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In: Zdeněk Frolík (ed.): Abstracta. 5th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1977. pp. 21–22.

Persistent URL: http://dml.cz/dmlcz/701080

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The theory of analytic spaces or sets is extended to a wider class of spaces containing also some non-Lindelöf spaces, particularly all complete metric spaces and their products with compact spaces. In the hyperanalytic spaces defined below the technique of upper semi-continuous compact-valued mappings, and also the technique used in non-separable metric spaces can be used. Definition 2 therefore unables to study the descriptive theory in non-separable spaces like an extension of the separable theory which was studied separately till now.

Definition 1. Say that a family $\{X_a\}_{a\in A}$ is σ-d.d. (σ-discretely descomposable) in the uniform space X if there are sets X_{an}, $a\in A$, and $n\in\omega$, such that $X_a = \bigcup_{n\in\omega} X_{an}$, and the families $\{X_{an}\}_{a\in A}$ are discrete.

Definition 2. Say that the uniform space H is hyperanalytic if there are a complete metric space M, and an upper semi-continuous compact-valued mapping f from M onto H, such that images of σ-d.d. families in M are σ-d.d. in H.

The hyperanalytic spaces have a lot of very special properties, for example they are paracompact and absolutely Souslin in some sence. We describe the strongest properties in Theorems 1 and 2.

Theorem 1. Let $\{X_a\}$ be a point-finite completely hyperanalytic-additive family of sets (subspaces) of the uniform space X (e.g. every subunion of $\langle X_a\rangle$ is hyperanalytic in X). Then, $\langle X_a\rangle$ is σ-d.d. in X.

Theorem 2. Let H be hyperanalytic, and S Souslin subsets of the uniform space X. If $H\cap S = \emptyset$, then there is a hyper-Baire set B

in X, such that $H \subset B \subset X-S$.

Remark. The ideas of the described theory follows Hansell's ideas for complete metric spaces, and the technique of upper semi-continuous compact-valued mappings introduced by the first author.

The properties of hyperanalytic spaces, proofs of Theorems 1 and 2, and further theorems will be published elsewhere.