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AN EXAMPLE OF J.W. ROBERTS OF A CONVEX COMPACT SUBSET IN A  
LINEAR METRIC SPACE WITH NO EXTREME POINTS

by

P. MANKIEWICZ

Very recently J.W. Roberts has constructed a convex compact subset  $K$  of a linear metric space  $(X, d)$  (obviously, non-locally convex) with no extreme points. This answers a well known problem of an existence of such a set (cf. for example the book of R.R. Phelps "Lectures on Choquet theory"). The construction of the author can be summarized in the following way:

Let  $E$  be the linear space of all real-valued step functions defined on the unit interval of the form  $f = \sum a_i \chi_{[\alpha_i, \beta_i]}$  where  $\alpha_i, \beta_i$  are binary rational numbers. In the space  $E$  consider the set

$$C = \{f \in E: f \geq 0, \int f dt \leq 1\}.$$

Using some delicate finite dimensional arguments, one can prove (the proof is relatively complicated) that there exists a linear metric  $d$  on  $E$  such that if  $X$  and  $\tilde{C}$  are the completions of  $E$  and  $C$  (respectively) in the metric  $d$  then  $\tilde{C}$  is a convex compact cone in  $X$  with only one extreme point (namely - the origin).

To obtain the desired example it suffices to define

$$K = \tilde{C} - \tilde{C}.$$