mathcal{U} = \left\{U \in \mathbb{R}^2 : DF(U) \text{ has distinct real eigenvalues}\right\}, \tag{6}$$

the "strictly hyperbolic" region. For $U \in \mathcal{U}$, let $\lambda_1(U) < \lambda_2(U)$ denote the eigenvalues of $DF(U)$, and let $\ell_i(U)$, $r_i(U)$ denote corresponding left and right eigenvectors with $\ell_i(U)\, r_j(U) = \delta_{ij}$.

A *rarefaction* of *type* R_i is a differentiable map $U : [a,b] \to \mathcal{U}$ such that $U'(\xi)$ is a multiple of $r_i\big(U(\xi)\big)$ for $a \leq \xi \leq b$, and $\xi = \lambda_i\big(U(\xi)\big)$. The definition implies that if $U = U(\xi)$ for some ξ in $[a,b]$, then

$$D\lambda_i(U)\, r_i(U) = \ell_i(U) D^2F(U)\big(r_i(U),\, r_i(U)\big) \neq 0\,.$$

It also implies that if $U_- = U(a)$ and $U_+ = U(b)$, then $\lambda_i(U_-) < \lambda_i(U_+)$. We define the *speed* s of a rarefaction as follows: for a rarefaction of type R_1, $s = \lambda_1(U_+)$; for a rarefaction of type R_2, $s = \lambda_2(U_-)$.

For $U_\pm \in \mathcal{U}$, there is a *shock* from U_- to U_+ with *speed* s provided the ordinary differential equation

$$\dot{U} = F(U) - F(U_-) - s(U - U_-) \tag{7}$$

has an equilibrium at U_+ and a heteroclinic solution from U_- to U_+. (Recall that we are taking $D = I$ in (4) and (5).)

For any equilibrium $U \in \mathcal{U}$ of (7), note that the eigenvalues of the linearization at U are $\lambda_i(U) - s$. We shall use the following terminology for equilibria $U \in \mathcal{U}$ of (7):

Name	Symbol	Eigenvalues	
Repeller	R	+	+
Repeller–Saddle	RS	0	+
Saddle	S	−	+
Saddle–Attractor	SA	−	0
Attractor	A	−	−

For $U_\pm \in \mathcal{U}$, an *elementary wave* with speed s from U_- to U_+,

$$w : U_- \xrightarrow{s} U_+ ,$$

is a rarefaction or a shock. The type of a rarefaction (R_1 or R_2) has already been defined; a shock is of *type* $R \cdot S$ if it is represented by a heteroclinic orbit from a repeller to a saddle, etc. There are 16 types of shocks with $U_\pm \in \mathcal{U}$ (a shock cannot start at an attractor, nor end at a repeller).

Associated with each elementary wave is a *speed interval* σ: for a shock of speed s, $\sigma = [s, s]$; for a rarefaction of type R_i, $\sigma = [\lambda_i(U_-), \lambda_i(U_+)]$. If σ_1 and σ_2 are intervals, we write $\sigma_1 \leq \sigma_2$ if $s_1 \leq s_2$ for every $s_1 \in \sigma_1$ and $s_2 \in \sigma_2$.

A *Riemann problem solution* for (1), (2), with U_L, $U_R \in \mathcal{U}$, is a sequence of elementary waves

$$U_0 \xrightarrow{s_1} U_1 \xrightarrow{s_2} \cdots \xrightarrow{s_n} U_n \tag{8}$$

with $U_0 = U_L$, $U_n = U_R$, and

$$\sigma_1 \leq \sigma_2 \leq \cdots \leq \sigma_n ; \tag{9}$$

here σ_i is the speed interval of the ith wave of (8). The reason the speed intervals must form a nondecreasing sequence can be seen from Figure 1. A sequence of elementary waves (8) is *allowed* provided it satisfies (9) and in addition no two successive waves are rarefactions of the same type. There is no loss of generality in requiring that Riemann problem solutions be allowed sequences of elementary waves, and we shall do so.

We shall sometimes denote the wave sequence (8) by $(w_1, \ldots, w_n)$, where w_i is $U_{i-1} \xrightarrow{s_i} U_i$.

