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STRICT φ-DISCONJUGACY OF N-TH ORDER LINEAR DIFFERENTIAL EQUATIONS WITH DELAYS*

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Abstract. A generalization of the strict disconjugacy (of n-th order linear differential equations with delays) is given. It is shown that for a class of vector function $\boldsymbol{\varphi}$ the interval of strict disconjugacy of each differential equation does not degenerate into a one-point set. The relation between strict $\boldsymbol{\varphi}$-disconjugacy and the existence of solutions of multipoint boundary value problems is discussed.

Key words. Linear differential equations with delays, initial value problem for differential equations with delays, multipoint boundary value problem for linear differential equation with delays

AMS subject classifications. 34C10, 34K10

Disconjugate differential equations play an important role in the theory of ordinary differential equations. There is an extensive literature on this topic (see, e.g. [1]). The notion of disconjugacy for differential equations with delay was introduced in [3], [4] and then it was generalized for vector differential equations with delays (see [6]), differential inclusion with delay (see [8], [14]) and differential equations of neutral type (see [7]). The generalized disconjugacy (strict $\boldsymbol{\varphi}$-disconjugacy) of differential equation with delay was introduced in [9] for second order differential equations of the form

$$x'' + N(t)x(t) + M(t)x(t - \Delta(t)) = 0.$$

The purpose of this paper is to generalize the notions of conjugate points and strictly disconjugate differential equation with delays, to show that the interval of generalized disconjugacy (strict $\boldsymbol{\varphi}$-disconjugacy) of each n-order linear differential equation with delays does not degenerate into one-point set and to show the connection between the strict $\boldsymbol{\varphi}$-disconjugacy and the solvability of a multipoint boundary value problem.

Let us consider the n-th order linear differential equation with delays

$$x^{(n)}(t) + \sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}(t)x^{(n-i)}\big(t - \Delta_{ij}(t)\big) = 0, \quad n \geq 1, \tag{1}$$

with continuous coefficients $a_{ij}(t)$ and delays $\Delta_{ij}(t) \geq 0$ on an interval $\mathbf{I}=\langle t_0, T)$, $T \leq +\infty$, $(i = 1, \ldots, n\,; \;\; j = 1, \ldots, m)$.

The fundamental initial value problem (**FIVP**) for equation (1) is defined as follows:

Let $a \in \langle t_0, T)$ and let a continuous initial value vector function $\mathbf{\Phi}(t) = \big(\phi_0(t), \ldots, \phi_{n-1}(t)\big)$ be given on the initial set $E_a := \bigcup_{i=1}^{n}\bigcup_{j=1}^{m} E_a^{ij} \cup \{a\}$,
where $E_a^{ij} := \{t - \Delta_{ij}(t) : \; t - \Delta_{ij}(t) < a\,, \; t \in \mathbf{I}\}$, $\;i = 1, \ldots, n\,; \; j = 1, \ldots, m$.

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We have to find the solutions $x(t) \in \mathbf{C}^{\mathbf{n}}(\mathbf{I})$ of equation (1) satisfying initial value conditions:

$$\begin{aligned} & x^{(k)}(a) = \phi_k(a) = x_a^k\,, \qquad \left(x_a^0, x_a^1, \dots, x_a^{n-1}\right) \neq (0,0,\dots,0) \\ & x^{(k)}\left(t - \Delta_{ij}(t)\right) = \phi_k\big(t - \Delta_{ij}(t)\big)\,, \qquad \text{if } \ t - \Delta_{ij}(t) < a \\ & (\, k = 0,1,\dots,n-1\,;\ i = 1,\dots,n\,;\ j = 1,\dots,m\,)\,. \end{aligned} \tag{2}$$

By the derivative $x^{(k)}(a)$, $k = 1,\dots,n-1$ at the point a of the interval $\langle a, T)$ we shall mean the right-hand point derivative and instead $x^{(k)}(a+0)$, we shall simply write $x^{(k)}(a)$.

Under the above assumptions the **FIVP** (1), (2) has exactly one solution defined on $\langle a, T)$ (see [5], [10], [11]), which we shall denote by $x_\Phi\big(t, a, x_a^0, x_a^1, \dots, x_a^{n-1}\big)$.

Besides **FIVP** for (1) we shall consider the homogenous initial value problem (**HIVP**): Let $a \in \langle t_0, T)$ and let a bounded continuous vector function $\mathbf{\Phi}(t) = \big(\phi_0(t), \dots, \phi_{n-1}(t)\big)$,

$$\phi_k(a) = 1 \qquad (k = 0,1,\dots,n-1) \tag{3}$$

be defined on the initial set E_a.

