

W. Barit

Contraction of some spaces of homeomorphisms

In: Josef Novák (ed.): General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the Third Prague Topological Symposium, 1971. Academia Publishing House of the Czechoslovak Academy of Sciences, Praha, 1972. pp. 63--64.

Persistent URL: <http://dml.cz/dmlcz/700760>

Terms of use:

© Institute of Mathematics AS CR, 1972

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* <http://project.dml.cz>

CONTRACTION OF SOME SPACES OF HOMEOMORPHISMS

W. BARIT

Amsterdam

Let $Q = \prod_{i>0} I_i$ and $s = \prod_{i>0} I_i^o$ where $I_i = [-1, 1]$ and $I_i^o = (-1, 1)$. Let l_2 denote separable Hilbert space. Following Anderson [1], we say a set K in X is a Z -set if K is closed and for each non-empty homotopically trivial open set U in X , $U \setminus K$ is non-empty and homotopically trivial. Some examples of Z -sets in l_2 are closed σ -compact subsets and closed sets whose projection in infinitely many directions is a point. Let $H(X)$ be the space of homeomorphisms of X onto X with the compact-open topology. Let $H_K(X) = \{h \in H(X) \mid h|_K = \text{id}\}$. The main result is the following:

Theorem 1. *Let $X = Q, s, \text{ or } l_2$, and let K be a compact Z -set in X . Then $H_K(X)$ is contractible.*

As background to this theorem, Wong [4] showed that any homeomorphism of X is isotopic to the identity. Renz [3] observed that this process is continuous and in fact contracts $H(X)$. In a later paper [5] Wong showed that any homeomorphism of X which is the identity on a compact Z -set K , is isotopic to the identity with each level of the isotopy being the identity on K . The proof of Theorem 1 requires a non-trivial modification of Wong's technique and the use of a canonical homeomorphism extension theorem due to Chapman [2]. We also obtain the following theorem.

Theorem 2. *Let $X = s \text{ or } l_2$, and let K be a Z -set in X . If h is a homeomorphism of X such that $h|_K = \text{id}$, then h is isotopic to the identity via $\{H_t\}_{t \in [0,1]}$ where for each t , $H_t|_K = \text{id}$.*

Theorem 2 shows that the compactness condition is not required for K , and thus answers a question posed in Wong's paper [5]. The methods used here do not show, however, that $H_K(s)$ is contractible for K a non-compact Z -set, and this question is still open. The requirement that K be a Z -set is necessary, and some examples are given.

References

- [1] *R. D. Anderson*: On topological infinite deficiency. *Michigan Math. J.* *14* (1967), 365—383.
- [2] *T. A. Chapman*: Canonical Extensions of Homeomorphisms. (Preprint.)
- [3] *P. Renz*: The contractibility of the homeomorphism group of some product spaces by Wong's method. *Math. Scand.* (to appear).
- [4] *R. Y. T. Wong*: On homeomorphism of certain infinite-dimensional spaces. *Trans. Amer. Math. Soc.* *28* (1967), 140—153.
- [5] *R. Y. T. Wong*: Stationary isotopes of infinite-dimensional spaces. (Preprint.)