Jan Pelant On some combinatorial problems in uniform spaces

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ON SOME COMBINATORIAL PROBLEMS IN UNIFORM SPACES

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0. We have solved some problem in the theory of uniform spaces using a combinatorial representation of them. The following easy observation is very important for our purpose:

Observation: Let (X, \mathcal{U}) be a uniform space. Let \mathcal{P}, \mathcal{Q} be \mathcal{U} -covers of X such that $\mathcal{P} \stackrel{*}{\rightarrow} \mathcal{Q}$. Then $\bigcup_{X \in \mathcal{Q}} \{P \in \mathcal{P} \mid X \in \mathcal{P}\} \supset \{P \in \mathcal{P} \mid P \supset Q\} \supset$

 $\bigcup_{x \in Q} \{P \in \mathcal{F} \mid \text{st } (x, \mathcal{Q}) \subset P\} \text{ for each } Q \in Q \ . \\ \text{Using Observation we have constructed uniform spaces as complicated as possible, namely we exhibited a class <math>\mathcal{K}$ of simply described uniform spaces (see [P]) that projectively generate the category UNIF. So these spaces resemble to \mathcal{L}_{∞} 's and really, each member of \mathcal{K} is uniformly homeomorphic to the positive part of the unit sphere of some \mathcal{L}_{∞} (\mathcal{L}_{∞} is endowed with the metric uniformity).

Results:

1. Point-character of a uniform space Definition: (X, \mathcal{U}) is a uniform space. A point-character $pc(X, \mathcal{U})$ is the least cardinal m such that there is a base \mathcal{B} of \mathcal{U} such that card $\{P \in \mathcal{F} \mid x \in P\}$ is less than m for each xe I and each Pe B.

Theorem: pc $\mathcal{L}_{\infty}(m) > m$ for each infinite cardinal m. Theorem implies that $\mathcal{L}_{\infty}(\omega_0)$ has no point-finite base. Under Generalized Continuum Hypothesis [GCH], Theorem is the best possible result as pc $\mathcal{L}_{\infty}(m) \leq 2^m$ in general.

2. Cardinal reflections

 (X,\mathcal{U}) is a uniform space, ∞ is an infinite cardinal. We define $p_{\mathcal{C}} \mathcal{U} = \{ \mathcal{O} \in \mathcal{U} \mid \text{card } \mathcal{P} < \infty \}$ If [GCH] holds or if (X,\mathcal{U}) has a point-finite base, then $p_{\mathcal{O}} \mathcal{U}$ is a uniformity for each ∞ . But applying Baumgartner's theorems on almost-disjoint sets (see [B]) we receive that it is consistent with ZFC to suppose that $p_{\mathcal{O}} \mathcal{U}$ is not a uniformity for any $\infty \geq \omega_2$ ($p_{\mathcal{O}} \mathcal{U}$ is always a uniformity if $\infty = \omega_0, \omega_1$)

3. Modification preserving completeness We are looking for a modification (i.e. a reflection preserving underlying sets) which preserves completeness. We know one : identity: UNIF \rightarrow UNIF but no other (and maybe, there is no other). The following theorem indicates the complexity of the problem (the problem is due to Z. Frolík). Theorem: Let r: UNIF \rightarrow UNIF be a modification. If there is a cardinal m such that pc(rK)<m for K e X (X is mentioned sub 0) then r does not preserve completeness. Example: The distal modification does not preserve completeness.

References:

- [B] Baumgartner J.E.: Almost-disjoint sets, the dense-set problem and the partition calculus, to appear in Annals of Math. Log.
- [P] Pelant J.: Cardinal reflections and point-character of uniformities, Seminar Uniform Spaces 1973-1974 directed by Z. Frolík, MÚ ČSAV.

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