Let x_a^k $(k = 0,1,\dots,n-1)$ be arbitrary real numbers. We have to find the solution $x(t)$ of (1) satisfying:

$$\begin{aligned} & x^{(k)}(a) = x_a^k\,, \qquad \left(x_a^0, x_a^1, \dots, x_a^{n-1}\right) \neq (0,0,\dots,0) \\ & x^{(k)}\big(t - \Delta_{ij}(t)\big) = x_a^k\, \phi_k\big(t - \Delta_{ij}(t)\big), \qquad \text{if } \ t - \Delta_{ij}(t) < a \\ & \big(k = 0,1,\dots,n-1\,;\ i = 1,\dots,n\,;\ j = 1,\dots,m\big). \end{aligned} \tag{4}$$

As a consequence of the existence and uniqueness theorem for **FIVP** we have the existence and uniqueness theorem for **HIVP** (see [12, Theorem 1]).

Remark 1. If the initial vector function $\mathbf{\Phi}$ is fixed, then the set of all solutions of the **HIVP** (1), (4) is an n-dimensional vector space which we shall denote by $V_\Phi^n(a)$. The base of $V_\Phi^n(a)$ are any n solutions $u_1(t), \dots, u_n(t) \in V_\Phi^n(a)$ such that

$$W\big(u_1(a), \dots, u_n(a)\big) = \begin{vmatrix} u_1(a) & \dots & u_n(a) \\ u_1'(a) & \dots & u_n'(a) \\ \dots & \dots & \dots \\ u_1^{(n-1)}(a) & \dots & u_n^{(n-1)}(a) \end{vmatrix} \neq 0$$

(see [11, pp. 68]).

Let us consider the following **HIVP**:
Let $\boldsymbol{\varphi}(t) = \big(\varphi_0(t), \dots, \varphi_{n-1}(t)\big)$ be a bounded continuous vector function such that

$$\begin{aligned} & \varphi_k : (-\infty, t_0\rangle \longrightarrow \mathbb{R}, \\ & \varphi_k(t_0) = 1\,, \\ & \big|\varphi_k(t)\big| \le B_k, \quad t \in (-\infty, t_0\rangle, \\ & \big(k = 0,1,\dots,n-1\big). \end{aligned} \tag{5}$$

Let $a \in \langle t_0, T)$ and $x_a^k \in \mathbb{R}$ $\big(k = 0, 1, \ldots, n-1\big)$. We have to find the solutions $x(t)$ of (1) satisfying

$$\begin{aligned} & x^{(k)}(a) = x_a^k\,, \quad \big(x_a^0, x_a^1, \ldots, x_a^{n-1}\big) \neq (0, 0, \ldots, 0) \\ & x^{(k)}\big(t - \varDelta_{ij}(t)\big) = x_a^k\, \varphi_k\big(t - \varDelta_{ij}(t) - a + t_0\big), \quad \text{if } t - \varDelta_{ij}(t) < a \\ & \big(\, k = 0, 1, \ldots, n-1\,;\; i = 1, \ldots, n\,;\; j = 1, \ldots, m\,\big)\,. \end{aligned} \tag{6}$$

By Remark 1 to any $a \in \langle t_0, T)$ the n-dimensional vector space $V_{\boldsymbol{\varphi}}^n(a)$ of solutions **HIVP** (1), (5), (6) is associated.

Let $x(t) \in V_{\boldsymbol{\varphi}}^n(a)$, $x(t) \not\equiv 0$ on interval $\langle a, T)$. The n-th consecutive zero (including multiplicity) of $x(t)$, to the right of a will be denoted by $\boldsymbol{\eta(x, a)}$.

