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A COMPACTNESS CRITERION FOR THE SPACE OF ALMOST PERIODIC FUNCTIONS

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In this note we give a compactness criterion with respect to the uniform topology for the space of almost periodic functions. By considering besides the uniform topology the compact open topology we are able to deduce the uniform criterion in a simple way. This criterion generalizes a criterion of Fink [4]. We state our results here without proofs. A paper containing full proofs will appear in "Mathematische Nachrichten".

Concerning some basic notions and results on almost periodic functions we refer to the book of BESICOVITCH [1]. The notion of almost periodicity of a function f from the reals to the real or complex numbers and most of the properties of such functions can easily be extended to the case that f takes values in an arbitrary BANACH space (see for instance FINK [4].

So, if R denotes the reals and Y is a BANACH space, let AP(R, Y) be the set of all almost periodic functions from R to Y. By C (R, Y) we denote the set of all continuous functions from R to Y. By the definition of almost periodicity we have $AP(R, Y) \subset C(R, Y)$. By the norm $\|g\| = \sup\{|g(t)| : t \in R\}$, where $|.|$ is the norm of the BANACH space Y, we have that AP(R, Y) is a normed space. Clearly the (metrizable) topology, induced by this norm on AP(R, Y) is the uniform topology τ_u.

For $f \in AP(R, Y)$ and $\tau \in R$ let $f_\tau : f_\tau(t) = f(t + \tau)$ for all $t \in R$. For each $\varepsilon > 0$ let be $T(f, \varepsilon) = \{\tau : \|f_\tau - f\| < \varepsilon\}$; moreover, if A is a family of almost periodic functions, then $T(A, \varepsilon) = \bigcap_{f \in A} T(f, \varepsilon)$.

The following notion is due to BOCHNER [2] (see also [4]).

The family $A \subset AP(R, Y)$ is said to be uniformly almost periodic (u.a.p.) iff for each $\varepsilon > 0$ the set $T(A, \varepsilon)$ is a relatively dense subset of real numbers.

By τ_{co} we denote the compact open topology. Concerning questions on τ_u and τ_{co}, especially on τ_{co}-compactness criteria we refer to POPPE [6] or KELLEY [5].

We can formulate the following theorem, including both a τ_{co}-criterion and a τ_u-criterion.

Theorem. Let $H \subset AP(R, Y)$; we consider the following conditions for H, where τ stands either for τ_{co} or for τ_u.

(1) H is relatively τ-compact in AP(R, Y)

(2) H is relatively sequentially τ-compact in AP(R, Y)

(3) (a) $H(t) = \{f(t) : t \in R\}$ is relatively compact in Y for each $t \in R$

(b) H is equicontinuous

(c) H is a u.a.p. family

I) In the case of compact open topology $\tau = \tau_{co}$ we then have:

(1) $\Longleftrightarrow$ (2)

(1) $\Longrightarrow$ (3), (a), (b)

(3) $\Longrightarrow$ (1)

II) In the case of uniform topology $\tau = \tau_u$ conditions (1), (2) and (3) are equivalent.

References

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