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Concerning the topological products of two Fréchet spaces


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Let \((X, \mathcal{U})\) be a Hausdorff space. Denote \(u^*\) the following operator: \(x \in u^*A\) if there are points \(x_k \in A\) such that each neighborhood of \(x\) contains \(x_k\) for all but a finite number of \(k\), i.e. 

if \(\lim \ x_k = x\). Let \(\{S_m\}\) be a twofold sequence, i.e. a sequence of sequences \(S_m\) of points of \(X\). If \(S'_m\) is a subsequence of \(S_m\), then we have twofold subsequence \(\{S'_m\}\) of \(\{S_m\}\). We define: \(\{S_m\}\) converges to \(x_o\) provided that \(x_o \in u^*S'_{m_i}\) for each subsequence \(\{S'_{m_i}\}\) of \(\{S_m\}\). Here \(S'_{m_i}\) denotes the set of all points of the sequence \(S_{m_i}\). A sequence \(\{x_k\}\) is a crosssequence in \(\{S_m\}\) provided that there is a subsequence \(\{m_k\}\) of \(\{m\}\) such that \(x_k \in S_{m_k}\).

Classify all points in a Hausdorff space into three (not necessarily disjoint) classes. We define the point \(x_o \in X\) to be a \(\mathcal{X}\) point provided that the following condition is fulfilled: if a twofold sequence \(\{P_m\}\) converges to \(x_o\), then there is a subsequence of \(\{P_m\}\) each crosssequence in which converges to \(x_o\). A point \(x_o\) is called a \(\mathcal{C}\) point if there is a twofold sequence \(\{R_m\}\) converging to \(x_o\) no crosssequence in which converges to \(x_o\). A point \(x_o\) is a \(\mathcal{G}\) point if there is a twofold sequence \(\{S_m\}\) converging to \(x_o\) in each subsequence of which there is a crosssequence converging to \(x_o\) and another one containing no subsequence converging to \(x_o\); moreover, if \(\lim S_m = x_o\) for each \(m\), then \(x_o\) is called a \(\mathcal{G}_1\) point and if \(\lim S_m = x_o\) and \(\lim x_m = x_o\) where \(x_m\) is one-to-one, then we have a \(\mathcal{G}_2\) point.

Let a twofold sequence \(\{S_m\}\) converge to \(x_o\) in \((X, \mathcal{U})\) and \(\{T_m\}\) converge to \(y_o\) in \((Y, \mathcal{V})\). The points \(x_o\) and \(y_o\) are said to be coupled if the following statement holds: If a crosssequence in \(\{S_m\}\)
converges to \( x_0 \), then the corresponding crosssequence in \( \{T_m\} \) does not converge to \( y_0 \) and vice versa: If a crosssequence in \( \{T_m\} \) converges to \( y_0 \), then the corresponding crosssequence in \( \{S_m\} \) does not converge to \( x_0 \).

**Theorem.** Let \((X,\mu)\) and \((Y,\nu)\) be Hausdorff Fréchet non isolated spaces and let \((X \times Y, w)\) be their topological product. Then \((X \times Y, w^*)\) is a Fréchet space iff there is no \( \rho \) point either in \( X \) or in \( Y \) and there are neither \( \sigma_1 \sigma_2 \) nor \( \sigma_2 \sigma_2 \) coupled points.

**References**


