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CHARACTERIZATION OF FOURIER-STIELTJES TRANSFORMS OF VECTOR-VALUED MEASURES

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Let G be a locally compact Abelian group, Γ its character group, $B(G)$ and $B(\Gamma)$ the system of Borel sets in G and Γ, respectively. Further, let X be a Banach space. The problem is to give conditions for a weakly continuous function $f : G \to X$ to be the Fourier-Stieltjes transform of some measure $\mu : B(\Gamma) \to X$, i.e.

$$f(x) = \int_\Gamma (x, \gamma)\,\mathrm{d}\mu(\gamma)\,, \quad x \in G\,. \tag{1}$$

Here (x, γ) denotes the value of a character $\gamma \in \Gamma$ in a point $x \in G$. We use the symbols M_a and M_d to denote the algebra of all absolutely continuous (with respect to Haar measure) and all discrete complex-valued measures on $B(G)$, respectively. For a measure $m \in M_a \cup M_d$, $\hat{m}$ denotes, as usual, the Fourier-Stieltjes transform of m, i.e. $\hat{m}(\gamma) = \int_G (x, \gamma)\,\mathrm{d}m(x)$, $\gamma \in \Gamma$.

Theorem. *There exists a measure $\mu : B(\Gamma) \to X$ such that (1) holds if and only if the set*

$$\left\{\int_G f(x)\,\mathrm{d}m(x) : \|\hat{m}\|_\infty \leqq 1,\ m \in M_a\right\} \tag{2}$$

or, equivalently, the set

$$\left\{\int_G f(x)\,\mathrm{d}m(x) : \|\hat{m}\|_\infty \leqq 1,\ m \in M_d\right\} \tag{3}$$

is relatively weakly compact in X.

The conditions of the theorem are closely related to conditions given by W. F. Eberlein [1] and others for a scalar-valued function to be a Fourier-Stieltjes transform.

If X is weakly sequentially complete, in particular reflexive, the relative weak compactness of the set (2) or (3) follows from its boundedness.

We consider the transformation I_f of $\hat{M}_a$ or $\hat{M}_d$ to X defined by $I_f(\hat{m}) = \int_G f(x)\,\mathrm{d}m(x)$ for $m \in M_a$ or $m \in M_d$, respectively. This transformation can be extended by continuity to the uniform closure $\hat{M}_a^-$ or $\hat{M}_d^-$ of $\hat{M}_a$ or $\hat{M}_d$. Considered on real functions in $\hat{M}_a^-$ or $\hat{M}_d^-$, respectively, it is a vector Daniell integral. The condition of relative weak compactness of the set (2) implies the existence of a measure $\mu : B(\Gamma) \to X$ such that $I_f(\hat{m}) = \int_\Gamma \hat{m}(\gamma)\,d\mu(\gamma)$ (see [2] and [3]). It is easy to prove that (1) holds. Detailed proofs will be given in [4].

References

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