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On $m$-adic spaces


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S. Mrówka [4] generalizing the notion of dyadicity, introduced the class of $m$-adic spaces. Denoting by $A_m$ the one-point compactification of a discrete space of cardinality $m$, a $T_2$-space $X$ is said to be $m$-adic if it is a continuous image of a suitable topological power of $A_m$. It is not difficult to prove that a space is dyadic iff it is $\aleph_0$-adic.

S. Mrówka also proposed the following generalization of $m$-adicity: let us denote by $W(\xi + 1)$ the order-topological space of the ordinal numbers $<\xi + 1$ for an ordinal $\xi$. A Hausdorff space which is a continuous image of some topological power of $W(\xi + 1)$ will be called a $\xi$-adic space. This class of spaces is wider than that of $m$-adic spaces; indeed $A_m$ is a continuous image of $W(\xi + 1)$, where $\xi$ is any ordinal of cardinality $m$.

S. Mrówka raised the following question as an open problem: is it true that an $m$-adic space with character $\leq n$ ($n \leq m$) is necessarily $n$-adic? Our aim is to give an affirmative answer to this question; indeed, the following more general theorem holds:

**Theorem 1.** The weight and the character of a $\xi$-adic space are equal.

The method of the proof is very similar to a method of N. A. Shanin (the "calibers" [5]).

**Definition.** Let $n$ denote an infinite cardinality. A topological space $X$ is said to have the property $B(n)$ if for any family $\{G_\alpha; \alpha \in A\}$, $|A| = n$, of non-empty open subsets of $X$ a set $B \subseteq A$, $|B| = n$, and a point $p \in X$ can be selected such that each neighbourhood of $p$ meets almost all sets $G_\beta$ in the sense that

$$|\{\beta \in B; V \cap G_\beta = \emptyset\}| < n$$

for each neighbourhood $V$ of $p$.

Our main tool for the investigation of $\xi$-adic spaces is the following theorem:

**Theorem 2.** An arbitrary product of spaces with property $B(n)$ has this property as well.
The continuous image of a space with property $B(n)$ also has this property and the spaces $W(\xi + 1)$ obviously have the property $B(n)$, hence we have

**Corollary.** *If the space $X$ is $\xi$-adic then $X$ has the property $B(n)$ for each infinite cardinality $n$.*

Using this Corollary and some other theorems of R. Engelking [4] and R. Marty [3] Theorem 1 can be proved.

Our Corollary implies also some related theorems for $\xi$-adic spaces. The following results are direct generalizations of two theorems of R. Engelking and A. Pelczynski [2].

**Theorem 3.** *If the Stone-Čech compactification of a Tychonoff space $T$ is $\xi$-adic for an ordinal $\xi$, then $T$ is pseudocompact.*

**Theorem 4.** *There is no infinite extremally disconnected $\xi$-adic Hausdorff space.*

To prove these two theorems it is enough to apply our Corollary to the case $n = \aleph_0$.

Using a different method, applying an argument due to Efimov [1] for a more general situation, we obtain

**Theorem 5.** *Let $X$ be a $\xi$-adic space. If $X$ has a dense set each point of which has a character $\leq n$ and $|\xi| \leq n$, then the weight of $X \leq n$."

**Corollary.** *If the Tychonoff space $X$ has a $\xi$-adic compactification $\alpha X$ for some ordinal $\xi$, then the weight of $\alpha X$ does not exceed the weight of $X$.*

**Problem.** Has each metrizable space $M$ an $m$-adic compactification? (By Theorem 5, if such an $m$ exists, then it can be chosen as the weight of the space $M$.)


References