

Toposym 3

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In: Josef Novák (ed.): General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the Third Prague Topological Symposium, 1971. Academia Publishing House of the Czechoslovak Academy of Sciences, Praha, 1972. pp. 149--150.

Persistent URL: <http://dml.cz/dmlcz/700785>

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ON MONOTONE DECOMPOSITIONS OF SMOOTH CONTINUA

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The notion of smoothness of fans, dendroids, and hereditarily unicoherent continua has been discussed in [1], [4], and [6], respectively. We shall define a class of continua, called smooth, which contains the class of smooth hereditarily unicoherent continua, and we shall discuss some of the basic properties of such continua.

A *continuum* is a compact, connected, metric space. A continuum is said to be *hereditarily unicoherent at the point p* provided that the intersection of any two subcontinua, each of which contains p , is connected. Clearly a continuum M is hereditarily unicoherent at p if and only if given any point x in M there exists a unique subcontinuum which is irreducible between p and x . If the continuum M is hereditarily unicoherent at p , and q is a point of M , then pq will denote the unique subcontinuum which is irreducible between p and q .

A continuum M is said to be *smooth at the point p* if M is hereditarily unicoherent at p , and for each convergent sequence of points a_n of M the condition $\lim a_n = a$ implies that the sequence of continua pa_n is convergent and $\text{Lim } pa_n = pa$. The set of points at which a continuum M is smooth is called the *initial set* of M and is denoted by $I(M)$. If $I(M) \neq \emptyset$, then M is said to be *smooth*.

Theorem 1. *If M is a smooth continuum then (i) M is locally connected at each point of $I(M)$, (ii) M is a dendrite if and only if $I(M) = M$, (iii) M is unicoherent, and (iv) every indecomposable subcontinuum of M has void interior.*

Theorem 2. *If M is a smooth continuum then there exists a decomposition D of M (called the canonical decomposition) such that (i) D is upper semicontinuous, (ii) the elements of D are continua, (iii) the decomposition space of D is arcwise connected, and (iv) if E is a decomposition satisfying (i), (ii), and (iii) then D refines E . Moreover, the decomposition space of D is a smooth dendroid and each element of D has void interior.*

The decomposition of Theorem 2 is similar to the decomposition obtained for λ -dendroids in [2]; however, the canonical decomposition of a λ -dendroid may consist of a single element [3] while the canonical decomposition of a smooth continuum is never degenerate.

For a detailed discussion of these results including generalizations to compact Hausdorff continua, see [5].

References

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