Definition 1. Let $a \in \langle t_0, T)$. By the **adjoint point** to the point a with respect to (1) and $\boldsymbol{\varphi}$ we shall mean the point

$$\boldsymbol{\alpha(a)} := \inf\big\{\, \eta(x, a) \;:\; x(t) \in V_{\boldsymbol{\varphi}}^n(a) \text{ and } \; x(t) \not\equiv 0 \,\big\}\,. \tag{7}$$

Definition 2. The equation (1) is said to be **strictly $\boldsymbol{\varphi}$-disconjugate** on an interval **I**, iff

$$a \in \mathbf{I} \implies \alpha(a) \notin \mathbf{I}. \tag{8}$$

Theorem 1. *Let $\boldsymbol{J} = \langle \alpha, \beta \rangle$ be a compact interval. Then the equation* (1) *is strictly $\boldsymbol{\varphi}$-disconjugate on every subinterval $\boldsymbol{J_1} \subseteq \boldsymbol{J}$, whose lenght is less than*

$$\delta = \min\left\{ 1,\, \frac{1}{n\,K} \right\}, \tag{9}$$

where

$$K := \max_{1 \leqslant i \leqslant n}\ \max_{t \in \boldsymbol{J}} \left\{ B_i \sum_{j=1}^{m} \big|\, a_{ij}(t)\, \big| \right\}\,. \tag{10}$$

Proof. We shall proof this theorem by contradiction.
We assume that the lenght of $\boldsymbol{J_1}$ is less than δ and the equation (1) is not strictly $\boldsymbol{\varphi}$-disconjugate on $\boldsymbol{J_1}$. Then there is a point $a \in \boldsymbol{J_1}$ and a solution $x(t) \in V_{\boldsymbol{\varphi}}^n(a)$, which has at least n zeros (including multiplicity) on an interval $\boldsymbol{J_2} = \langle a, \infty) \cap \boldsymbol{J_1}$. Thus by the Mean Value Theorem $x^{(k)}(t)$ has at least $(n-k)$ zeros on interval $\boldsymbol{J_2}$ $\big(k = 1, \ldots, n-1\big)$.

Let for all t from the interval $\boldsymbol{J_2}$ the inequality $t - \Delta_{ij}(t) < a$, $(i \in \{1, \ldots, n\}$, $j \in \{1, \ldots, m\})$ holds. Then using (5) and (6) we obtain for $k = 0, \ldots, n-1$

$$\Big|\, x^{(k)}\big(t - \Delta_{ij}(t)\big) \Big| \leqslant \big|\; x_a^k \;\big|\; B_k \leqslant B_k \max_{t\, \in \boldsymbol{J_2}} \Big| x^{(k)}(t) \Big|\;.$$

Otherwise, by the inequality $t - \Delta_{ij}(t) \geqslant a$ we have $t - \Delta_{ij}(t) \in \boldsymbol{J_2}$ and this implies

$$\Big|\, x^{(k)}\big(t - \Delta_{ij}(t)\big) \Big| \leqslant \max_{t\, \in \boldsymbol{J_2}} \Big| x^{(k)}(t) \Big|\;.$$

The assumption (5) yields $B_k \geqslant 1$ and from the last inequalities we get the inequality

$$\max_{t\in \boldsymbol{J_2}} \left| x^{(k)}\big(t-\Delta_{ij}(t)\big) \right| \leqslant B_k \max_{t\in \boldsymbol{J_2}} \left| x^{(k)}(t) \right| \\ \big(k=0,1,\dots,n-1\,;\ i=1,\dots,n\,;\ j=1,\dots,m\big)\,. \tag{11}$$

We denote

$$\mu_k := \max_{t\in \boldsymbol{J_2}} \left| x^{(k)}(t) \right|, \qquad k=0,1,\dots,n \qquad \big(x^{(0)}(t) := x(t) \ \ t\in \boldsymbol{J_2}\big)\,. \tag{12}$$

Since $x(t)$ is continuous function, from the existence at least n zeros on $\boldsymbol{J_2}$ we obtain by Mean Value Theorem

$$|\,x(t)\,| = |\,x(t)-x(\xi)\,| = |\,x'(\eta)(t-\xi)\,| \le \mu_1\ |\,t-\xi\,| \quad \forall\, t\in \boldsymbol{J_2},$$

where ξ is the zero of the solution $x(t)$ and η is any point on the nondegenerate interval with the end points t and ξ. Therefore

$$\mu_0 \le \mu_1\delta.$$

Likewise we obtain

$$\mu_k \le \mu_{k+1}\delta, \quad k=1,\dots,n-1.$$

If $\mu_k > 0$ then

$$\mu_k < \mu_{k+1}\delta\ .$$

Since $x(t) \not\equiv 0$ and the inequality $\mu_0 > 0$ holds, we get

$$0 < \mu_k < \delta^{n-k}\,\mu_n, \quad k=0,1,\dots,n-1. \tag{13}$$

On the other hand, from (1), (9), (11) and (13) we have

$$\mu_n \le \sum_{i=1}^{n}\sum_{j=1}^{m} \big|\,a_{ij}(t)\big)\,\big|\ B_i\,\mu_{n-i} \le K\sum_{i=1}^{n}\mu_{n-i} < K\left(\delta+\delta^2+\dots+\delta^n\right)\mu_n \le n\,K\,\delta\,\mu_n,$$

i.e.

