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In: (ed.): General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the symposium held in Prague in September 1961. Academia Publishing House of the Czechoslovak Academy of Sciences, Prague, 1962. pp. [270]--271.

Persistent URL: http://dml.cz/dmlcz/700920

COLLARED SETS

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The results presented here were obtained jointly with MORTON BROWN; the important Corollary 3 was obtained by him alone before our collaboration began.

A subset $A \subset X$ is *collared in* X if there exists a homeomorphism h from $A \times [0, 1)$ onto an open $U \supset A$ such that $h(a, 0) = a$ for all $a \in A$. Moreover, A is *locally collared in* X if each $a \in A$ has a neighborhood in A which is collared in X.

Theorem 1. *A locally collared subset of a metric space is collared.*

Corollary 1. *The boundary of a manifold with boundary is collared.*

Now call $A \subset X$ *bi-collared in* X if $[0, 1)$ is replaced by $(-1, 1)$ in the above definition of collared, and similarly for *locally bi-collared*. The "equator" of a Möbius band shows that a locally bi-collared set need not be bi-collared (although see Theorem 3). However, we have

Corollary 2. *A locally bi-collared compact* $(n - 1)$*-manifold in* E^n *is bi-collared.*

Combining this result with the "Generalized Schönfliess Theorem" of M. BROWN and M. MORSE, one obtains

Corollary 3. (M. Brown.) *A locally bi-collared* $(n - 1)$*-sphere in* E^n *can be sent onto the unit sphere by an autohomeomorphism of* E^n.

Now call $A \subset X$ *multicollared in* X if there exists an $f : \tilde{A} \twoheadrightarrow A$ (the double arrow means onto) such that

(a) f is continuous, closed, and (compact, 0-dimensional)-to-one,

(b) there exists a homeomorphism h from M'_f (the "decapitated" mapping cylinder of f) onto an open $U \supset A$ such that, considering $A \subset M'_f$, we have $h(a) = a$ for all $a \in A$.

We denote the set of all such f by $M(A, X)$.

Theorem 2. *If* $A \subset X$ *metric, and* $f_i : \tilde{A}_i \twoheadrightarrow A$ $(i = 1, 2)$ *are in* $M(A, X)$, *then there exists a homeomorphism* $h : A_1 \twoheadrightarrow A_2$ *such that* $f_1 = f_2 \circ h$.

Call $A \subset X$ *double-collared* in X if there exists an f in $M(A, X)$ which is a (possibly trivial) double covering of A. (In the trivial case, this reduces to bi-collared. The equator of a Möbius band is double-collared without being bi-collared; locally, however, these two concepts coincide.)

Theorem 3. *A locally multicollared subset of a metric space is multicollared. Similarly for double-collared.*

Suppose now that A is a multicollared subset of E^n, and that $f : \tilde{A} \twoheadrightarrow A$ is in $M(A, E^n)$. Note that a closed interval or triod are both multicollared in the plane, with $\tilde{A}$ a circle. In general, every component of $\tilde{A}$ must be a manifold (compact if A is) if $n \leqq 3$. S. Jaworowski has shown that, for A compact, $\tilde{A}$ and $E^n - A$ have the same number of components.

Now consider a finite, connected $(n - 1)$-subcomplex K of E^n, all of whose simplices are faces of $(n - 1)$-simplices. In general, K need not be multicollared in E^n, although it is if $n = 2$, or $n = 3$ and the star of each vertex is connected. Nevertheless, one can always canonically define an $(n - 1)$-complex $\tilde{K}$, and a simplicial, finite-to-one $f : \tilde{K} \twoheadrightarrow K$, such that *if* K is multicollared in E^n, then f is in $M(K, E^n)$.

Here is a problem:

Is the union of all multicollared subsets of a finite-dimensional metric space again multicollared?