

A. Bellen; A. Volčič

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## NON-CYCLIC TRANSFORMATIONS AND UNIFORM CONVERGENCE OF PICARD SEQUENCES

A. BELLEN and A. VOLČIČ

Trieste

**Definition.** A continuous transformation of a topological space  $S$  into itself is said to be *non-cyclic* iff  $f(x) \neq x$  implies  $f^2(x) \neq x$ .

In a recent paper, extending a theorem due to S. C. Chu and R. D. Moyer, the second author proved that, if  $f$  is a continuous transformation of a compact and connected space  $S$  into itself, whose topology is deduced from a total ordering, then all the Picard sequences converge iff  $f$  is non-cyclic (and this last property is proved to be equivalent to five other properties).

In this paper the authors characterize (under the same hypotheses on  $S$ ) the non-cyclic transformations for which the Picard sequences converge uniformly with respect to  $x \in S$ . Besides, some partial answers to the same problem in a more general setting are given.

Notations:  $F(f)$  indicates the set of all fixed points of  $f$ .  $F^*(f)$  indicates the set  $\bigcap_N f^n(S)$ .

Obviously we have  $F^*(f) \supset F(f)$ .

**Theorem.** *If  $S$  is totally ordered, connected and compact in the order topology, and  $f$  a non-cyclic transformation of  $S$  into itself, then the following properties are equivalent:*

- (a)  $F(f)$  is connected;
- (b)  $F^*(f) = F(f)$ ;
- (c) the convergence of the Picard sequences is uniform.

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