

# Toposym 3

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## ON MONOTONE DECOMPOSITIONS OF SMOOTH CONTINUA

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The notion of smoothness of fans, dendroids, and hereditarily unicoherent continua has been discussed in [1], [4], and [6], respectively. We shall define a class of continua, called smooth, which contains the class of smooth hereditarily unicoherent continua, and we shall discuss some of the basic properties of such continua.

A *continuum* is a compact, connected, metric space. A continuum is said to be *hereditarily unicoherent at the point  $p$*  provided that the intersection of any two subcontinua, each of which contains  $p$ , is connected. Clearly a continuum  $M$  is hereditarily unicoherent at  $p$  if and only if given any point  $x$  in  $M$  there exists a unique subcontinuum which is irreducible between  $p$  and  $x$ . If the continuum  $M$  is hereditarily unicoherent at  $p$ , and  $q$  is a point of  $M$ , then  $pq$  will denote the unique subcontinuum which is irreducible between  $p$  and  $q$ .

A continuum  $M$  is said to be *smooth at the point  $p$*  if  $M$  is hereditarily unicoherent at  $p$ , and for each convergent sequence of points  $a_n$  of  $M$  the condition  $\lim a_n = a$  implies that the sequence of continua  $pa_n$  is convergent and  $\text{Lim } pa_n = pa$ . The set of points at which a continuum  $M$  is smooth is called the *initial set* of  $M$  and is denoted by  $I(M)$ . If  $I(M) \neq \emptyset$ , then  $M$  is said to be *smooth*.

**Theorem 1.** *If  $M$  is a smooth continuum then (i)  $M$  is locally connected at each point of  $I(M)$ , (ii)  $M$  is a dendrite if and only if  $I(M) = M$ , (iii)  $M$  is unicoherent, and (iv) every indecomposable subcontinuum of  $M$  has void interior.*

**Theorem 2.** *If  $M$  is a smooth continuum then there exists a decomposition  $D$  of  $M$  (called the canonical decomposition) such that (i)  $D$  is upper semicontinuous, (ii) the elements of  $D$  are continua, (iii) the decomposition space of  $D$  is arcwise connected, and (iv) if  $E$  is a decomposition satisfying (i), (ii), and (iii) then  $D$  refines  $E$ . Moreover, the decomposition space of  $D$  is a smooth dendroid and each element of  $D$  has void interior.*

The decomposition of Theorem 2 is similar to the decomposition obtained for  $\lambda$ -dendroids in [2]; however, the canonical decomposition of a  $\lambda$ -dendroid may consist of a single element [3] while the canonical decomposition of a smooth continuum is never degenerate.

For a detailed discussion of these results including generalizations to compact Hausdorff continua, see [5].

#### References

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