Béla Szökefalvi-Nagy

On some properties of the function class H^{∞} and applications to Hilbert space operators

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Béla Szőkefalvi-Nagy (Szeged):

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Let H^{∞} denote the Banach algebra of bounded analytic functions in the unit disc. An element u of H^{∞} is called <u>inner</u> if $|u(e^{it})| = 1$ almost everywhere on the unit circle. For any family of elements $u_a \in H^{\infty}$ let $\bigwedge_a u_a$ denote the largest common inner divisor if not all u_a equal 0, and let it be 0 if all $u_a = 0$.

A useful "arithmetic property" of H^{∞} is established by the following "Main Lemma":

Let u_{ik} be an $n \times m$ matrix over H^{∞}, where n and m may be finite or ∞; and suppose $\|u_{ik}\|_{\infty} \leq M$. Let w be an inner function. Then there exists a sequence of complex numbers $x_1=1, x_2, x_3, \ldots$, with $|x_2|+|x_3|+\ldots$ as small as we wish, so that we have, for $i=1,\ldots,n$,

$$\sum_{k=1}^{m} u_{ik}\, x_k = h_i \cdot \bigwedge_{k=1}^{m} u_{ik}, \text{ with } h_i \in H^{\infty},\ h_i \wedge w = 1.$$

The proof uses the canonical factorization of functions in H^{∞}, a lemma of M. Sherman (the special case $n=1$, $m=2$ of the above statement), function theoretic reasonings (Vitali-Montel), and the Baire category theorem for the space ℓ^1.

The lemma is used to extend a result of E. Nordgren on the "quasi-equivalence" of finite matrices over H^{∞}, to their diagonalized form, to the case of semi-infinite or (at least partly) to the case of infinite matrices.

When applied to the "characteristic matrix function" Θ of a contraction operator T on a Hilbert space H, such that $T^{*n} \to 0$ as $n \to \infty$, and $\dim (I - TT^{*})H = m < \infty$, the results imply sort of similarity relations between T and its "Jordan model".

For detailed exposition, and references, see

Béla Sz.-Nagy: Diagonalization of matrices over H^{∞}, <u>Acta Sci. Math.</u>, **38**(1976), 223-238.