E., Jr. Makai

The space of bounded maps into a Banach space

In: Josef Novák (ed.): General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the Third Prague Topological Symposium, 1971. Academia Publishing House of the Czechoslovak Academy of Sciences, Praha, 1972. pp. 289.

Persistent URL: http://dml.cz/dmlcz/700777

THE SPACE OF BOUNDED MAPS INTO A BANACH SPACE

E. MAKAI, JR.

Budapest

Let D be a real B-space for which

(1) D is strictly convex,

(2) for every $v \in D^*$, $\|v\| = 1$, and $0 < \delta < 1$ there is a number γ such that the set

$$\{w, w \in D, \|w\| \leqq 1, v(w) = 1 - \delta\}$$

contains a u for which

$$\{w, w \in D, \|w - u\| \leqq \gamma, v(w) = 1 - \delta\} \subset \{w, w \in D, \|w\| \leqq 1, v(w) = 1 - \delta\},$$

and for a fixed v $\gamma/\delta \to \infty$ if $\delta \to 0$,

(3) D has no proper subspace isometrically isomorphic to D,

(4) D is not finite dimensional.

Let X_j, $j = 1, 2$ be realcompact spaces. $C^*(X_j, D)$ denotes the B-space of the bounded continuous functions from X_j to D. For any linear isometry ψ of $C^*(X_1, D)$ onto $C^*(X_2, D)$ there exist a homeomorphism $\varphi : X_2 \to X_1$ and a continuous map A from X_1 to the isometrical isomorphies of D to itself (these taken in the strong operator topology) such that $(\psi f)(x_2) = A(\varphi(x_2)) . f(\varphi(x_2))$.

Let X_j be compact, let D have property (2). A similar statement holds (with $A(x_1) \equiv$ identity) for the pairs $(i_j, C^*_w(X_j, D))$ where $C^*_w(X_j, D)$ denotes the B-space of bounded weakly continuous functions from X_j to D, $i_j : D \to C^*_w(X_j, D)$, $(i_j d)(x_j) = d$ for every $x_j \in X_j$.

For further development S-compact spaces should be considered, where S is the unit sphere of a B-space (of measurable cardinality), or the unit sphere of a (non-reflexive) B-space with the weak topology.

References

[1] *M. Jerison:* The space of bounded maps into a Banach space. Ann. of Math. *52* (1950), 309—327.

[2] *E. Makai, Jr.:* The space of bounded maps into a Banach space. Publ. Math. Debrecen (to appear).

MATHEMATICAL INSTITUTE OF THE HUNGARIAN ACADEMY OF SCIENCES, BUDAPEST