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ON GROTHENDIECK SPACES OF TYPE $C(K)$

By

Jürgen FLACHSMEYER

In his fundamental work on weakly compact operators Grothendieck presented the following theorem (see *Canad. J. Math.* 5 (1953)):

For a Banach space E the following properties are equivalent:

- (1) Every continuous linear map $u: E \rightarrow Y$ from E into some separable Banach space is weakly compact, i.e. u transmits the unit ball into some relative weakly compact set.
- (2) Weak- $*$ -convergence and weak convergence for sequences coincide in the dual E' .

Banach spaces with these properties (1) and (2) are called Grothendieck spaces (see for example J. Diestel: Grothendieck spaces and vector measures. In: Vector and operator valued measures and applications. Ed. by D.H. Tucker, H.B. Maynard, Acad. Press. Inc. 1973, 97-108.)

The following problem (problem 3 in Diestel's paper is) unsolved: Characterize those compact Hausdorff spaces K for which the Banach space $C(K)$ of all continuous real-valued functions on K is a Grothendieck space.

What is known about this problem?

Let be K a compact Hausdorff space. We will write $K \in G$ iff $C(K)$ is Grothendieck.

Grothendieck (1953): (i) K Stonian (=extremally disconnected) \Rightarrow

Ando (1961): (ii) K \mathcal{C} -extremally disconnected $\Rightarrow K \in G$.

Semadeni (1964): by another approach received (ii).

Seever (1968): (iii) K an F -space $\Rightarrow K \in G$.

H. Schaefer (1971) also proved (ii). Of course, (iii) \Rightarrow (ii) \Rightarrow (i).

By the Riesz representation theorem the dual $C'(K)$ can be identified with the space $M(K)$ of bounded signed Radon measures on K .

Using this approach $K \in G$ gets equivalent to the following.

For every sequence (μ_n) of bounded Radon measures holds:

$$\mu_n(f) \rightarrow 0 \quad \forall f \in C(K) \Rightarrow \mu_n(g) \rightarrow 0 \quad \forall \text{ bounded Borel functions } g.$$

Thus, a necessary condition for $K \in G$ is that K must be sequentially discrete.

The lecture now explains the following result:

For every infinite compact F -space K the Alexandrov-double $K \circledast K$ is never an F -space.

For every $K \in G$ the Alexandrov-double $K \circledast K$ belongs to G . Thus a good deal of non- F -spaces are in G .

(Remark: The extension of the class G in a suitable way to non-compact spaces was treated in a thesis (Greifswald 1976) by Nguyen Doan Tien).