$$1 < n\,K\,\delta\ ,$$

which is a contradiction with (9) and thus proof of the theorem is complete. □

Corollary 1. *If* $\boldsymbol{\varphi}(t) = \big(\varphi_0(t),\dots,\varphi_{n-1}(t)\big)$, $\varphi_k(t) \equiv 1$, $t\in(-\infty,t_0\rangle$, $k=0,1,\dots,n-1$, *then the notions of strictly* $\boldsymbol{\varphi}$*-disconjugate differential equation with delay and strictly disconjugate differential equation with delay coincide (see [5], [7]).*

Corollary 2. *If* $\Delta_{ij}(t) \equiv 0$, $t\in\langle t_0,T)$, $i=1,\dots,n$, $j=1,\dots,m$, *then the notions of strictly* $\boldsymbol{\varphi}$*-disconjugate differential equation with delay and disconjugate differential equation without delay (see [1]) coincide.*

Let us define the multipoint boundary value problem (**BVP**) for the equation (1): Let $\tau_0 \in \langle t_0, T)$,

$$\tau_1, \tau_2, \dots, \tau_p \in (\tau_0, T), \text{ where } \tau_0 < \tau_1 < \tau_2 < \dots < \tau_p;\ (p \le n), \tag{14}$$

$$r_1 + \dots + r_p = n\,, \quad r_1, \dots, r_p \in \mathbb{N} \tag{15}$$

and let

$$\beta_1^1, \dots, \beta_1^{r_1}, \dots, \beta_p^1, \dots, \beta_p^{r_p} \in \mathbb{R}. \tag{16}$$

The problem is to find a solution $x : \langle t_0, T) \to \mathbb{R}$ of the equation (1) which satisfies the conditions:

$$x^{(\nu_l - 1)}(\tau_l) = \beta_l^{\nu_l}; \quad \nu_l = 1, \dots, r_l; \quad l = 1, \dots, p. \tag{17}$$

THEOREM 2. *The equation* (1) *is strictly φ-disconjugate on an interval $\boldsymbol{I}$, iff each (**BVP**) has exactly one solution $x(t)$, such that $x(t) \in V_{\varphi}^n(\tau_0)$.*

Proof. Any solution $x(t) \in V_{\varphi}^n(\tau_0)$ can be written in the form

$$x(t) = \sum_{k=1}^{n} \alpha_k\, u_k(t, \tau_0),$$

where $u_k(t, \tau_0) \in V_{\varphi}^n(\tau_0)$; $k = 1, \dots, n$ such that

$$W\big(u_1(\tau_0,\tau_0), \dots, u_n(\tau_0,\tau_0)\big) = \begin{vmatrix} u_1(\tau_0,\tau_0) & \dots & u_n(\tau_0,\tau_0) \\ u_1'(\tau_0,\tau_0) & \dots & u_n'(\tau_0,\tau_0) \\ \dots & \dots & \dots \\ u_1^{(n-1)}(\tau_0,\tau_0) & \dots & u_n^{(n-1)}(\tau_0,\tau_0) \end{vmatrix} \ne 0\,,$$

(see REMARK 1).

We denote

$$\mathbf{A} = \begin{pmatrix} u_1(\tau_1,\tau_0) & \cdots & u_n(\tau_1,\tau_0) \\ & \dots\dots\dots & \\ u_1^{(r_1-1)}(\tau_1,\tau_0) & \cdots & u_n^{(r_1-1)}(\tau_1,\tau_0) \\ u_1(\tau_2,\tau_0) & \cdots & u_n(\tau_2,\tau_0) \\ & \dots\dots\dots & \\ u_1^{(r_p-1)}(\tau_p,\tau_0) & \cdots & u_n^{(r_p-1)}(\tau_p,\tau_0) \end{pmatrix}, \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \\ \vdots \\ \\ \alpha_n \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} \beta_1^1 \\ \vdots \\ \beta_1^{r_1} \\ \beta_2^1 \\ \vdots \\ \beta_p^{r_p} \end{pmatrix}.$$

Then we have to choose $\boldsymbol{\alpha}$ such that

$$\mathbf{A}\,\boldsymbol{\alpha} = \boldsymbol{\beta}\,. \tag{18}$$

This is possible for each $\boldsymbol{\beta}$ if and only if the corresponding homogenous system

$$\mathbf{A}\,\boldsymbol{\alpha} = \mathbf{0} \tag{19}$$

has only trivial solution. □

This occurs if and only if the differential equation (1) is strictly $\boldsymbol{\varphi}$-disconjugate on $\mathbf{I}$ (then the trivial solution is the only solution $x(t) \in V_{\boldsymbol{\varphi}}^n(\tau_0)$ which has n zeros on $\mathbf{I}$ (including multiplicity), see [5], [7]).