In the literature, Riemann problem solutions are usually pictured by fixing U_L and drawing the U_R–plane, which is divided into regions in which different types of solutions occur. The classical work of L a x [La], which treats U_R close to U_L, leads to Figure 2. If $U_R = U_L$ (the dot at the center of the picture), the solution is constant. If U_R lies on one of the curves drawn through U_L, the solution contains a single wave: a 1– or 2–rarefaction (R_1 or R_2), a 1–shock

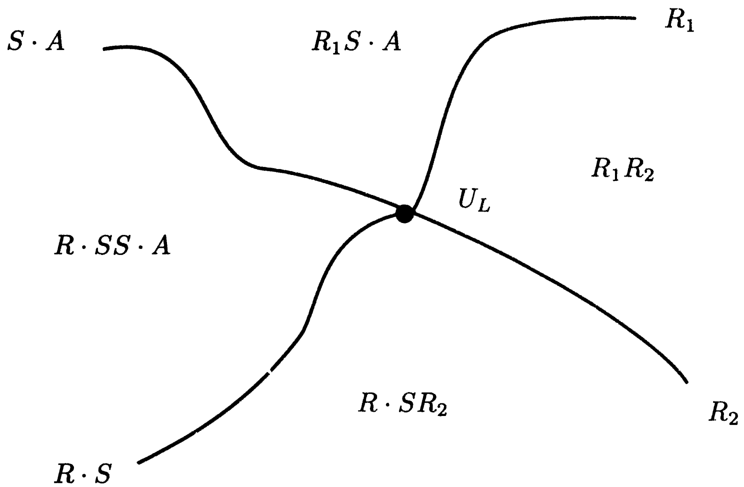


FIGURE 2. Different Riemann problem solutions for fixed U_L in a neighborhood of $U_R = U_L$ in the U_R-plane.

$(R \cdot S)$, or a 2–shock $(S \cdot A)$. If U_R lies in one of the open regions separated by the curves, the Riemann problem solution has two waves, as indicated.

This figure is the starting point for the literature on Riemann problems. Far more complicated diagrams arise in the literature, and there is at present a desire among workers in the field for organizing principles that will bring some order to the profusion of examples.

The approach to Riemann problem solutions that we sketch here can be explained in the context of Figure 2. This figure can be viewed as a bifurcation diagram. If U_R lies in one of the open regions, the Riemann problem solution is stable to perturbation, in the sense that if we vary U_L, U_R, and F a little, the Riemann problem solution is a sequence of the same number of waves, of the same types. (This notion of stability is in principle independent of time-asymptotic stability.) Points U_R on the curves through $U_R = U_L$ in Figure 2 correspond to codimension one bifurcations of the Riemann problem solution. At the point $U_R = U_L$ there is a codimension two bifurcation.

In bifurcation theory or singularity theory, one normally analyses first the stable problems, then the codimension one problems, etc. From this point of view, the classical approach to Riemann problems, which takes as its starting point the codimension two Riemann problem $U_R = U_L$, is somewhat perverse. We therefore propose to restart the study of Riemann problem solutions at the codimension zero solutions.

In order to define stability to perturbation of a Riemann problem solution more precisely, let

$$U_0^* \xrightarrow{s_1^*} U_1^* \xrightarrow{s_2^*} \cdots \xrightarrow{s_n^*} U_n^* \tag{10}$$

be a Riemann problem solution for

$$U_t + F^*(U)_x = 0\,.$$

Let $K \subset \mathbb{R}^2$ be a compact set such that

(1) $U_i^* \in \operatorname{Int} K, \quad i = 0, \ldots, n;$

(2) For $i = 1, \ldots, n$, the differential equation

$$\dot{U} = F^*(U) - F^*(U_{i-1}^*) - s_i^*(U - U_{i-1}^*)$$

has a heteroclinic solution from U_{i-1}^* to U_i^* that lies in Int K.

Let $\mathscr{B}$ be the space of C^k functions $F : K \to \mathbb{R}^2$, with the C^k norm, $k \geq 2$. $\mathscr{B}$ is a Banach space. In the following we will think of F^* as an element of $\mathscr{B}$, but the results will not depend on the choices of K and k.

