R. Huff

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For considering extensions of the Bishop-Phelps theorem [1], Lindenstrauss [5] studied the following two possible properties for a real Banach space X.

PROPERTY A. *For every Banach space* Y *the set*

$$P(X,Y) = \{T \in L(X,Y) : \|T\| = \|Tx\| \text{ for some } x \text{ in } X \text{ with } \|x\| = 1\}$$

is norm dense in the space $L(X,Y)$ *of all bounded linear operators on* X *to* Y.

PROPERTY B. *For every Banach space* Y *the set* $P(Y,X)$ *is norm dense in* $L(Y,X)$.

For a recent study, see [4]. Here we proved

THEOREM [3]. *If* X *fails to have the Radon-Nikodým property, then there exist equivalent norms* $\|\cdot\|$ *and* $|||\cdot|||$ *on* X *such that the identity operator is not in the closure of* $P((X,\|\cdot\|),(X,|||\cdot|||))$. *In particular,* $(X,\|\cdot\|)$ *does not have Property A and* $(X,|||\cdot|||)$ *does not have Property B.*

The proof is obtained by modifying a proof in [2] where it is shown that if X has the RNP then it satisfies an apprently much stronger property than Property A.

An open question is: *Is Property A an isomorphic property?* (Equivalently: *Is Property A equivalent to the RNP?*)

References

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