

# Toposym 3

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## THE SPACE OF BOUNDED MAPS INTO A BANACH SPACE

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Let  $D$  be a real  $B$ -space for which

- (1)  $D$  is strictly convex,  
 (2) for every  $v \in D^*$ ,  $\|v\| = 1$ , and  $0 < \delta < 1$  there is a number  $\gamma$  such that the set

$$\{w, w \in D, \|w\| \leq 1, v(w) = 1 - \delta\}$$

contains a  $u$  for which

$$\{w, w \in D, \|w - u\| \leq \gamma, v(w) = 1 - \delta\} \subset \{w, w \in D, \|w\| \leq 1, v(w) = 1 - \delta\},$$

and for a fixed  $v$   $\gamma/\delta \rightarrow \infty$  if  $\delta \rightarrow 0$ ,

- (3)  $D$  has no proper subspace isometrically isomorphic to  $D$ ,  
 (4)  $D$  is not finite dimensional.

Let  $X_j$ ,  $j = 1, 2$  be realcompact spaces.  $C^*(X_j, D)$  denotes the  $B$ -space of the bounded continuous functions from  $X_j$  to  $D$ . For any linear isometry  $\psi$  of  $C^*(X_1, D)$  onto  $C^*(X_2, D)$  there exist a homeomorphism  $\varphi : X_2 \rightarrow X_1$  and a continuous map  $A$  from  $X_1$  to the isometrical isomorphisms of  $D$  to itself (these taken in the strong operator topology) such that  $(\psi f)(x_2) = A(\varphi(x_2)) \cdot f(\varphi(x_2))$ .

Let  $X_j$  be compact, let  $D$  have property (2). A similar statement holds (with  $A(x_1) \equiv \text{identity}$ ) for the pairs  $(i_j, C_w^*(X_j, D))$  where  $C_w^*(X_j, D)$  denotes the  $B$ -space of bounded weakly continuous functions from  $X_j$  to  $D$ ,  $i_j : D \rightarrow C_w^*(X_j, D)$ ,  $(i_j d)(x_j) = d$  for every  $x_j \in X_j$ .

For further development  $S$ -compact spaces should be considered, where  $S$  is the unit sphere of a  $B$ -space (of measurable cardinality), or the unit sphere of a (non-reflexive)  $B$ -space with the weak topology.

### References

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