We shall say that (10) is *stable to perturbation* if there are neighborhoods $\mathcal{U}_i$ of U_i^*, $\mathcal{I}_i$ of s_i^*, $\mathcal{F}$ of F^*, and a smooth map

$$G\colon \mathcal{U}_0 \times \mathcal{I}_1 \times \mathcal{U}_1 \times \cdots \times \mathcal{I}_n \times \mathcal{U}_n \times \mathcal{F} \to \mathbb{R}^{3n-2}$$

such that

(P1) $G(U_0, s_1, U_1, \ldots, s_n, U_n, F) = 0$ implies that

$$U_0 \xrightarrow{s_1} U_1 \xrightarrow{s_2} \cdots \xrightarrow{s_n} U_n$$

is a Riemann problem solution for

$$U_t + F(U)_x = 0\,,$$

with successive waves of the same types as those of (10);

(P2) $DG(U_0^*, s_1^*, \ldots, s_n^*, U_n^*, F^*)$, restricted to the space of vectors $(\dot{U}_0, s_1, \ldots, \dot{s}_n, \dot{U}_n, \dot{F})$ with $\dot{U}_0 = \dot{U}_n = 0$ and $\dot{F} = 0$ (which has dimension $3n - 2$), is an isomorphism onto $\mathbb{R}^{3n-2}$.

The map G will be said to *exhibit* the stability to perturbation of (10). Of course (P2) implies, by the implicit function theorem, that $G^{-1}(0)$ is a graph over $\mathcal{U}_0 \times \mathcal{U}_n \times \mathcal{F}$.

If $U_-^* \xrightarrow{s^*} U_+^*$ is an elementary wave of type T for $U_t + F^*(U)_x = 0$ that satisfies some nondegeneracy conditions, then there are neighborhoods $\mathcal{U}_\pm$ of $U_\pm^*$, $\mathcal{I}$ of s^*, $\mathcal{F}$ of F^*, and a map $G_T : \mathcal{U}_- \times \mathcal{I} \times \mathcal{U}_+ \times \mathcal{F} \to \mathbb{R}^e$ (e depends only on the type T of the wave) such that $G_T(U_-, s, U_+, F) = 0$ if and only if $U_- \xrightarrow{s} U_+$ is an elementary wave of type T for $U_t + F(U)_x = 0$. The system $G_T = 0$ is a set of *defining equations* for waves of type T. Figure 3 shows phase portraits for several types of shock waves, and their defining equations.

In (10), let $T(i)$ denote the type of the ith wave. We consider Riemann problem solutions whose stability to perturbation is exhibited by a map G of the form $G = (G_1, \ldots, G_n)$, where

$$G_i(U_0, s_1 \ldots, s_n, U_n, F) = G_{T(i)}(U_{i-1}, s_i, U_i, F).$$

Here $G_{T(i)} = 0$ is a set of defining equations for waves of type T in a neighborhood of (U_{i-1}^*, s^*, U_i^*), so $G_i = 0$ implies that the ith wave $U_{i-1} \xrightarrow{s_i} U_i$ is of the correct type. We conjecture that if a Riemann problem solution is stable to perturbation, then the stability can be exhibited by a map G of this form.

Suppose G_i maps into $\mathbb{R}^{e(i)}$ (i.e., the number of equations required to define a wave of type $T(i)$ is $e(i)$). Then in view of the requirement that G map into $\mathbb{R}^{3n-2}$, a necessary condition for $G = (G_1, \ldots, G_n)$ to exhibit the stability to perturbation of (10) is

$$\sum_{i=1}^{n} e(i) = 3n - 2,$$

i.e.,

$$\sum_{i=1}^{n} \big(3 - e(i)\big) = 2. \tag{11}$$

We are therefore led to define the *Riemann number* of an elementary wave w to be

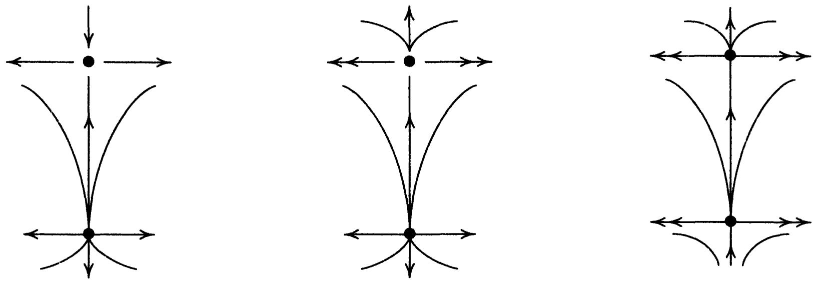
$$\rho(w) = 3 - e(w),$$

where $e(w)$ is the number of equations needed to define a wave of the type of w. From (11) we are led to concentrate our attention on allowed sequences of elementary waves $(w_1, \ldots, w_n)$ with $\sum_{i=1}^{n} \rho(w_i) = 2$.

