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ABSOLUTE BOREL SETS

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This talk gives a survey of results, many of them due to present and former students (Freiwald, Hansell, Michaels, Willard). It is concerned with spaces which are metrisable and absolutely Borel, but not necessarily separable. The problem of classifying them under Borel isomorphism or generalised homeomorphism has in principle been solved, except that it is still unknown whether every Borel isomorphism is necessarily itself a generalised homeomorphism. The significant properties of a space X, for this classification, are the weight of X and the property of being σ-locally of weight $< \mathfrak{k}$ for various cardinals $\mathfrak{k}$. The results for maps between such spaces are much more fragmentary. A simple proof can be given for Taimonov's theorem, that a closed map cannot lower the Borel class of a space, by using two results which characterize absolute G_α and F_α spaces X, respectively, in terms of βX (or other compactifications). Every absolute Borel set X of multiplicative class $\alpha + 1$ is an image of a standard space by a generalised homeomorphism of class $(0, \alpha)$, which maps a suitable base of X onto a σ-locally-finite collection of sets. Bimeasurable maps, in the separable case, have been dealt with by a theorem of Purvis; this theorem has been partially generalised to nonseparable spaces. The apparently more natural, but more intractable, problem of finding intrinsic topological characterizations of the various Borel classes has been solved for $F_{\sigma\delta}$ sets.

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