

Toposym 1

Marshall Harvey Stone Commemoration of Eduard Čech

In: (ed.): General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the symposium held in Prague in September 1961. Academia Publishing House of the Czechoslovak Academy of Sciences, Prague, 1962. pp. 26--28.

Persistent URL: <http://dml.cz/dmlcz/700925>

Terms of use:

© Institute of Mathematics AS CR, 1962

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* <http://project.dml.cz>

M. H. STONE

Mr. President, ladies and gentlemen,

I feel greatly honoured to be asked to speak today on this occasion. The only reason why I should have been asked is that mathematical colleagues interested in work done independently by Prof. E. ČECH and myself have been good enough to join our names to designate a theorem to which we both made contributions.

The paper in which Professor Čech discussed this very important theorem was published in USA under the title "On bicomact spaces", in 1937. This work has a close relation, at least historically, to the fundamental memoir of Alexandrov and Urysohn under the title "Mémoire sur les espaces topologiques compacts" which was published in 1929. The link between this work and the work of Prof. Čech was an equally well known paper of the Russian mathematician Tichonov "Über die topologische Erweiterung von Räumen", published in 1930. In the memoir of Alexandrov and Urysohn the general properties of topological spaces, and in particular of compact topological spaces, were studied in a very deep and detailed fashion. Many new problems were proposed there for the first time. The paper of Tichonov gave a resolution to one of these important new problems: Under what circumstances does a topological space possess an embedding, homeomorphic embedding, into a compact space, and in particular onto a dense subset of a compact space? The solution given by Tichonov to this problem actually contained information which Tichonov either did not observe or did not choose to emphasize. It was just this point which was the foundation of Prof. Čech's memoir "On bicomact spaces". He saw that by using the technique of Tichonov one could construct a maximal compact envelope for those spaces which are called completely regular. In his paper Prof. Čech studied many properties of such a space, he characterized the compact envelope, and he established its uniqueness. Speaking for myself, I may say that I found particularly interesting his observations on the question of when a space is determined by its compact envelope. This question in certain situations has a very simple answer. Equally interesting was his use of the compact envelope to define for topological spaces in general a notion of completeness and to relate this definition to the standard concept of completeness for metric spaces.

Perhaps I may be forgiven if I use this opportunity to say a few words about the relation of this work of Prof. Čech to the work which I was doing at the same time.

We worked quite independently. It was, I believe, at the *World Topological Congress* held in Moscow in 1935 that I first met Prof. Čech. At the Congress we both presented ideas which had a bearing on the problem of the compact envelope. My own concern was of very different nature from Prof. Čech's. For various reasons I had glimpsed a possibility of studying some of the problems raised by Alexandrov and Urysohn in the fundamental memoir which I have cited, by considering them as special cases of the general problem of enlarging a topological space. The essential notion which lies at the root of such a study is one to which we now refer by the name of filter. Filters had been introduced in the early thirties by Henri Cartan, and the methods which I developed for the study of the problem of compactification implicitly involved the notion of filter. However, since my attention to this problem had grown out of an interest in some algebraic structures which occur in topology, I used algebraic language in order to provide the machinery which could equivalently have been set up by the use of filters. It seems to me correct to say that by this type of discussion it was possible to attain a general understanding of the problem of adjoining new points to topological spaces so as to extend or enlarge these spaces in topologically interesting ways. It was also possible to solve some of the problems formulated in the memoir of Alexandrov and Urysohn. When I spoke at the Congress in Moscow in 1935, I had not yet tried to specialize this kind of consideration to particular spaces. It was only following that Congress that I reviewed the general structure of my work and saw what it involved for certain special kinds of space. From this review there emerged a demonstration of the existence, uniqueness and general characteristic properties of the compact envelope which Prof. Čech discovered at approximately the same time. A little later — and this was before the publication of Prof. Čech's paper — I realized that this specialization could be accomplished by the method of Tichonov. In later years I published such a demonstration, which is essentially only a modification of that given by Prof. Čech.

I have been at some pains to discuss these matters here because I think it is only in the light of some such comment that one can pass to a consideration of the consequences which have followed from this important contribution of Prof. Čech.

The explicit study of the compact envelope for completely regular spaces has occupied a good many mathematicians since that time, and a good deal has been added to the already detailed comments that were made by Prof. Čech in his fundamental paper. The use of the compact envelope for various topological purposes has been exploited to some extent, but I believe that its potentialities have not yet been exhausted. There remain many problems which can still be attacked by means of it: some which have to do with the dimensionality, some which have to do with the different kinds of compactification which one meets in problems of analysis, and so on. As for the more general procedures introduced by my papers it may be noted that they have not yet been exploited very vigorously. However, only recently some of the more complex techniques which I developed have found applications at the hands of Prof. Oxtoby. One reason why attention has been to some extent turned away from

these matters is that we have had to concentrate our efforts on an attempt to understand the algebraic structure of the family of continuous functions on a completely regular space. This aspect of the topology of completely regular spaces, which is implicit in the work of Prof. Čech and explicit in my own work, has inspired many interesting contributions to mathematics. In particular it had a great deal to do with the modern study of normed rings and algebras, especially those defined over the real and the complex fields. It is for this reason, perhaps, that mathematicians with applications to analysis in mind have tended to concentrate their own topological studies upon what happens in the case of normed rings, especially those which appear in various mathematical situations. It may be noted that the general topological features which are the object of this sort of investigation still offer a rather mysterious but important field of study. I am sure that if Prof. Čech had lived he would himself have added to our knowledge of this particular domain. However, it was quite sufficient for a man who had written so many other important papers on topology that he should have made a single such basic contribution to such an interesting and important field as this.

In conclusion, I may say how deeply grateful I am for the privilege of expressing in this meeting organized as a memorial to Prof. Čech the sincere homage of all American mathematicians. — Thank you, Mr. President.