For a rarefaction, $\rho = 1$, while for a shock, ρ is given by Table 1.

From the definition of an allowed sequence of elementary waves, such a sequence can contain only the wave successions given in Table 2.

Some of these wave successions apparently do not occur in Riemann problem solutions that are stable to perturbation. The wave successions in Table 3 are termed *good*.



R·S

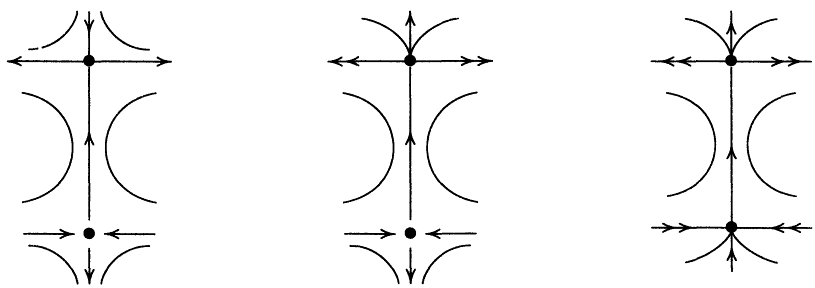
$$F(U_+)-F(U_-)-s(U_+-U_-)=0$$

R·RS

$$F(U_+)-F(U_-)-s(U_+-U_-)=0$$
$$\lambda_1(U_+)-s=0$$

RS·RS

$$F(U_+)-F(U_-)-s(U_+-U_-)=0$$
$$\lambda_1(U_+)-s=0$$
$$\lambda_1(U_-)-s=0$$



S·S

$$F(U_+)-F(U_-)-s(U_+-U_-)=0$$
$$S(U_-,s)=0$$

S·RS

$$F(U_+)-F(U_-)-s(U_+-U_-)=0$$
$$S(U_-,s)=0$$
$$\lambda_1(U_+)-s=0$$

SA·RS

$$F(U_+)-F(U_-)-s(U_+-U_-)=0$$
$$S(U_-,s)=0$$
$$\lambda_1(U_+)-s=0$$
$$\lambda_1(U_-)-s=0$$

FIGURE 3. Types of shocks and defining equations. The lower equilibrium is U_-, the upper is U_+. S is a "separation function" that represents the separation between invariant manifolds of U_- and U_+.

TABLE 1. Riemann numbers of shock waves

from	to	RS	S	SA	A
R		0	1	0	1
RS		−1	0	−1	0
S		−1	0	0	1
SA		−2	−1	−1	0

TABLE 2. Wave successions in allowed sequences of elementary waves

w_i	w_{i+1}	R_1	$RS \cdot *$	$S \cdot *$	$SA \cdot *$	R_2
R_1			✓	✓	✓	✓
$* \cdot RS$		✓	✓	✓	✓	✓
$* \cdot S$				✓	✓	✓
$* \cdot SA$					✓	✓
R_2					✓	

TABLE 3. Good wave successions

w_i	W_{i+1}
R_1	$RS \cdot RS$, $RS \cdot S$, $S \cdot *$, R_2
$* \cdot RS$	R_1
$S \cdot SA$, $SA \cdot SA$	R_2
$* \cdot S$	$S \cdot *$, R_2
R_2	$SA \cdot *$

We can now state

THEOREM 1. *Let $(w_1, \ldots, w_n)$ be an allowed sequence of elementary waves. Then*

(1) $\sum_{i=1}^{n} \rho(w_i) \leq 2$.

(2) $\sum_{i=1}^{n} \rho(w_i) = 2$ *if and only if*

(a) *all wave successions are good,*
and

(b) *w_1 is of type $R \cdot RS$, $R \cdot S$ or R_1; w_n is of type $SA \cdot A$, $S \cdot A$, or R_2.*

Theorem 1 can be proved by induction on n, using nothing more than Tables 1, 2, and 3.

We now give a more conceptual description of the allowed sequences of elementary waves with $\sum_{i=1}^{n} \rho(w_i) = 2$. First we state some more definitions.

