ON NUMERICAL AND NON-NUMERICAL ECART

D. KUREPA

Beograd

Historically, the notion of ecart evolved from distance obtained by measurements; in this respect the theorem of Pythagoras is of paramount importance. It is to be observed that for a very long time there were no analytical generalizations of this theorem. In this respect we have to mention the names of Laguerre (1853), Minkowskï (1896), Hilbert (about 1912) connected with some distances they introduced in the quoted years respectively.

Fréchet (1905) defined the most general numerical ecart and examined it in a series of important spaces (e.g. space of holomorphic functions, space of measurable functions, etc.). Every numerical ecart is a certain function having a domain of the form $E \times E$ and an antidomain contained in $R\ [0, \infty)$; E is any set. For non-numerical ecarts the domain remains of the same form but the antidomain varies considerably. One of the first examples (Kurepa, 1934) considered totally ordered ecarts of a type $(\omega_\alpha + 1)^*$, ω_α being a regular initial ordinal number; D_α-spaces or pseudo-metric spaces were introduced in such a way. In 1936 Kurepa considered the case in which the range of ecart was any structure, topological or non topological, used to topologize a given set E. The question was examined and studied by a series of authors (K. Menger, B. Price, G. K. Kalisch, A. Weil, J. Colmez, R. Doss, M. Fréchet, A. Appert, P. Papić, Zl. Mamuzić, M. Antonovskij, V. Boltjanskij, T. Sarímsakov, L. Dokas, J. Schröder, S. Ciampa, etc.). In particular, some kinds of ecarts were examined in connection with various kinds of spaces and structures, allowing that particular ecarts have various additional properties. E.g. the general ecart enables us to reconstruct any topological space; on the other hand, a general D_1-space is neither metric nor orderable. Uniform spaces are a particular case of spaces definable by non-numerical set-ecart, or by means of points of a semi-field. In particular, every completely regular T_2-space as well as any proximity space is definable by means of a distance over a semifield. According to an oral communication of R. E. DeMarr every T_2-space is definable by means of an ecart over a semi-group with unity.

In each case we essentially consider any structure M and any mapping f from E^2 into M, and associate with every point a of a given set E a certain family $R(a)$ of sets, called radii connected with a and serving to define the spheroids $E(a, r) = \{x; x \in E, f(a, x) \in r\}$ for any $r \in R(a)$; of course r is a subset of M.

Bibliography

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