

R. Huff

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In: Zdeněk Frolík (ed.): Abstracta. 5th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1977. pp. 29–30.

Persistent URL: <http://dml.cz/dmlcz/701083>

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FIFTH WINTER SCHOOL (1977)

Asplund Spaces and the RNP

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A real Banach space X is an Asplund space provided for every open convex set $U \subset X$, every continuous convex functional $\phi : U \rightarrow \mathbb{R}$ is (Frechet-) differentiable on a dense G_δ subset of U [3]. The main result discussed is the following.

THEOREM (Asplund, Namioka, Phelps, Stegall). The following two statements are equivalent:

- (1) X is an Asplund space.
- (2) X^* has the Radon-Nikodym property (RNP).

A proof was given by breaking the result into two parts, the first of which is

THEOREM ([3], [4]). Statement (2) above is equivalent to

- (3) For every closed bounded set $A \subset X^*$, the set

$$\{x \in X : \tilde{x} \text{ strongly exposes } A\}$$

is a dense G_δ subset of X .

The second part, of course, is to show that (3) and (1) are equivalent; a proof can be found in [3], but here we gave a new proof via the following two easy lemmas.

DUALITY LEMMA. Let A be any closed bounded subset of $X^* \times \mathbb{R}$, and let
 $\varphi(x) = \sup\{f(x) + \alpha : (f, \alpha) \in A\}$ $(x \in X)$. Then φ is a convex functional on
 X satisfying a Lipschitz condition. Moreover, φ is differentiable at x
(with gradient g) iff the map on $X^* \times \mathbb{R}$ sending (f, α) into $f(x) + \alpha$ strongly
exposes A (at the point $(g, \varphi(x) - g(x))$).

REDUCTION LEMMA. Let U be any open convex subset of X and $\varphi : U \rightarrow \mathbb{R}$
any continuous convex functional. For each $n = 1, 2, \dots$, let

$$A_n = \{(f, \alpha) \in X^* \times \mathbb{R} : f(x) + \alpha \leq \varphi(x), \text{ all } x \in U, \|f\| \leq n, |\alpha| \leq n\},$$

and let $\varphi_n(x) = \sup\{f(x) + \alpha : (f, \alpha) \in A_n\}$. For every x in U there exist
 $\delta > 0$ and N such that

$$\|y - x\| < \delta \Rightarrow y \in U \quad \text{and} \quad \varphi(y) = \varphi_n(y) \quad \text{for all } n \geq N.$$

References

1. E. Asplund, Frechet differentiability of convex functions, Acta Math. 121 (1968), 31-47
2. R. Huff, preprint.
3. I. Namioka and R.R. Phelps, Banach spaces which are Asplund spaces, Duke Math. J. 42 (1975).
4. C. Stegall, in preparation.