

## Toposym 4-B

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Franklin D. Tall

Some topological applications of a generalized Martin's axiom

In: Josef Novák (ed.): General topology and its relations to modern analysis and algebra IV, Proceedings of the fourth Prague topological symposium, 1976, Part B: Contributed Papers. Society of Czechoslovak Mathematicians and Physicist, Praha, 1977. pp. [453].

Persistent URL: <http://dml.cz/dmlcz/700623>

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Some topological applications of a generalized Martin's Axiom

F.D. Tall

Toronto

R. Laver and J. Baumgartner have independently established the consistency of generalizations of variants of Martin's Axiom. We deduce several topological applications of Baumgartner's version. Details will appear elsewhere.

Definition. A partial order  $P$  is countably closed if every countable descending sequence has a lower bound. A subset of  $P$  is linked if any two members of it are compatible.  $P$  is  $\aleph_1$ -linked if  $P$  is the union of  $\aleph_1$  linked subsets.  $P$  is well-met if any two compatible elements have an inf. Baumgartner's Axiom is the assertion that for each countably closed  $\aleph_1$ -linked well-met partial order  $P$  and each collection  $D$  of  $< 2^{\aleph_1}$  dense subsets of  $P$ , there is a filter meeting each element of  $D$ . Theorem (Baumgartner).  $\text{Con}(\text{ZF}) \rightarrow \text{Con}(\text{ZFC} + \text{BA} + \text{CH} + 2^{\aleph_1} > \aleph_2)$ . Theorem.  $\text{BA} + \text{CH} + 2^{\aleph_1} = \kappa^+ \rightarrow$  there is an L-space such that every uncountable subspace has weight  $\kappa$ . Theorem.  $\text{BA} + \text{CH} \rightarrow \aleph\text{N-N}$  is not the union of  $< 2^{\aleph_1}$  nowhere dense sets. Theorem.  $\text{BA} + \text{CH} \rightarrow$  if  $|X| < 2^{\aleph_1}$  and  $\pi(X) \leq \aleph_1$  and  $X$  has caliber  $\aleph_1$ , then  $X$  is separable. Theorem.  $\text{Con}(\text{ZF}) \rightarrow \text{Con}(\text{ZFC} + \text{BA} + \text{CH} \cdot 2^{\aleph_1} > \aleph_2 +$  every normal space of character  $\leq \aleph_1$  is  $\aleph_1$ -collectionwise Hausdorff, but there is a normal space of character  $\aleph_1$  which is not  $\aleph_2$ -collectionwise Hausdorff.)