Definition 3. Let $\boldsymbol{\Psi}(t) = \big(\psi_0(t), \psi_1(t), \dots, \psi_{n-1}(t)\big)$ be an admissible vector function (continuous and bounded) defined on the E_{τ_0}. Then

$$\boldsymbol{H}\big(\boldsymbol{\varphi}, \tau_0, \boldsymbol{\Psi}\big) := \{ \; (\, \psi_0(t) + c_0 \, \varphi_0(t - \tau_0 + t_0), \psi_1(t) + c_1 \, \varphi_1(t - \tau_0 + t_0), \dots \\ \dots, \psi_{n-1}(t) + c_{n-1} \, \varphi_{n-1}(t - \tau_0 + t_0)) \; , \; c_0, c_1, \dots, c_{n-1} \in \mathbb{R} \; \} \, .$$

Let $x(t)$ be a solution of (1). Then we shall write

$$x(t) \in \boldsymbol{H}\big(\boldsymbol{\varphi}, \tau_0, \boldsymbol{\Psi}\big)$$

iff there are constants $\bar{c}_0, \bar{c}_1, \dots, \bar{c}_{n-1} \in \mathbb{R}$ such that $x(t)$ is a unique solution of **FIVP** for equation (1) which is determined by the initial vector function

$$\big(\, \psi_0(t) + \bar{c}_0 \, \varphi_0(t - \tau_0 + t_0), \psi_1(t) + \bar{c}_1 \, \varphi_1(t - \tau_0 + t_0), \dots, \psi_{n-1}(t) + \bar{c}_{n-1} \, \varphi_{n-1}(t - \tau_0 + t_0)\big),$$

$$t \in E_{\tau_0}$$

and constants

$$x^{(k)}(\tau_0) = x_{\tau_0}^{\,k} = \psi_k(\tau_0) + \bar{c}_k \, \varphi_k(t_0) \, , \quad k = 0, 1, \dots, n-1. \tag{20}$$

Theorem 3. *Differential equation* (1) *is strictly* $\boldsymbol{\varphi}$*-disconjugate on the interval* $\boldsymbol{I} = \langle t_0, T)$ *if and only if for each* $\tau_0 \in \boldsymbol{I}$ *that satisfies* (14) *and for each admissible vector function* $\boldsymbol{\Psi}(t)$ *defined on the initial set* E_{τ_0} *(continuous and bounded), every boundary value problem* (1), (17) *has exactly one solution* $x(t)$ *such that*

$$x(t) \in \boldsymbol{H}\big(\boldsymbol{\varphi}, \tau_0, \boldsymbol{\Psi}\big).$$

Proof. Denote by $x\,(t, \tau_0, \psi_0, \psi_1, \dots, \psi_{n-1})$ the solution of (1) determined by the initial vector function $\boldsymbol{\Psi}(t) = \big(\psi_0(t), \psi_1(t), \dots, \psi_{n-1}(t)\big)$. Now Theorem 3 follows from the uniqueness of the solution of **FIVP**, Theorem 2 and from the identity

$$\begin{aligned} & x\big(\,t, \tau_0, \psi_0(t) + c_0\varphi_0(t - \tau_0 + t_0), \psi_1(t) + c_1\varphi_1(t - \tau_0 + t_0), \dots \\ & \qquad\qquad \dots, \psi_{n-1}(t) + c_{n-1}\varphi_{n-1}(t - \tau_0 + t_0)\big) \\ = \; & x\big(\,t, \tau_0, \psi_0(t), \psi_1(t), \dots, \psi_{n-1}(t)\big) \\ & + x\big(t, \tau_0, c_0\varphi_0(t - \tau_0 + t_0), c_1\varphi_1(t - \tau_0 + t_0), \dots, c_{n-1}\varphi_{n-1}(t - \tau_0 + t_0)\big) \, . \end{aligned}$$

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