A *1–wave group* is either a single $R \cdot S$ wave, or a sequence of elementary waves of the form

$$(R \cdot RS)(R_1\ RS \cdot RS) \cdots (R_1\ RS \cdot RS)\, R_1\, (RS \cdot S)$$

where the terms in parentheses are optional.

A *transitional wave group* is either a single $S \cdot S$ wave, or a sequence of elementary waves of the form

$$S \cdot RS(R_1\ RS \cdot RS) \cdots (R_1\ RS \cdot RS)\, R_1\, (RS \cdot S)\,, \tag{12}$$

or of the form

$$(S \cdot SA)\, R_2\, (SA \cdot SA\, R_2) \cdots (SA \cdot SA\, R_2)\, SA \cdot S\,, \tag{13}$$

where in cases (12) and (13) the terms in parentheses are optional.

A *2–wave group* is either a single $S \cdot A$ wave, or a sequence of elementary waves of the form

$$(S \cdot SA)\, R_2\, (SA \cdot SA\, R_2) \cdots (SA \cdot SA\, R_2)\, (SA \cdot A)\,,$$

where the terms in parentheses are optional.

With these definitions, we have

THEOREM 2. *Let* (10) *be an allowed sequence of elementary waves with* $\sum_{i=1}^{n} \rho(w_i) = 2$.

(*1*) *Suppose* (10) *includes no* $SA \cdot RS$ *waves. Then* (10) *consists of one 1–wave group, followed by an arbitrary number of transitional wave groups (in any order), followed by one 2–wave group.*

(*2*) *Suppose* (10) *includes* $m \geq 1$ $SA \cdot RS$ *waves. Then they separate* $m + 1$ *wave sequences* $g_0, \ldots, g_m$. *Each* g_i *is exactly as in* (1), *except:*

   (*a*) *If* $i < m$, *the last wave in the group is* R_2.

   (*b*) *If* $i > 0$, *the first wave in the group is* R_1.

The condition $\sum_{i=1}^{n} \rho(w_i) = 2$ simply ensures that the map $G = (G_1, \ldots, G_n)$ maps into $\mathbb{R}^{3n-2}$. In order to ensure that G also satisfies (P1) and (P2), we impose three additional types of conditions:

(1) On each wave we impose certain *wave nondegeneracy conditions.*

(2) In the absence of $SA \cdot RS$ waves, we impose one *wave group interaction condition* on how the different wave groups are related. If there are $m \geq 1$ $SA \cdot RS$ waves, we impose $m+1$ wave group interaction conditions, one on each of the $m+1$ wave sequences $g_0, \dots, g_m$. Roughly speaking, these conditions say that certain wave curves are transverse.

(3) If w_i is a $* \cdot S$ wave and w_{i+1} is an $S \cdot *$ wave, we require that $s_i < s_{i+1}$.

Then we have

THEOREM 3. *Let (10) be an allowed sequence of elementary waves with* $\sum_{i=1}^{n} \rho(w_i) = 2$. *Assume:*

(H1) *Each wave satisfies the appropriate wave nondegeneracy conditions.*

(H2) *The wave group interaction conditions are satisfied.*

(H3) *If w_i is a $* \cdot S$ wave and w_{i+1} is an $S \cdot *$ wave, then $s_i < s_{i+1}$.*

Then (10) is stable to perturbation.

Proofs of Theorems 1, 2, and 3 will be given in a forthcoming paper with Brad Plohr and Dan Marchesin.

The classical approach to Riemann problems implicitly poses the following question: if step 1 is to understand the codimension two bifurcation at $U_R = U_L$, what is step 2? The literature provides various possible answers: (1) extend the wave curves (i.e., the codimension one bifurcation curves in Figure 1) through various subsequent codimension two bifurcations [W]; (2) identify classes of flux functions F for which some analog of Lax's construction works [SmJ, Li]; (3) study "interesting" examples [ShSchaMP-L]. Of course, there is no obvious step 2.

In contrast, our approach does have an obvious step 2: analyze how the Riemann problem solution bifurcates when exactly one of the assumptions that lead to stability is violated. This program provides an organized approach to understanding codimension one Riemann problem solutions, such as the one-wave solutions in Figure 2. I am presently working on this program with Plohr and Marchesin; here we only remark that many of the codimension one situations can lead to two solutions or no solution of nearby Riemann problems. The significance of such multiple solutions is a completely open problem.

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